

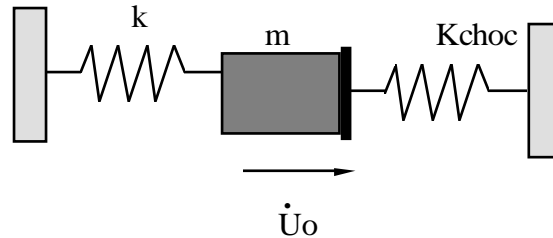
SDND101 - To release of a system masses spring with shock

Summarized

This problem corresponds to a transient analysis by modal recombination of a nonlinear discrete system to a degree of freedom. Non-linearity consists of a contact with shock on a rigid level. The mass is launched with a non-zero initial velocity against the obstacle. Initial clearance between the material point and the obstacle is null. This problem makes it possible to test the postprocessing of the forces of impact: velocity impact, period of shock...

1 Problem of reference

1.1 Geometry



1.2 Properties of the materials

the system makes up of a mass m and a spring of stiffness k . The thrust of shock has a stiffness equalizes with K_{choc} .

Mass	$m = 100 \text{ kg}$
Stiffness	$k = 10^4 \text{ N/m}$
normal Stiffness of shock	$K_{choc} = 10^6 \text{ N/m}$

1.3 Initial conditions

the system is initially in position at rest ($U_0 = 0$) and has an initial velocity $\dot{U}_0 > 0$. One will choose for the application an initial velocity $\dot{U}_0 = 1 \text{ m/s}$.

2 Reference solution

2.1 Method of calculating used for the reference solution

During the phase of impact, the system is solution of the differential equation:

$$m \cdot \ddot{u} + k \cdot u + K_c \langle u \rangle^+ = 0 \text{ with } u_0 = 0 \text{ and } \dot{u}_0 = \dot{U}_0 .$$

$\langle x \rangle^+$ indicate the positive value of x .

The analytical solution of this problem is:

$$u = \frac{\dot{U}_0}{\omega_c} \sin(\omega_c t) \text{ where } \omega_c = \sqrt{\frac{k + K_c}{m}} .$$

The velocity is cancelled for $t_{\dot{u}=0} = \frac{\pi}{2\omega_c}$.

The shock force is then maximum and is worth $F_{\max} = K_c u(t_{\dot{u}=0}) = K_c \frac{\dot{U}_0}{\omega_c}$.

By construction, the period of the shock is worth $T_{choc} = 2t_{\dot{u}=0}$.

The system returns to the position $u=0$ with the velocity $-\dot{U}_0$.

In the field $u < 0$ the system has as an equation $m \cdot \ddot{u} + k \cdot u = 0$ with for initial conditions $u_1 = 0$ and $\dot{u}_1 = -\dot{U}_0$.

Its solution is $u = -\frac{\dot{U}_0}{\omega_0} \sin(\omega_0 t')$ where $\omega_0 = \sqrt{\frac{k}{m}}$.

The velocity is cancelled for: $t'_{\dot{u}=0} = \frac{\pi}{2\omega_0}$.

By construction, the time of coasting flight is worth: $T_{vol} = 2t'_{\dot{u}=0}$.

The system is thus periodic with alternatively a phase of time of shock of period T_{choc} when the system describes an arch of sine in the field of $u > 0$ and a phase of coasting flight of period T_{vol} when the system describes an arch of sine in the field Des. $u < 0$

the impulse with each impact is worth: $I = \int_0^{T_{choc}} K_c u(t) dt = 2 K_c \frac{\dot{U}_0}{\omega_c^2} = \frac{2m\dot{U}_0}{1 + \frac{k}{K_c}}$.

2.2 Results of reference

the results taken for reference are the values of times of maximum force, the value of maximum force, the period of the time of shock, the value of the impulse and impact speed as well as the elementary number of impact for the first two oscillations of the system.

2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical references

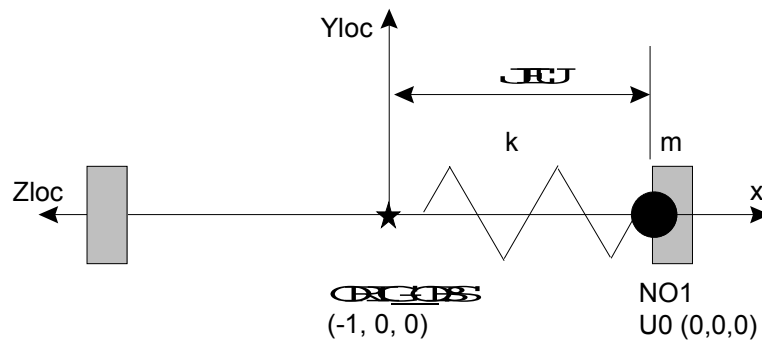
- G.JACQUART: Postprocessing of computations of heart and interns REFERENCE MARK under seismic request - HP-61/95/074/A.

3 Modelization A

3.1 Characteristic of the modelization

the spring-mass system are modelled by element of type a POI1 with the node *NO1*. It is fixed to move according to the axis *x*. The node *NO1* is positioned in $O=(0,0,0)$.

An obstacle of the type *PLAN_Z* (two parallel planes separated by a clearance) is used to simulate the possible shocks of the spring-mass system against a rigid plane. One chooses to take the axis *Oy* for norm with the plane of shock, that is to say *NORM_OBST*: (0. , 1. , 0.). Not to be constrained by the rebound of the oscillator on the symmetric level, one very pushes back this one far (cf [Figure. 3.1-a]). One thus chooses to locate the origin of the obstacle in *ORIG_OBS*: (- 1. 0. 0.).



Appear 3.1-a: Modelled geometry

It stays to define the parameter *JEU* which gives the half-spacing between the planes in touch. One wishes a real clearance here no one, from where $JEU : 1$. If one wishes a real clearance of *j*, it is necessary, in the case of figure presented, to impose $JEU : 1 + j$.

Temporal integration is carried out with the algorithm of Eulerian and time step of $5 \cdot 10^{-4} s$. All the computation steps are filed. It is considered that reduced damping ξ_i for all the calculated modes is null.

3.2 Characteristics of the mesh

The mesh consists of a node and a mesh of the type POI1.

3.3 Quantities tested and results

For the first two shocks, one compared to analytical values computed values of the time when the impact occurs, of the maximum force of shock, the time of shock, the impulse and the impact speed. One also tests the value of the absolute extremum of the force of impact.

First shock:

Time (s)	Reference
INST	1,5630E-02
F_MAX	9,9500E+03
T_CHOC	3,1260E-02
IMPULSE	1,9805E+02
V_IMPACT	- 1.

Second shock:

Time (s)	Reference
INST	3,6100E-01
F_MAX	9,9500E+03
T_CHOC	3,1260E-02
IMPULSE	1,9805E+02
V_IMPACT	- 1,0000E+00

Time (s)	Reference
F_MAX_ABS	9,95E+03

4 Summary of the results

One notes, on all the quantities, a very good agreement with the produced analytical solution. The quantities the least best represented are the period of shock and the time of shock (with better than 1% however). This problem is not related on the accuracy of computation but to the only fact that time step of integration of $5 \cdot 10^{-4} s$ was selected what over periods as short as $0,03 s$ already produced a temporal inaccuracy of 1,66%. To supplement this synthesis, one could carry out a test of convergence by decreasing the computation step.