SDNV104 - Dynamic response of a rigid shoe rubbing subjected to a pressure and a recoil force

Summarized

One considers a mass in contact rubbing with a rigid plane. It is retained by a spring and one imposes a side pressure to him. Friction is modelled by the model of Coulomb. The computation is a direct dynamic computation.

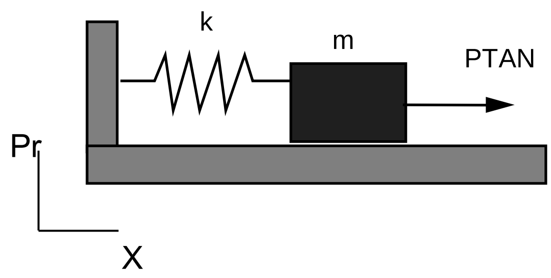
The reference solution is analytical.

The modelizations suggested use `DYNA_NON_LINE` with an elastic constitutive law in 2D, for two solvers. The contact is managed by various methods available in `AFFE_CHAR_MECA`.

1 Problem of reference

1.1 Geometry

the system considered consists of a shoe: square of 1m on 1m, posed on a support. It is subjected to its weight, by the strength of recall of a come out from stiffness K and with a side pressure. The contact is a rubbing contact.



1.2 of the model Masses

there:	$7.10^3 kg$
Stiffness of spring:	$24.10^3 N/m$
Coefficient of Coulomb:	$0,3$
Gravity:	$70\,000\,Pa$
Side pressure:	$200\,000\,Pa$
Young modulus of the shoe:	$2,1 . 10^{11} Pa$
Young modulus of the solid mass:	$1,0 . 10^{11} Pa$
Poisson's ratio:	0

1.3 Boundary conditions, initial conditions and loadings

the mass rests on the rigid level with the dimension $x=0$.

The loadings of weight and side pressure are applied with a slope which reaches its maximum into 0,07 second.

The support is embedded in x and in y.

2 Reference solution

the reference solution is analytical.

If the shoe is considered as sufficiently rigid one can compare it to a point mass, there then exists an analytical solution with this problem of rubbing mass-spring. The assumption is made that the application of the loading is done immediately (not slope).

As $\frac{F_t}{F_n} > \mu$ there is never phase of dependancy but only of the sliding. The frictional force is thus worth $f = \mu F_n$

One can write the equation of motion as follows:

$$m\ddot{x} + kx = F_t \pm f$$

One notes: $\omega = \sqrt{\frac{k}{m}}$

Stage 1: The frictional force is opposed to the motion which is carried out initially according to x the positive ones

$$m\ddot{x} + kx = F - f$$

with a X-coordinate and an initial velocity null. There is then the following solution after taking into account of these initial conditions:

$$x(t) = \frac{F - f}{k}(1 - \cos(\omega t))$$

this result is valid as long as the velocity remains positive is $\dot{x} \geq 0$, i.e. until $\omega . t = \pi$.

The first extremum of the curve $x(t)$ is $x_1 = 2.\frac{F - f}{k}$.

Stage 2: The frictional force changes sign to be opposed to the motion which is done now according to x negative

$$m\ddot{x} + kx = F + f$$

the initial X-coordinate is worth x_1 , and the initial velocity is null. One has then, by posing the new X-coordinate of times like origin with π / ω :

$x(t) = \frac{F + f}{k} + \frac{F - 3f}{k}\cos(\omega t)$, until $\omega . t = \pi$.

The second extremum of the curve $x(t)$ is $x_2 = \frac{4f}{k}$.

Stage $2n - 1$ and $2n$:

One separates motion according to the sign the velocity. For an odd stage motion is done according to x the positive ones. For an even stage, motion is done according to x the negative ones.

One shows by recurrence result according to:

$$x_{2p-1}=x\left((2p-1)T\right)=\frac{2\left(F-(2p-1)f\right)}{k}$$
$$x_{2p}=x\left(2pT\right)=\frac{4pf}{k}$$

with $T=\frac{\pi}{\omega}$ and $\omega=\sqrt{\frac{k}{m}}$.

The stop of motion occurs when x_n is understood enters $\frac{F_t-f}{k}$ and $\frac{F_t+f}{k}$.

2.1 Results of reference

modelization proposed Ci below correspond to the analytical solution until the stop of the shoe. By preoccupations with a time-saving of computation, one tests only the two first extremum.

Time (s)	Displacement in x (m)
1,697	14,917
3,393	3,500

2.2 Uncertainty on the solution

the analytical solution gives result exact under the assumption of infinitely rigid bodies. It is also supposed that the loading is applied directly (it does not depend on time).

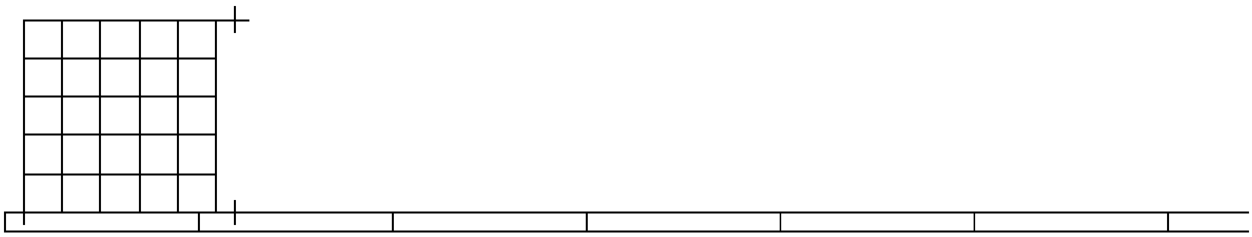
3 Modelization A

3.1 Characteristic of the modelization

the problem is D_PLAN. The shoe and the support are modelled by surfaces with a grid in QUAD4. An element 2D_DIS_T represents spring, its component non-zero is in the direction x .

One uses operator DYNA_NON_LINE to carry out dynamic computation. The forces of contact are taken into account by AFFE_CHAR_MECA / CONTACT, with the method LAGRANGIAN.

3.2 Characteristics of the mesh



Many nodes:	42	number of meshes and types:	26 QUAD4 26 SEG2

3.3 Values tested for the method LAGRANGIAN, MULT_FRONT

t	Reference	Aster	% difference
1,697	14,91	14,84	-0,5%
3,393	3,50	3,62	3,6%

3.4 Values tested for the method LAGRANGIAN, LDLT

t	Reference	Aster	% difference
1,697	14,91	14,83	-0.5%
3,393	3,50	3,62	3,6%

4 Modelization B

4.1 Characteristic of the modelization

the problem are D_PLAN. The shoe and the support are modelled by surfaces with a grid in QUAD4. An element 2D_DIS_T represents spring, its component non-zero is in the direction x .

One uses operator DYNA_NON_LINE to carry out dynamic computation. The forces of contact are taken into account by AFFE_CHAR_MECA / CONTACT, with the method PENALIZATION.

4.2 Characteristics of the mesh

Many nodes:	42	number of meshes and types:	26 QUAD4 26 SEG2

4.3 Values tested for the method PENALIZATION, MULT_FRONT

t	Reference	*Aster*	% difference
1,697	14,91	14,84	-0.5%
3,393	3,50	3,62	3,6%

4.4 Values tested for the method PENALIZATION, LDLT

t	Reference	*Aster*	% difference
1,697	14,91	14,83	-0,5%
3,393	3,50	3,62	3,6%

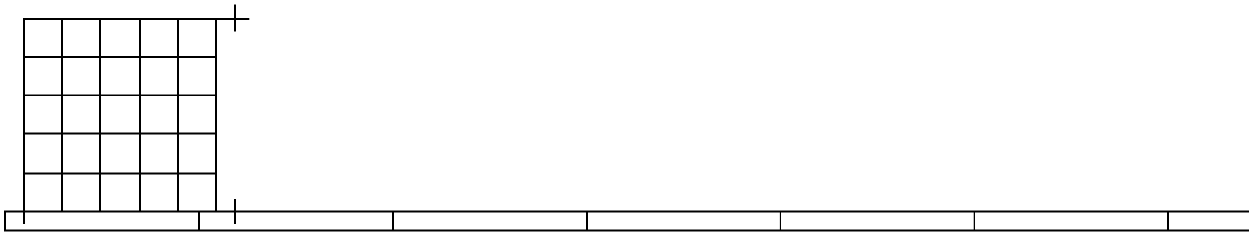
5 Modelization C

5.1 Characteristic of the modelization

the problem are D_PLAN. The shoe and the support are modelled by surfaces with a grid in QUAD4. An element 2D_DIS_T represents spring, its component non-zero is in the direction x .

One uses operator DYNA_NON_LINE to carry out dynamic computation. The forces of contact are taken into account by AFFE_CHAR_MECA / CONTACT, with the continuous method .

5.2 Characteristics of the mesh



Many nodes:	42	number of meshes and types:	26 QUAD4 26 SEG2

5.3 Values tested for the continuous method , MULT_FRONT

t	Reference	*Aster*	% difference
1,697	14,91	14,84	-0,5%
3,393	3,50	3,62	3,6%

5.4 Values tests for the continuous method , LDLT

t	Reference	*Aster*	% difference
1,697	14,91	14,83	-0,5%
3,393	3,50	3,62	3,6%

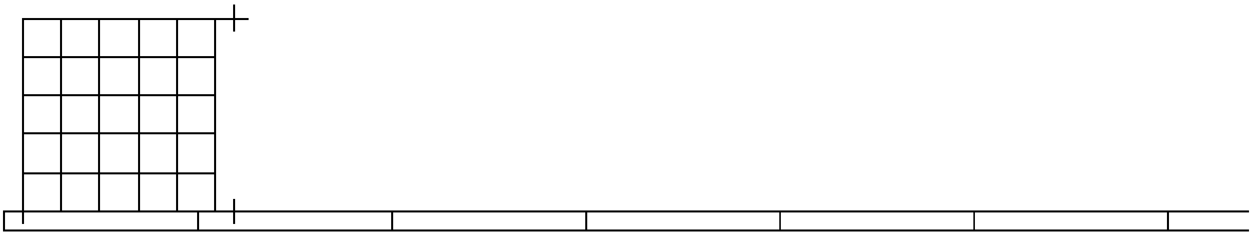
6 Modelization D

6.1 Characteristic of the modelization

the problem are D_PLAN. The shoe and the support are modelled by surfaces with a grid in QUAD8. An element 2D_DIS_T represents spring, its component non-zero is in the direction x .

One uses operator DYNA_NON_LINE to carry out dynamic computation. The forces of contact are taken into account by AFFE_CHAR_MECA / CONTACT, with the method LAGRANGIAN.

6.2 Characteristics of the mesh



Many nodes:	110	number of meshes and types:	26 QUADS 8 24 SEG3 2 SEG2

6.3 Values tested for the method LAGRANGIAN, MULT_FRONT

t	Reference	*Aster*	% difference
1,697	14,91	14,83	-0. , 5%
3,393	3,50	3,62	3,6%

6.4 Values tested for the method LAGRANGIAN, LDLT

t	Reference	*Aster*	% difference
1,697	14,91	14,83	-0. , 5%
3,393	3,50	3,62	3,6%

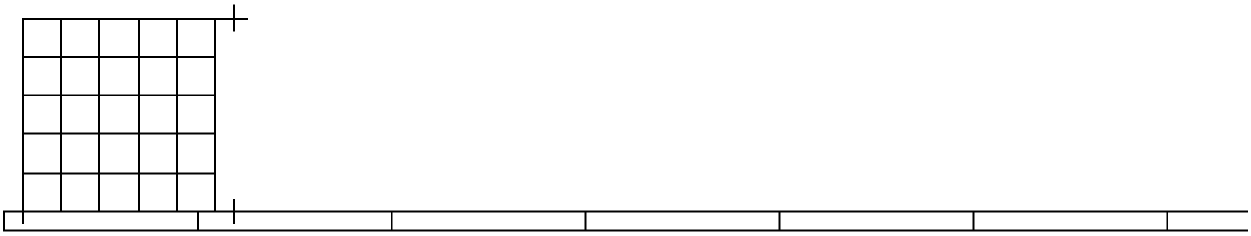
7 Modelization E

7.1 Characteristic of the modelization

the problem are D_PLAN. The shoe and the support are modelled by surfaces with a grid in QUAD8. An element 2D_DIS_T represents spring, its component non-zero is in direction X.

One uses operator DYNA_NON_LINE to carry out dynamic computation. The forces of contact are taken into account by AFFE_CHAR_MECA / CONTACT, with the method PENALIZATION.

7.2 Characteristics of the mesh



Many nodes: 110 number of meshes and types: 26 QUADS 8
24 SEG3
2 SEG2

7.3 Values tested for the method **PENALIZATION**, **MULT_FRONT**

t	**Reference**	***Aster***	**% difference**
1,697	14,91	14,83	-0.50%
3,393	3,50	3,62	3,6%

7.4 Values tested for the method **PENALIZATION**, **LDLT**

t	**Reference**	***Aster***	**% difference**
1,697	14,91	14,83	-0.50%
3,393	3,50	3,62	3,6%

8 Summary of the results

the results got on the group of this case test are satisfactory, as well into linear as into quadratic. The values obtained are with less than 1% from/to each other; and less than 4% of the reference solution.

It is noted that the value of reference of the second point is lower than the two others, which increases the percentage of error artificially.

The choice of the coefficients of the penalized method is delicate. But it is noted that once the coefficients chosen, result is stable with respect to the choice of the finite elements and of the solver.