

SDND106 – Shoe rubbing with coefficients of static friction and dynamics

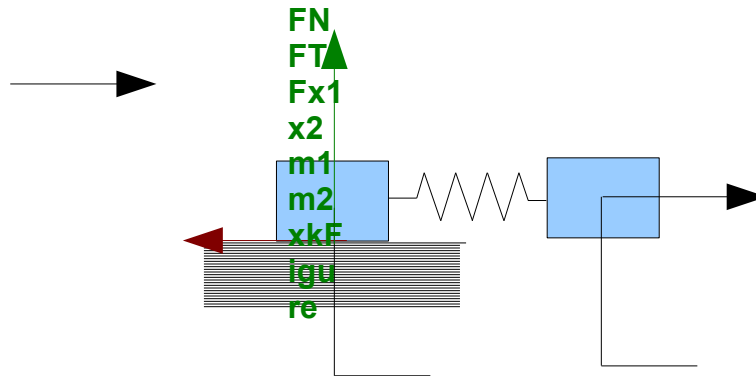
Summarized:

The purpose of this benchmark is validating the functionality of nonlinearity of friction by penalization, with the use of two coefficients of kinetic friction, a static (for the phases of dependency) and a dynamics (for the phases of sliding).

The studied system contains two degrees of freedom. Its evolution contains a transition dependency – sliding and a sliding transition – dependency.
The reference solution is analytical.

1 Problem of reference

1.1 Geometry



1.1-a : Diagram of the benchmark.

1.2 Properties of the material

the two masses are identical and are worth $m_1 = m_2 = m = 5 \text{ kg}$. The stiffness placed between these masses is of $k = 10^4 \text{ N/m}$.

1.3 Boundary conditions and loadings

the masses m_1 and m_2 move according to \vec{x} only.

The modulus of the normal force F_N is constant equal to $F_N = 10^4 \text{ N}$.

The mass m_1 is affected of a condition of contact – friction. Coefficients of kinetic friction 5 are:

- in the static case: $\mu_S = 0,3$,
- in the dynamic case: $\mu_D = 0,2$.

In this modelization, the system passes by three phases:

- A phase of dependency, during which the force exerted on the mass m_2 is constant: $F = 3 \cdot 10^3 \text{ N}$.
- A phase of sliding, beginner with t_1 , when the tangential force cannot compensate for the tensile force exerted any more by spring on m_1 . At the end of an arbitrary time ($t_2 = 0,2 \text{ s}$), the imposed force is put at 0 and the kinetics of the system tends then towards 0.
- One second phase of dependency, beginner with t_3 , when the velocity of the mass m_1 is cancelled.

1.4 Initial conditions

At time $t = 0$, the two masses are at rest (displacement and a null velocity).

2 Reference solution

2.1 Method of calculating

the pulsation of the system is noted ω_0 and is such as $\omega_0^2 = \frac{k}{m}$.

Phase of dependancy

At time $t=0$, the mass m_1 checks

$$x_1(t)=0$$

$$\dot{x}_1(t)=0$$

and the mass m_2 checks

$$x_2(t)=0$$

$$\dot{x}_2(t)=0.$$

The basic principle of the dynamics applied to the mass m_2 makes it possible to write the following equation

$$\ddot{x}_2 + \omega_0^2 x_2 = \frac{F}{m},$$

and on the mass m_1 the following equation

$$F_T = -k x_2$$

the general solution of x_2 is form:

$$x_2 = \tilde{A} \cos(\omega_0 t) + \tilde{B} \sin(\omega_0 t) + \frac{F}{(m\omega_0^2)}$$

where \tilde{A} and \tilde{B} are constants.

By taking account of the initial conditions, the displacement of the mass m_2 is thus written

$$x_2 = \frac{F}{k} [1 - \cos(\omega_0 t)].$$

This statement is valid until

$$\|F_T\| = \mu F_N$$

In other words, until time t_1 checking the following statement

$$F [1 - \cos(\omega_0 t_1)] = \mu_s F_N$$

the phase of sliding thus begins with t_1 definite with

$$t_1 = \frac{1}{\omega_0} \arccos\left(1 - \mu_s \frac{F_N}{F}\right)$$

Phase of sliding

At time $t=t_1$, the mass m_1 checks

$$x_1(t_1)=0$$

$$\dot{x}_1(t_1)=0$$

the basic principle of the dynamics applied to the mass m_1 makes it possible to write the equation

$$\ddot{x}_1 + \omega_0^2 x_1 = \frac{-\mu_D F_N}{m} + \omega_0^2 x_2,$$

and on the mass m_2 the equation

$$\ddot{x}_2 + \omega_0^2 x_2 = \frac{F}{m} + \omega_0^2 x_1.$$

By making the change of variables

$$\begin{cases} X = x_1 + x_2 \\ Y = x_1 - x_2 \\ \Omega_0 = \sqrt{2} \omega_0 \end{cases}$$

the preceding system is written

$$\begin{aligned} \ddot{X} &= \frac{F - \mu_D F_N}{m} \\ \ddot{Y} + \Omega_0^2 Y &= -\frac{F + \mu_D F_N}{m} \end{aligned}$$

For $t \geq t_1$, the general solution of X is form

$$X = \frac{(F - \mu_D F_N)}{2m} (t - t_1)^2 + \tilde{C} (t - t_1) + \tilde{D}.$$

and that of Y is form

$$Y = \tilde{E} \cos(\Omega_0(t - t_1)) + \tilde{F} \sin(\Omega_0(t - t_1)) - \frac{F + \mu_D F_N}{2k}.$$

By taking account of the initial conditions, the statements of the constants \tilde{C} , \tilde{D} , \tilde{E} and \tilde{F} are worth:

$$\begin{aligned} \tilde{C} &= \dot{x}_2(t_1) \\ \tilde{D} &= x_2(t_1) \\ \tilde{E} &= \frac{F + \mu_D F_N}{2k} - x_2(t_1) \\ \tilde{F} &= -\frac{\dot{x}_2(t_1)}{\Omega_0} \end{aligned}$$

Displacements x_1 and x_2 are deduced from the preceding statements.

Phase of sliding with a loading $F=0$ from t_2 (arbitrary but higher than t_1) until time t_3 (urgent of return to the phase dependency)

the basic principle of the dynamics presented previously is always valid. For the mass m_1 , the checked equation is the same one:

$$\ddot{x}_1 + \omega_0^2 x_1 = \frac{-(\mu_D F_N)}{m} + \omega_0^2 x_2,$$

and for the mass m_2

$$\ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2 x_1$$

With the same change of variable as previously, the system is written

$$\ddot{X} = -\frac{\mu_D F_N}{m}$$

$$\ddot{Y} + \Omega_0^2 Y = -\frac{\mu_D F_N}{m}$$

For $t > t_2$, the general solution of X is form

$$X = -\frac{\mu_D F_N}{2m} (t - t_2)^2 + \tilde{G} (t - t_2) + \tilde{H}$$

the general solution of Y is form

$$Y = \tilde{I} \cos(\Omega_0(t - t_2)) + \tilde{J} \sin(\Omega_0(t - t_2)) - \frac{\mu_D F_N}{2k}$$

By taking account of the initial conditions, the statements of the constants \tilde{G} , \tilde{H} , \tilde{I} and \tilde{J} are worth:

$$\tilde{G} = \dot{x}_1(t_2) + \dot{x}_2(t_2)$$

$$\tilde{H} = x_1(t_2) + x_2(t_2)$$

$$\tilde{J} = \frac{\mu_D F_N}{2k} + x_1(t_2) - x_2(t_2)$$

$$\tilde{I} = \frac{\dot{x}_1(t_2) - \dot{x}_2(t_2)}{\Omega_0}$$

Displacements x_1 and x_2 are deduced from the preceding statements.

2.2 Quantities and results of reference

the quantities tested are the kinematics of the masses m_1 and m_2 with various times in the various phases of the modelization. Are also tested following times of transition:

- transition of the phase of dependency to the phase of sliding (urgent t_1);
- transition of the phase of sliding to the phase of dependency (time t_3).

2.3 Uncertainties on the solution

exact analytical Solution.

2.4 Bibliographical references

[1] E. BOYERE: Modelization of the shocks and friction in transient analysis by modal recombination. Documentation of reference of R5.06.03 Code_Aster. September 2009.

3 Modelization A

3.1 Characteristic of the modelization

In this modelization, the force exerted on the mass m_2 is cancelled in $t_2=0,2s$.

3.2 Characteristics of the mesh

The mesh contains 2 meshes discrete to a node (for each mass) and 1 discrete mesh for spring.

3.3 Quantities tested and Urgent

Standard	results (s)	Quantity	Reference	Aster	Difference (%)
Analytical	0.02	x_1	0.	2.0621E-05	
Analytical	0.02	x_2	0.11221	0.11221	0
Analytical	0.15	x_1	1.35332	1.35329	-0.002
Analytical	0.15	x_2	1.80751	1.80751	0
Analytical	0.34	\dot{x}_1	0.	-3.3802E-5	
Analytical	0.34	x_2	3.96813	3.97012	0.05

Standard	Quantity (S)	Reference	Aster	% Analytical
difference	t_1	0.03512	0.03520	0.23
Analytical	t_3	0.31492	0.31500	0.03

3.4 Remarks

the difference in % is not given for the cases where the value of reference is worth 0, but it is observed that the absolute difference is about 10^{-5} .

4 Summary of the results

This benchmark shows that the transitions dependency – sliding and sliding – dependency are well collected. It validates in addition the capacity of operator `DYNA_VIBRA` to integrate problems in large displacements in translation.