SDNV105 - Swinging of a block on an array

Summarized:

This benchmark is used to validate the capacity of DYNA_NON_LINE and the methods of contact in *Code_Aster* available to dealing with problems of nonregular dynamics in the presence of collisions with implicit resolution. The problem consists in analyzing the dynamic response of a heavy homogeneous rectangular block posed on an array, subjected to various loadings (to release, harmonic excitation): the swinging led to successive collisions with the array.

One evaluates various ways treat the contact, like various choices of temporal integration:

1. processing of the contact by Lagrangian method and temporal integration by implicit scheme Newmark-HHT in displacement (modelization A, in 2D);
2. processing of the contact by method by penalized shock absorbers and temporal integration by implicit scheme Newmark-HHT in displacement (modelization B, in 2D);
3. processing of the contact by continuous method and temporal integration by implicit scheme Newmark-HHT in displacement (modelization C, in 2D);
4. processing of the contact by Lagrangian method and temporal integration by θ - diagram formulated in displacement (modelization D, 2D.

The got results are in relatively good agreement with the results of reference. These results of reference are of three natures:

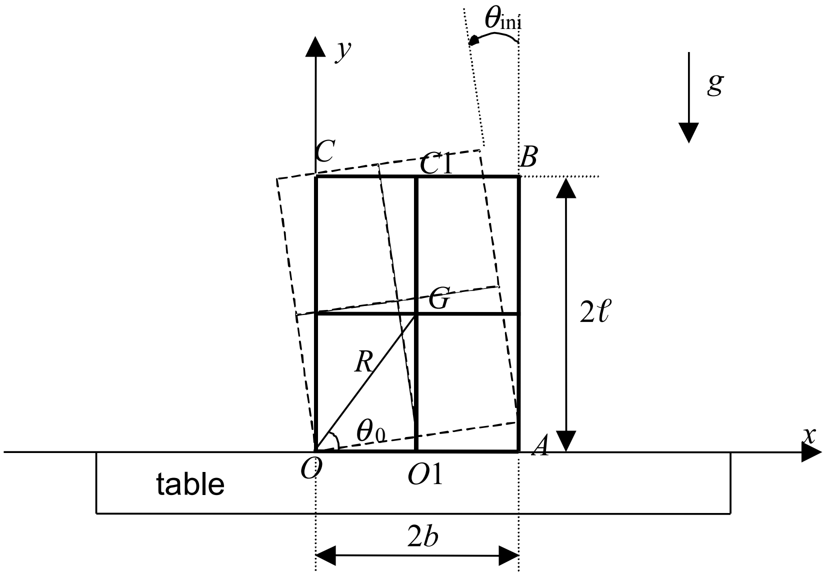
1. benches analytically with the assumption of rigid body without rebound, while simulation *Code_Aster* is made with elastic bodies, which induces a little distant results;
2. benches analytically starting from the coefficient of restitution of average shock raised on simulation *Code_Aster*, which makes it possible to ensure the accuracy of several dynamic variables: velocities, times of collision, energies kinetic, reactions, percussions;
3. obtained numerically with software LMGC90 of laboratory LMGC (University of Montpellier), with one θ - diagram of velocity.

However, it will be noted that this dynamic problem is very sensitive and thus that the accuracy can be degraded quickly during the transient; moreover, the choice of the algorithm (method of contact, temporal integration) has a great influence. Also the tolerances on the values tested grow during the transient. One advises the use of θ - diagram which is at the same time more robust and cheaper.

1 Problem of reference

1.1 Geometry

One considers a heavy homogeneous parallelepipedic block resting initially on a rigid array (null clearance).



Appear 1.1-a: Front view of the block on the array.

Geometrical data:
Face of the block: $2b \times 2l$: $0{,}360\ m \times 0{,}800\ m$. Thickness: $e=1\ m$. One notes $R=\sqrt{b^2+l^2}$ and θ_0 the angle $(\vec{OQ}, \vec{OG})$. The array has one thickness vertical of $0{,}08\ m$.

1.2 Material properties

Characteristic	block	counts
Young's modulus	$6{,}0\,10^{11}\ Pa$	$1{,}0\,10^{14}\ Pa$
Poisson's ratio	0,2	0,3
Density	$1450\ kg/m^3$	$2500\ kg/m^3$
AMOR_ALPHA	$0.0001\ s$	$0.001\ s$
AMOR_BETA	0.0.0.0	

the array is selected “tough”. The dry coefficient of kinetic friction of Coulomb between the block and the array is: $\mu=0{,}9$. This high coefficient makes privilege situations where rotation around the corners of the block is done without sliding. However, during the installation of the heavy block on the array, one admits that there is sliding free: $\mu=0{,}0$. The mass of the block is $417{,}600\ kg$.

One adds a damping of Rayleigh to introduce a certain dissipation high frequency material using key word AMOR_ALPHA (see above), from where forces: $\mathbf{C}\,\dot{\mathbf{U}}=\alpha\,\mathbf{K}\,\dot{\mathbf{U}}$.

1.3 Boundary conditions and loadings

Boundary conditions

the translations of the array according to the axes y and z are blocked on the lower face of this one.
The translation of the array according to the axis x is blocked at the point $S1$ in with respect to $O1$.
The translations of the block according to the axis z are blocked.
The point $O1$ of the block, medium of its base is constrained by: $dx=0$ only for the initial phase of phased introduction of the loading of gravity.
The acceleration of gravity is worth: $g=9.81\,m/s^2$.
A condition of contact-friction is ensured between the base of the block and the upper face of the array.

To release after slope of the block, constant gravity remaining

the condition of contact-friction is ensured between lower face of the block and upper face of the array. The acceleration of gravity is worth: $g=9.81\,m/s^2$.

Loading of harmonic excitation of the array

Under investigation, not restored.

1.4 Initial conditions

Loading of gravity

the block and the array are initially at rest: at $t=0$ $dx(0)=0$, $dx/dt(0)=0$ in any point.

Loading to release

the point $C1$, medium on the higher side of the block is constrained: $dx=-0.008\,m$, that is to say an angle of 10^{-2} . whereas the point $O1$ is fixed in x . One expects then the stabilization of vibrations, to carry out to release it under gravity, since this position drawn aside at initial velocity null in any point.

Loading of harmonic excitation of the array

Under investigation.

2 Reference solution

the results of reference are of two natures:

- benches analytically with the assumption of rigid body, without rebound, while simulation *Code_Aster* is made with elastic bodies, which induces a little distant results;
- benches analytically starting from the average ratio of loss of kinetic energy raised during the collisions on simulation *Code_Aster*, which makes it possible to ensure the accuracy of several dynamic variables: velocities, times of collision, energies kinetic, reactions, percussions;
- obtained numerically with software LMGC90 of laboratory LMGC (University of Montpellier), with one θ - diagram of velocity, cf [bib3, bib4].

2.1 Method of calculating used for the reference solution

For more details on the solution with the assumption of rigid body, to refer to [bib1]. The mass of the block is: $M=\rho V=4\rho b l e$. A possible driving acceleration of the array is considered: $(\ddot{u}_{ent}, \ddot{v}_{ent})$. The parameter of configuration is noted θ. The inertia of rotation at the center of gravity is $J_G=\frac{1}{3}M\left(b^2+l^2\right)$, that with the corner O is $J_O=J_G+MR^2$. The equations of the motion of the block, in the frame of the modelization in rigid body are:

$$\begin{cases} rocking\ about\ O\ :\ J_O\ddot{\theta}+MR\left(\ddot{v}_{ent}+g\right)\cos\left(\theta_0+\theta\right)-MR\,\ddot{u}_{ent}\sin\left(\theta_0+\theta\right)=0 \quad with\ \ \theta\in\mathbb{R}^+ \\ rocking\ about\ A\ :\ J_A\ddot{\theta}-MR\left(\ddot{v}_{ent}+g\right)\cos\left(\theta_0-\theta\right)-MR\,\ddot{u}_{ent}\sin\left(\theta_0-\theta\right)=0 \quad with\ \ \theta\in\mathbb{R}^- \end{cases}$$

The pulsation of swinging is: $\omega_{rO}=\sqrt{MgR/J_O}$, that is to say $\omega_{rO}=\sqrt{3g/4R}$; with the geometrical data: $\omega_{rO}=4.0956\,s^{-1}$.

If one analyzes the head-on collision of the base of the block on a rigid wall, the total period of contact τ (time of return ticket of the elastic wave) is given by:

$$\tau=4l\sqrt{\frac{\rho}{E}}$$

that is to say here: $\tau\approx 1.5731\ 10^{-4}\ s$.

What is directly associated with the frequency with the mode with vertical extension with the block, which is: $f_1=6357\,Hz$. One can make the same analysis for the array: $f_1=625\,kHz$. The frequency of the first mode of bending transverse of the block, considered as a beam of Eulerian, rotulée at its base and free at its top, is: $f_f=8100\,Hz$. This value gives an idea of the type of dynamic response of the block likely to occur at the time of the collision after releasing.

The total period of contact τ, during a frontal shock, makes it possible to estimate the stiffness of shock absorbers to be placed at the lower corners of the block, when one of the contact considers a technique of processing by shock absorbers: it is the case of the modelization B. One has as follows:

$$K_{res}=M_{bloc}\frac{\pi^2}{\tau^2}=\frac{\pi^2 ES}{16\,\ell}$$

This value corresponds to a longitudinal wave propagation in a continuum. If one considers the response of a finite element linear elastic into unidimensional, with consistent mass matrix, length $2l$, one a:

$$K_{res}=M_{bloc}\frac{3E}{\rho l^2}=\frac{3ES}{l}$$

For this configuration one finds respectively: $9.253\,10^{11}\,N/m$ and $4.5\,10^{12}\,N/m$.

2.1.1 Case to release from a position inclined at rest: rigid body

Like $\cos^2\theta_0>\frac{2}{3} \Leftrightarrow l/b<\sqrt{2}/2$, the block balances alternatively corner on the other, cf [bib1]. One notes θ_{in} the initial slope, presumedly weak. The balance equation for the phases of "coasting flight" (valid for small angles, with the 1st order) is reduced to $\frac{4}{3}MR^2\ddot{\theta}+Mbg-M\ell g\theta=0$. The period of the coasting flight t_{coll} around O before collision on the other corner A is thus solution of:

$$0=\frac{b}{l}+\left(\theta_{in}-\frac{b}{l}\right)\cosh\left(t_{coll}\sqrt{\frac{3gl}{4R^2}}\right)$$

For $\theta_{in}=10^{-2}$, and the dimensions of the block considered, one a: $t_{coll}\approx 0,05440978\ s$. This time corresponds to the first impact with the corner A .

Kinetic energy right before the collision is: $E^-_{kin}=\frac{1}{2}J_O\dot{\theta}^2=Mbg\,\theta_{in}$. It is worth: $7.37398\,J$.

One from of also deduced the angular velocity before the first collision:

$$\dot{\theta}(t_{coll})=\frac{1}{R}\sqrt{\frac{3}{2}bg\,\theta_{in}}\approx-\frac{3}{4}bgt_{coll}/R^2$$

the vertical equilibrium gives the vertical reaction in O :

$$F_O=Mg+Mb\,\ddot{\theta}\approx Mg\left(1-\frac{3b^2}{4R^2}\right)$$

the horizontal equilibrium gives the horizontal reaction:

$$H_O=M\,l\,\ddot{\theta}\approx-\frac{3Mgb\,l}{4R^2}$$

Their values are: $F_O\approx 3579,25\ N$ and $|H_O|\approx 1149,79\ N$.

Their ratio is thus $\frac{3bl}{4R^2-3b^2}\approx 0.3212$.

This ratio is weaker than the selected coefficient of kinetic friction: one thus does not expect sliding.

The balance equations in percussion of shock at the times of the impacts provide for their part:

$$I_y = -\frac{4R^2 - 3b^2}{2R^2} Mb\,\omega_- \quad \text{while} \quad \frac{|I_x|}{|I_y|} = \frac{3b\,l}{4R^2 - 3b^2} \approx 0.3212$$

This ratio is weaker than the selected coefficient of kinetic friction: one thus does not expect sliding.

To each impact a share of kinetic energy is transmitted block to the array; since, at the time of the collision, it is admitted that there is no rebound (i.e. just at the time of the collision, the impacted point has a built-in position), one identifies an equivalent "coefficient of restitution then" connecting the ratio angular velocities before and after the collision (one points out that in phase "coasting flight", accelerations are quasi constant, therefore the velocities quasi closely connected):

$$\omega_+ = \frac{2\,R^2 - 3\,b^2}{2\,R^2}\,\omega_- \approx 0.7474\ \omega_-$$

This same ratio corresponds to that of the time intervals between two successive collisions (with the 1st order).

One of deducted the ratio enters kinetic energies after and before each collision: 0.5586086 (because of the square). This same ratio corresponds to that of the swing angles of the block after and before each collision, because it is admitted that there is no dissipation of energy during free rotation between two collisions, which provides the values of maximum vertical displacements of the corners of the block.

The time interval between two successive collisions $k \rightarrow k+1$ is given by the maximum swing angle θ_k reached at the time of the phase right after the collision k (or θ_{k-1} reached at the time of the phase right before the collision k):

$$\Delta\,t_{k\rightarrow k+1} \approx \frac{8\,R^2}{3\,bg}\,\dot{\theta}_k^+ = 4\,R\sqrt{\frac{2\,\theta_k}{3\,bg}} = \frac{4\,R^2 - 6\,b^2}{R}\sqrt{\frac{2\,\theta_{k-1}}{3\,bg}}$$

As the ratio θ_k / θ_{k-1} with two successive collisions is identical (with the 1st order) to that of kinetic energies $E_{kin}^{+k} / E_{kin}^{-k}$, then one also has:

$$\Delta\,t_{k\rightarrow k+1} / \Delta\,t_{k-1\rightarrow k} \approx \sqrt{E_{kin}^{+k} / E_{kin}^{-k}} = \omega_+ / \omega_-$$

2.1.2 Case to release from a position inclined at rest: elastic body

One does not have a complete theoretical reference under this assumption. On the other hand, by admitting the weakness of the energy stored in vibratory form, there are the following results.

The phase of "coasting flight" (rotation around the corners) is still characterized by the balance equation of the rigid body in rotation (valid for small angles):

$$\frac{4}{3} MR^2\,\ddot{\theta} = -Mbg + M\,l\,g\,\theta$$

From where reactions: $F_O = Mg + Mb\,\ddot{\theta} \approx Mg\left(1 - \frac{3\,b^2}{4\,R^2}\right)$ and $H_O = M\,l\,\ddot{\theta} \approx -\frac{3\,Mgb\,l}{4\,R^2}$, that is to say: $F_O \approx 3579{,}25\ N$ and $|H_O| \approx 1149{,}79\ N$. These values are valid for all the phases of coasting flight between two collisions.

One obtains also the angular velocity right before the first collision: $\dot{\theta}\left(t_{coll}\right)=\frac{1}{R}\sqrt{\frac{3}{2}bg\,\theta_{in}}$ and kinetic energy: $E_{kin}^{-}\left(t_{coll}\right)=Mbg\,\theta_{in}=\frac{2}{3}MR^2\,\dot{\theta}_{-}^2\left(t_{coll}\right)$. The time of the first collision must be the same one as analyzes some in rigid body: $0{,}05440978\ s$; kinetic energy with the first collision being also: $7{,}373981\ J$.

The contact being maintained over a certain period in computational simulation with the assumption of deformable bodies, contrary to the case of the rigid bodies, the restitution of shock does not have same phenomenology.
Also decides one to take the value found by simulation by *Code_Aster* to identify a coefficient of restitution, which is employed to reconstitute the succession of the later collisions.

One thus decides to take by the mean value of $\sqrt{E_{kin}^{+k}/E_{kin}^{-k}}$ on the 5 studied collisions the coefficient of restitution: 0.79, that one chooses for the successive predictions. This value higher than that is obtained in assumption of rigid body without rebound, cf [§ 2.1.1], which is legitimate.

While following the assumption of absence of vibration lasting the phase of coasting flight of swinging, the time interval between two successive collisions $k\rightarrow k+1$ is given by the maximum swing angle θ_k reached at the time of the phase right after the collision k :

$$\Delta t_{k\rightarrow k+1}\approx\frac{8R^2}{3bg}\dot{\theta}_k^{+}=4R\sqrt{\frac{2\theta_k}{3bg}}$$

In addition, it is observed numerically that the phase of rebound “delays” the restarting of the rocking movement to each collision. One then adopts a mean value of the relationship between the time intervals between two successive collisions of: 0.77 .

The results which one then draws from this value are listed in table 2.3.

The vertical percussion is the integral over the period of contact at the time of the collision of the vertical reactions of shock:

$I_y=\int_{t_0}^{t_1}f_y(t)\,dt\ \ (N.m)$ that one compared to $I_y=-\frac{4R^2-3b^2}{2R^2}Mb\,\omega_{-}$

As their exact obtaining by computational simulation is delicate, one will test also the relationship with the horizontal percussion $|I_x|/|I_y|\approx 0.3212$, which is weaker than the selected coefficient of kinetic friction (one should not thus have of sliding).

2.1.3 Harmonic case of training

Under investigation, not restored. For more details, to refer to [bib2].

2.2 Quantities and results of reference

Here the list of times of impact studied and the calculated quantities, under the assumption rigid body. One also compares with the results got by software LMGC90 (v2), of the University of Montpellier, employing one θ - diagram of temporal integration of velocity for one time step $\Delta t=10^{-4}\,s$, and a method of processing of the contact of velocity.

Phase Impact n°	Urgent (*S*)	vertical Displacement (*m*) maximum of the corner	vertical Velocity (*m/s*) maximum of the corner right before collision	Kinetic energy (*J*) maximum of the block right before collision
initialization	0.0000	0,003600 in A	0,0000	0,0000
$0 \Rightarrow 1$		0,002011 in O		
1	0.05440978		– 0,133572 in A	7,373981
LMGC90	0.0546	0,1898 in O (– 5.6%)	– 0,13325 (– 0.24%)	7.23255 (– 1.918%)
$1 \Rightarrow 2$		0,001123 in A		
2	0. 13574		– 0,099832 in O	4,119169
LMGC90	0.1324		– 0,10053 (0.71%)	4.04688 (1.756%)
$2 \Rightarrow 3$		0,000628 in O		
3	0. 196529		– 0,074615 in A	2,301003
LMGC90	0.1929			2.25311 (– 2.081%)
$3 \Rightarrow 4$		0,000351 in A		
4	0. 241961		– 0,055767 in O	1,28536
LMGC90	0.2359			1.26174 (– 1.841%)
$4 \Rightarrow 5$		0,000196 in O		
5	0.27592		– 0,04168 in A	0,718013
LMGC90	0.2693			

Table 2.1. Analytical solutions under the assumption rigid body, without rebound. Comparison with the solution obtained with software LMGC90 (v2), of the University of Montpellier.

Phase Impact n°	Urgent (S)	Ratio the angular velocities	vertical Reaction on the corner	horizontal Reaction on the corner	vertical I_y Percussion on the corner
$0 \Rightarrow 1$			formulates 3579, 25 in O	– 1149,8 N formulates O	
1	0.7474	48,73			of them N.s formulates 3579,25 A
$1 \Rightarrow 2$			into 1149,8 of them A	N formulates 0.13574 A	
2	36,42	of them			N.s formulates of them O
$2 \Rightarrow 3$			N in – 1149,8 O	N formulates 0.19652 9 O	
3	27,22	of them			N.s formulates 3579,25 A
$3 \Rightarrow 4$			into 1149,8 of them A	N formulates 0 A	
4	20,35 of them	N.s			formulates of them 3579,25 O
$4 \Rightarrow 5$			in – 1149,8 O	N formulates 0.27592 0.7474 O	
5	of them	N.s in			Table 2.2. Analytical A

solutions under the assumption rigid body, without rebound. Phase Impact N

° Instants (S) vertical	Displacement	(m) maximum of the corner vertical Velocity	(m/s) extreme of the corner right before collision Kinetic energy	(J) maximum of *the block* right before collision vertical Percussion	on I_y the corner initialization
0.0000 0,003600	in 0,0000	0,0000 A	formula	formulates	
$0 \Rightarrow 1$					
1	in	– 0,13357229 O	in 7,373981 48,73 A	N.s in formula	formulates 0.138201 A
$1 \Rightarrow 2$					
2	in –	0,1055221 in A	4,6021 37,52 N.s in O	formula	formulates 0. 202720 O
$2 \Rightarrow 3$					
3	in – 0,083362	in 2,8722 O	28,89 N.s in formula A		formulates 0. 252400 A
$3 \Rightarrow 4$					
4	– 0,065856	in 1,7925 A	22,24 N.s in formula O	formulates	0.29065 0,000341 O
$4 \Rightarrow 5$					
5	0,052026	in 1,1187 17,12 O	N.s in Table A	2.3. Solutions	interpreted A

analytically from average coefficients of restitution of shock estimated numerically with Code_Aster. Uncertainties on *the analytical*

2.3 solution Solution obtained in

assumption of rigid body without rebound; quasi-analytical elastic solution exploiting an energy value obtained by simulation, characterizing the restitution of shock. Bibliographical references

2.4 F. VOLDOIRE: VHTR 2015: Graphite

1 core seismic design: Non-linear dynamics of blocks; oscillations and contact-impacts. Bibliographical survey, analytical results. Note technical EDF/AMA HT-62/04/010/A, March 2005. F. VOLDOIRE: VHTR 2015: Graphite

2 core seismic design: Non-linear dynamics of blocks; oscillations and contact-impacts. First finite element modelling methodologies. Note technical EDF/AMA HT-62/05/018/A, February 2006. [P. Chabrand, O. Chertier, and

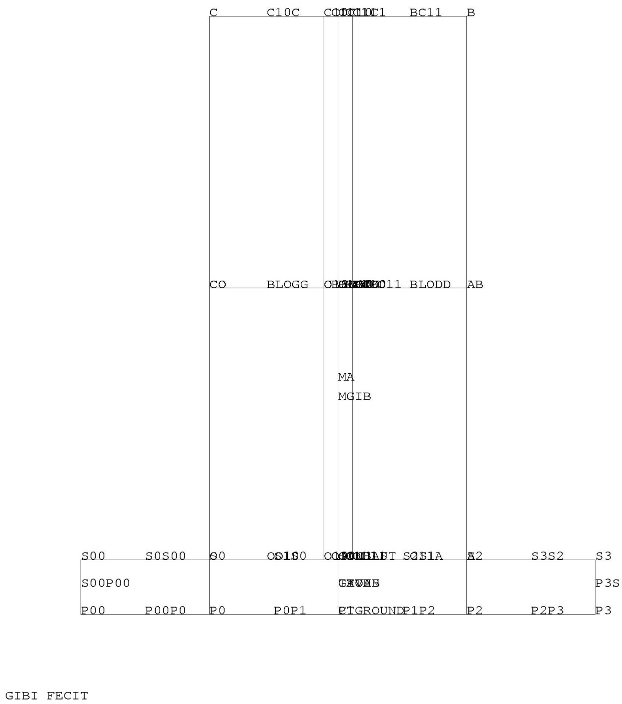
3 F. Dubois. Complementarity methods for multibody friction and contact problems in finite strains. Int. J. Num. Meth. Eng., 51:553-578, 2001. F. VOLDOIRE, Mr. Kham: Project

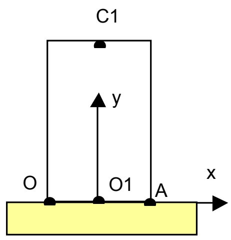
4 OMERSI. Deliverable T342: Benchmark of structures tilting and slipping into Code_Aster, coupling Code_Aster LMGC90. CR-AMA-07.289, 2/2008. Modelization A Characteristic

3 of the modelization

3.1 One chooses a plane

modelization, with of the finite elements 2D in plane stresses (modelization C_PLAN). Contact-friction `between` block and array is treated by the method of Lagrange. Appear 3.1-a: Modelization





and mesh Characteristics of the mesh

3.2 One the model invites Mo (see

fig. 3.1 - a) associated with the problem. The mesh made: 25 nodes , 34 meshes SEG2 and 12 meshes QUA4. Here the list of the nodes groups and meshes useful in the modelization: Name groups Contained O low left

lower	Node
of	the block A Node lower low right
of	the block O1 Nœud lower medium of
the block	C Node higher medium of
the block	S0 Nœud of the array in contact
	with S1 Nœud of the array in contact O
	with S2 Nœud of the array in contact $O1$
	with CTGROUND Nodes of the bottom of A
array CTBLO	Meshes SEG2 contour of
block	CONHAUT Meshes SEG2 bases Meshes
block	CONBAS SEG2 of the array
under	the base of the block BLOCK Meshes of block GROUN
Meshes	of the array Characteristics
	of the loadings

3.3 contact-friction between block

and array is treated by the method of Lagrange, pairing being by master-slave method, the selected norm being that of the Master: Group meshes main: CONBAS

- ; Group slaves meshes : CONHAUT
- . A geometrical reactualization

is adopted. The array is blocked in at

the point, and in on its basis x . Gravity $S1 \quad y$ is simulated by $CTGROUND$

a slope, precondition to the slope of the block, is operated by an imposed displacement. These phases are stabilized with their face value by a strong numerical damping, precondition under investigation dynamic. Characteristics of integration

3.4 in time One chooses a diagram, implicit

in time, of Newmark, modified average acceleration. For the phases of installation

under the action of vertical gravity and of initialization of releasing (by shift in initial rotation of the block), one chooses time step, and a diagram of integration $0.0125\,s$ in time in - method key word "HHT" θ : ALPHA=-0.30, MODI_EQUI = ' NON "`for gravity of 0 with (of` with), then one chose g ALPHA $t=-2.0\,s$ =-0.60 $t=-1.0\,s$ for the phase D" `slope` of the block (of with). The initialization of $t=-1.25\,s$ releasing $t=0 \quad s$

with t=0s is done from a velocity null in the block and of the displacement induced by the slope. For the phases of coasting flight (swinging), time step is regulated by where is a dimension characteristic $\Delta t=\sqrt{\frac{h}{50\,g}}$ h of the fall. One chooses: . One takes the parameters of $\Delta t=0.0025 \quad s$ Newmark of average acceleration: , via key word "HHT". $\alpha=0$. For the phases including

the collisions, one changes integration, while choosing a diagram of modified average acceleration: - method "HHT", with MODI α _EQUI=' NON' , and `one time step refined` $\alpha=-0.2$. Thus, there is the succession $\Delta t_r=0.000010 \quad s$: N° impact Interval (S) - method

Time step	0,0000	α - 0,0025	10-5 S 1 0,0525
	- 0,0575 10-5	$\alpha=-0.1$	S 2 0,1400
	- 0,1450 formula	$\alpha=-0.2$	10 -
5	S 3 0,2050 - 0,2125	$\alpha=-0.2$	10 -
5	S 4 0,2575 - 0,2650	$\alpha=-0.2$	10 -
5	S 5 0,2975 - 0,3075	$\alpha=-0.2$	10
-5	S One notes	$\alpha=-0.2$	rebound

shortly after each collision. It is necessary so that simulation is correct that this rebound is integrated with time step refined. Final moment is: , in order to have the first 5 $0.33\,s$ collisions. Non-linearities are integrated

with the method of Newton, with tangent matrix reactualized with each iteration. The necessary precise details on the equilibrium are: resi_glob_rela=10-6, resi_ glob _maxi=10-3 (what is rather strict!). The nombre of iterations of maximum Newton is: 22. One envisages subdivision

only for the initial phase of slope of the block. The nombre total of time step

for the study of swinging is of 3867 steps. Quantities tested and Case

3.5 results of release-swinging from

3.5.1 a position inclined at rest Several values are tested

not per times, but by extreme values on a time interval, because that is more relevant. Tests of NON-regression, with weak tolerance, were added in order to track the evolutions of the algorithms. Values of displacements (depl

), Type of **reference** : "ANALYTIQUE ": Identification Time (S) Reference

Aster	relative Error	% D_y group	_no	O (m) Value min
on [0, 0.33] S 0.000000	– 1.90 10.-10 0.00%	D_y group	_no O (m) Value	max
on [0, 0.33] S Rigid body	: 0.002011 With	retiming: 0.00225 0.00233034 15.9% 3.71% D_y	G roup_no A	(m) Value min
on [0, 0.33] S 0.00000	– 1.03 10.-10 0.00%	D_y group	_no A (m) Value	max
on [0, 0.33] S 0.00360	0.00359997 – 0.07%	Values	velocities	(quickly),

Type of reference : "ANALYTIQUE ": Identification Time (S) Reference

Aster	relative Error	% V_y group	_no	O (m/s) Value min
on [0, 0.33] S Rigid body	: – 0.09983 With	retiming: – 0.10552 – 0.10707 7.255% 1.47% V_	there group_no O	(m/s) Value min
on [0.17, 0.33]	S Rigid body: – 0.074614	With retiming: – 0.08336 – 0.06564 – 12.0% – 21.2%	V_y group	_no A (m/s) Value min
on [0, 0.33] S –	0.13357 – 0.13281 – 0.571%	Values	of reactions	to the corners

(vale_cont **), component** norm (RN) and tangential (X-ray), Type of reference: "ANALYTIQUE ": Identification Time (S) Reference

Aster	relative Error	% RN grou	_no	O (N) Value min on
[0, 0.33] S 0.0000	0.0000 0.0000 RN grou_	No O (N	) Value	max on
[0, 0.05] S 3580.0	3599.52 0.545% RN grou	_no A	(N) Value	min on
[0, 0.33] S 0.0000	0.0000 0.0000 RN grou_	No A (N	) Value	max on
[0.06, 0.12]	S 3580.0 3597.96 0.502% X-ray grou	_no O	(N) Value	min on
[0, 0.05] S –	1150.0 – 1137.1718 –	1.11% X-ray grou	_no A (N) Value	max on
[0.06, 0.12]	S 1150.0 1141.44	of	percussions	to the corners

– 0.744% Values

(vale_cont **integrated** around `times` of collision), Type of reference: “ANALYTIQUE ”: Identification `Time (S)` Reference

Aster	relative Error	% RN grou	*_no*	A (N) Value integrated
on [0.05440	, 0.05455] 48.73 41.8916 – 14.0% X-ray grou	_no	A (N) Value	integrated
on [0.05440	, 0.05455] 15.65 11.2831 – 27.9% Values	of	kinetic energy	

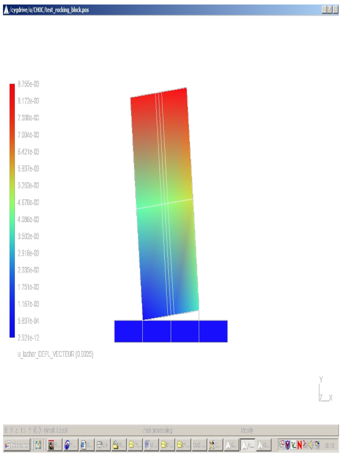
on the block **alone (total), Type** of reference: “`ANALYTIQUE` ”: Identification `Time (S)` Reference

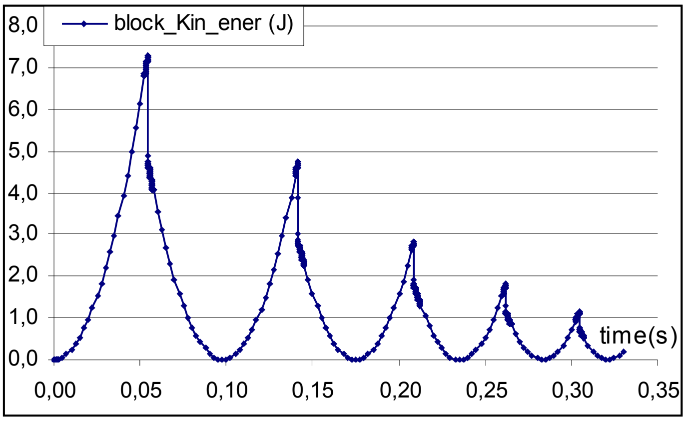
(J) Aster	relative	Error % Kinetic energy		(J) Value
max on [0.0, 0.33 *]*	S Rigid body: 7.37398 7.28974	– 1.14% Kinetic energy		(J) Value
max on [0.10, 0.33]	S Rigid body: 4.1192 With	retiming: 4.6021 4.73829 15.0% 2.96% Kinetic energy		(J) Value
max on [0.15, 0.33]	S Rigid body: 2.3010 With *retiming*	: 2.8722 2.85983 24.3% – 0.43% Kinetic energy		(J) Value
max on [0.22, 0.33]	S Rigid body: 1.2854 With *retiming*	: 1.7925 1.78097 38.5% – 0.64% Kinetic energy		(J) Value
max on [0.28, 0.33]	S Rigid body: 0.7180 With *retiming*	: 1.1187 1.08113 50.6% – 3.36% Values	of	times of collision

, Type **of reference: “ANALYTIQUE** ”: Identification `energy (J)` Reference

(S)	Aster relative	Error % Urgent	*1st*	collision (S
) Value max on [0.0, 0.33 *]*	S Rigid body: 0.0544098 *0.05441*	– 0.018% Time	2nd collision	(S
) Value max on [0.10, 0.33]	S Rigid body: 0.13574 With	retiming: 0.13820 0.14151 4.25% 2.39% Time	3rd	collision (S
) Value max on [0.15, 0.33]	S Rigid body: 0.196529 With	retiming: 0.20272 0.20905 6.37% 3.12% Time	4th	collision (S
) Value max on [0.22, 0.33]	S Rigid body: 0.241961 With	retiming: 0.25240 0.26224 8.38% 3.90% Time	5th collision	(S
) Value max on [0.28, 0.33]	S Rigid body: 0.27592 With	retiming: 0.29065 0.30306 10.04% 4.46% Remarks	Appear	3.6-a : Front

3.6 position



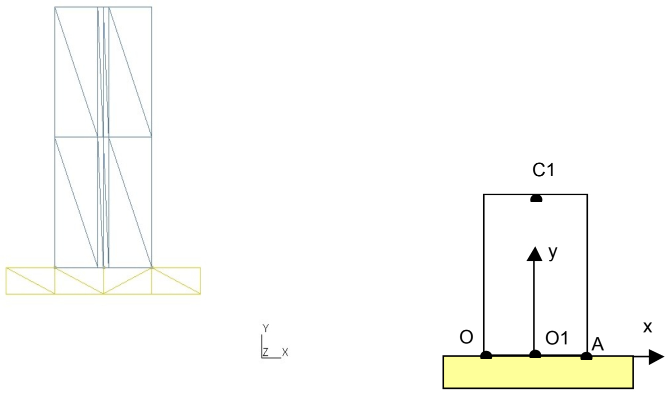


to release. Evolution of the kinetic energy (J) of the block. Modelization B Characteristic

4 of the modelization

4.1 One chooses a plane

modelization, with of the finite elements 2D in plane stresses (modelization C_PLAN) and of the specific `discrete elements`, spring seat of shock. Appear 4.1-a: Modelization



and mesh Characteristics of the mesh

4.2 The mesh made: 25 nodes

, 34 meshes SEG2 and 20 meshes TRIA6. Here the list of the nodes groups and meshes useful in the modelization: Name groups Contained O low left

lower	Node
`of`	the block and mesh-POINT A Node lower low right
`of`	the block and mesh-POINT O1 Nœud lower medium of
`the block`	and mesh-POINT C Node higher medium of
`block`	CTBLO Meshes SEG2 contour of
`block`	CONHAUT Meshes SEG2 bases Meshes
`block`	CONBAS SEG2 of the array
`under`	the base of the block BLOCK Meshes of the block One creates
`a new`	mesh

ma_poi meshes to add the 3 - `points` POI1 on the nodes O, A and `O1,` which will be support `of` the discrete elements of spring DIS_T. The array is not part of the built model. One calls Mo the model associated with the problem . Characteristics of the loadings

4.3 contact-friction between block

and array is treated by absorbers shock specific placed in O, A and O1. One assigns a material `DIS_CONTACT` and characteristics `DISCRET` and ORIENTATION to the elements of `springs (` to place them vertically). The selected characteristics are: in AFFE_CARA_ELEM: DISCRET

•avecCARA `= ' K_T_D_N',`

VALE= (30000000000.0 , 10000000.0 , 0.0,),) CARA=' A_T_D_N', VALE= (0.1, 1.0
, 0.0,),),), ORIENTATION avecCARA=' VECT_X_
Y', VALE= (0.0 , - 1.0 , 0.0, 1.0, 0.0, 0.0 ,),),); in DEFI_MATERIAU: DIS_CONTACT
•avecRIGI_NOR=30000000000.0
, RIGI_TAN=10000000.0, AMOR_NOR =5000000.0, COULOMB=
0.9, JEU=0.0. The normal stiffness is such as

3 springs in parallel gives again the value of spring determined with [§2.1]. Then, in dyna_non_line,

one will use BEHAVIOR MODEL DIS_CHOC on these elements. The discrete elements , being three-dimensional, are blocked in. Gravity is simulated by z

a slope, precondition to the slope of the block, is operated by an imposed displacement. These phases are stabilized with their face value by a strong numerical damping, precondition under investigation dynamic. Characteristics of integration

4.4 in time One chooses a diagram, implicit

in time, of Newmark, modified average acceleration. For the phases of installation

under the action of vertical gravity and of initialization of releasing (by shift in initial rotation of the block), one chooses time step, and a diagram of integration $0.0125\,s$ in time in - method key word “HHT” α: ALPHA=-0.30, MODI_EQUI = ' NON "for gravity of 0 to G (of with), then one chose ALPHA $t=-2.0\,s$ =-0.60 $t=-1.0\,s$ for the phase D” slope of the block (of with). The initialization of $t=-1.25\,s$ releasing $t=0\ s$

with is done from a velocity $t=0\ s$ null in the block and of the displacement induced by the slope. For the phases of coasting flight (swinging), time step is regulated by where is a dimension characteristic $\Delta t=\sqrt{\frac{h}{50\,g}}$ h of the fall. One chooses: . One takes the parameters of $\Delta t=0.00125\ s$ Newmark of average acceleration: , via key word “HHT”. $\alpha=0$. For the phases including

the collisions and the light immediately consecutive rebounds, induced by the penetration caused by the penalization of the contact (due to the elements of springs), one changes integration, while choosing a diagram of modified average acceleration: - method “HHT”, with MODI α _EQUI=' NON' or true diagram "HHT", with MODI_EQUI=' OUI' and (i.e. by introducing $\alpha=-0.1$ a light numerical damping), with one time step refined. Thus, there is the succession $\Delta t_r=0.000025\,\mathrm{s}$: N° impact Interval (S) method

value	of Time step	0,00000	0,00250	- method
	2.5 10 - 5 S 1 0,05375	α 0,05875	$\alpha=-0.1$	- method
	2.5 10 - 5 S 2 0,13375	α 0,15000	$\alpha=-0.1$	- method
	2.5 10 - 5 S 3 0,19250	α 0,21000	$\alpha=-0.1$	HHT 2.5
	10-5 S 4 0,23250	0,24250	$\alpha=-0.1$	HHT 2.5
	10-5 S 5 0,27000	0,27750	$\alpha=-0.1$	HHT 2.5
	10-5 S One notes	a light	$\alpha=-0.1$	rebound

shortly after each collision (and a light penetration). It is necessary so that simulation is correct that this rebound is integrated with time step refined. Moreover, the choice of time step is not independent of the value of the normal stiffness of shock and the normal damping of shock, cf [§ 4.3]. It is noted that times of collision arrive more precociously than with the method of contact of Lagrange not penalized, cf [§ 4.4]. Final moment is: , in order to

have the first 5 $0.285\,s$ collisions. Non-linearities are integrated

with the method of Newton, with tangent matrix reactualized with each iteration. The necessary precise details on the equilibrium are: resi_glob_rela=10-6, resi_ glob `_maxi=10-3 (what` is `rather strict!`). The nombre of iterations of maximum Newton is: 15. One envisages subdivision

only for the initial phase of slope of the block. The nombre total of time step for the study of swinging is of 2140 steps. Quantities tested and Case

4.5 results of release-swinging from

4.5.1 a position inclined at rest Several values are tested

not per times, but by extreme values on a time interval, because that is more relevant. Tests of NON-regression, with weak tolerance, were added in order to track the evolutions of the algorithms. Values of displacements (depl

), Type of **reference** “ANALYTIQUE”: Identification `Time (S)` Reference

Aster	relative Error	% D_y group	_no	O (m) Value min
on [0, 0.33] S 0.000000	– 3.0079 10 -6 0.00%	D_y	group_no O (m) Value	max
on [0, 0.33] S Rigid body	: 0.002011 With	retiming: 0.00225 0.00198832 – 1.13% – 11.51%	D_y group	_no A (m) Value min
on [0, 0.33] S 0.00000	– 4.94 10 -6 0.00%	D_y group	_no A (m) Value	max
on [0, 0.33] S 0.00360	0.0035958 – 0.117%	Values	velocities	(quickly),

Type of reference “ANALYTIQUE”: Identification `Time (S)` Reference

Aster	relative Error	% V_y group	_no	O (m/s) Value min
on [0, 0.33] S Rigid body	: – 0.09983 With	retiming: – 0.10552 – 0.098934 – 0.90% – 6.24%	V_y group	_no O (m/s) Value min
on [0.17, 0.33]	S Rigid body: – 0.074614	With retiming: – 0.08336 – 0.051964 – 30.36% –	37.66% V_y	group_no A (m/s) Value min
on [0, 0.33] S –	0.13357 – 0.132746 –	0.617% Values	of reactions	to the corners

: on **the discrete elements** of springs (sief_elga), Type of reference `“ANALYTIQUE`”: Identification `Time (S)` Reference

Aster	relative Error	% N grou	_no O	(N) Value max on
[0, 0.05] S 3580.0	3604.36 0.680% N grou	_no A (	N) Value	max on
[0.06, 0.12]	S 3580.0 3611.13 0.870% VY grou	_no O	(N) Value	min on
[0, 0.05] S –	1150.0 -1152.71 0.236% VY	grou_no A	(N) Value	max on
[0.07, 0.12]	S 1150.0 1258.71 9.45% Values	of kinetic energy		

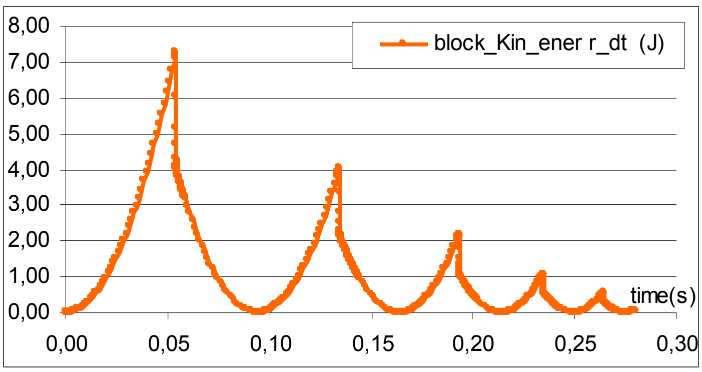
on the block **alone (total), Type** of reference `“ANALYTIQUE”`: Identification `Time (S)` Reference

(J) Aster	relative	Error % Kinetic energy		(J) Value
max on [0.0, 0.28] S	Rigid body: 7.37398 7.28314	– 1.23% Kinetic energy		(J) Value
max on [0.10, 0.28]	S Rigid body: 4.1192 With retiming	: 4.6021 4.04539 – 1.79% – 12.1%	Kinetic energy	(J) Value
max on [0.15, 0.28]	S Rigid body: 2.3010 With retiming	: 2.8722 2.17093 – 5.653% – 24.42%	Kinetic energy	(J) Value
max on [0.22, 0.28]	S Rigid body: 1.2854 With retiming	: 1.7925 1.11604 – 13.18% – 37.7%	Kinetic energy	(J) Value
max on [0. 26, 0.28	] S Rigid body : 0.7180 With retiming	: 1. 1187 0.57368 – 20.1% – 48.7%	Values	of times of collision

, Type of **reference “ANALYTIQUE”**: Identification `energy (J)` Reference

(S)	Aster relative	Error % Urgent	*1st*	collision (S)
Value max on [0.0, 0.33] S	Rigid body: 0.0544098 0.054375	– 0.064% Time	2nd collision	(S)
Value max on [0.10, 0.33]	S Rigid body: 0.13574 With	retiming: 0.13820 0.134900 – 0.619% – 2.388%	Time	3rd collision (S)
Value max on [0.15, 0.33]	S Rigid body: 0.196529 With	retiming: 0.20272 0.19365 – 1.465% – 4.474%	Time	4th collision (S)
Value max on [0.22, 0.33]	S Rigid body: 0.241961 With	retiming: 0.25240 0.23560 – 2.629% – 6.66%	Time	5th collision (S)
Value max on [0. 28, 0.33	] S Rigid body : 0.27592 With	retiming: 0.29065 0.26570 – 3.70% – 8.58%	Remarks	Figure 4.6 - have: Evolution

4.6 of L

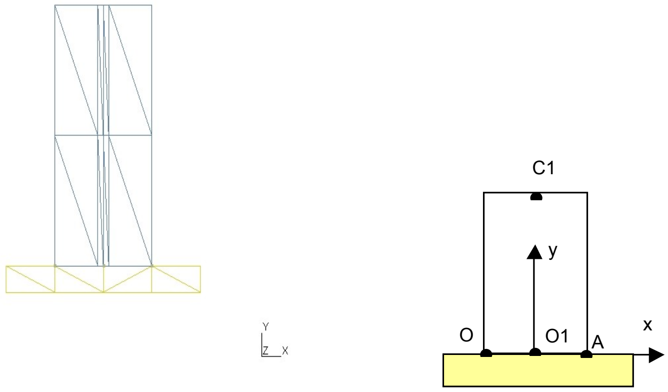


“kinetic energy (J) of the block. Modelization C Characteristic

5 of the modelization

5.1 One chooses a plane

modelization, with of the finite elements 2D in plane stresses (modelization C_PLAN). Contact-friction `between` block and array is treated by the method "continuous hybrid formulation", cf [R5.03.52] and a diagram D" integration implicit, formulated in displacement. Figure 5.1-a : Modelization



and mesh Characteristics of the mesh

5.2 One the model invites Mo (see

fig. 5.1 - `a)` associated with the problem. The mesh made: 25 nodes , 34 meshes SEG2 and 12 meshes QUA4. Here the list of the nodes groups and meshes useful in the modelization: Name groups Contained O low left

lower	Node
`of`	the block A Node lower low right
`of`	the block O1 Nœud lower medium of
`the block`	C Node higher medium of
`the block`	S0 Nœud of the array in contact
	with O S1 Nœud of the array in contact
	with O1 S2 Nœud of the array in contact
	with A CTGROUND Nodes of the bottom of
`array CTBLO`	Meshes SEG2 contour of
`block`	CONHAUT Meshes SEG2 bases Meshes
`block`	CONBAS SEG2 of the array
`under`	the base of the block BLOCK Meshes of block GROUN
`Meshes`	of the array Characteristics
	of the loadings

5.3 contact-friction between block

and array is treated by the method of Lagrange, pairing being by master-slave method, the selected norm being that of the Master : Group meshes main: CONBAS

•; Group slaves meshes : CONHAUT

•. A geometrical reactualization

is adopted. The array is blocked in X at

the S1 point, and in there on its basis CTGROUND. Gravity is simulated by
a slope, precondition to the slope of the block, is operated by an imposed displacement. These phases are stabilized with their face value by a strong numerical damping, precondition under investigation dynamic. Characteristics of integration

5.4 in time One chooses a diagram, implicit

in time, of Newmark, modified average acceleration. For the phases of installation

under the action of vertical gravity and of initialization of releasing (by shift in initial rotation of the block), one chooses time step, and a diagram of integration $0.0125\,s$ in time in - method key word "HHT" α: ALPHA=-0.30, MODI_EQUI = ' NON "for gravity of 0 to G (of with), then one chose ALPHA $t=-2.0\,s$ =-0.60 $t=-1.0\,s$ for the phase D" slope of the block (of with). The initialization of $t=-1.25\,s$ releasing $t=0\ s$
with is done from a velocity $t=0\,s$ null in the block and of the displacement induced by the slope. For the phases of coasting flight (swinging), time step is regulated by where is a dimension characteristic $\Delta t=\sqrt{\dfrac{h}{50\,g}}$ h of the fall. One chooses: . One takes the parameters of $\Delta t=0.0025\ s$ Newmark of average acceleration: , via key word "HHT". $\alpha=0$. For the phases including

the collisions, one changes integration, while choosing a diagram of modified average acceleration: - method "HHT", with MODI α _EQUI=' NON' , and one time step refined $\alpha=-0.2$. Thus, there is the succession $\Delta t_r=0.000010\ s$: N° impact Interval (S) - method

Time step	0,0000	α - 0,0025	10-5 S 1 0,0525
	- 0,0600 10-5	$\alpha=-0.1$	S 2 0,1400
	- 0,1450 10-5	$\alpha=-0.2$	S 3 0,2050
	- 0,2150 formula	$\alpha=-0.2$	10 -
5	S 4 0,2575 - 0,2650	$\alpha=-0.2$	10 -
5	S 5 0,2975 - 0,3100	$\alpha=-0.2$	10 -
5	S One notes	$\alpha=-0.2$	rebound

shortly after each collision. It is necessary so that simulation is correct that this rebound is integrated with time step refined. Final moment is: , in order to

have the first 5 $0.32\,s$ collisions. However in order to have a less expensive test, one stops after the second collision: one stops in practice at time. Like the solver, METHODE $0.145\,s$ "

LDLT" does not go with the continuous method , one takes METHODE "MULT_FRONT", with RENUM "MDA", because RENUM " METIS " can be expensive in System time according to the machines. Quantities tested and results

5.5 Several values are tested

not per times, but by extreme values on a time interval, because that is more relevant. Tests of NON-regression, with weak tolerance, were added in order to track the evolutions of the algorithms. One limits oneself here to three phases of coasting flight. Values of displacements (depl

), Type of **reference** "ANALYTIQUE": Identification Time (S) Reference

Aster	relative Error	% D_y group	_no	O (m) Value min
on [0, 0.33] S 0.000000	– 1.8916210 -10 0.00%	D_y	group_no O (m) Value	max
on [0, 0.33] S Rigid	: 0.002011 With	retiming:	group_no A	(m) Value

body		0.00225 0.00233214 15.97% 3.79% D_y		min
on [0, 0.33] S 0.00000	– 1.0265 10.-10 0.00%	D_y	group_no A (m) Value	max
on [0, 0.33] S	0.00360 0.0035999 – 0.06%	Values	velocities	(quickly),

Type of reference “ANALYTIQUE”: Identification `Time (S)` Reference

Aster	relative Error	% V_y group	_no	O (m/s) Value min
on [0, 0.33] S Rigid body	: – 0.09983 With	retiming: – 0.10552 – 0.107117 7.30% 4.16% V_	there group_no A	(m/s) Value min
on [0, 0.33] S –	0.13357 – 0.132807 –	0.571% Values	of reactions	to the corners

(vale_cont **), component** norm `(RN) and` tangential (X-ray), Type of reference “ANALYTIQUE”: Identification `Time (S)` Reference

Aster	Error relative	% RN grou	_no	O (N) Value max on
[0, 0.05] S 3580.0	3599.52 0.545% RN grou	_no A	(N) Value	max on
[0.06, 0.12]	S 3580.0 3597.66 0.493% X-ray grou	_no O	(N) Value	min on
[0, 0.05] S –	1150.0 – 1137.17 – 1.12%	X-ray grou	_no A (N) Value	max on
[0.07, 0.12]	S 1150.0 1141.47 – 0.742% Values	of	percussions	to the corners

(vale_cont **integrated** around `times` of collision), Type of reference “ANALYTIQUE”: Identification `Time (S)` Reference

Aster	relative Error	% RN grou	_no	A (N) Value integrated
on [0.05440	, 0.05455] 48.73 41.9301 – 13.9% X-ray grou	_no	A (N) Value	integrated
on [0.05440	, 0.05455] 15.65 11.2919 – 27.8% Values	of	kinetic energy	

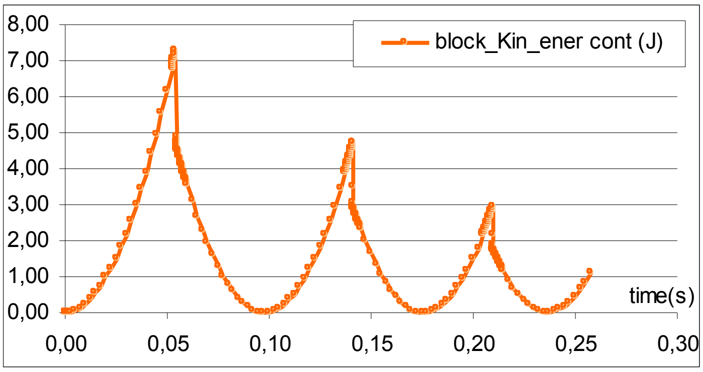
on the block **alone (total):** Identification Time (S) Reference

(J) Aster	relative	Error % Kinetic energy		(J) Value
max on [0.0, 0.33] S	Rigid body: 7.37398 7.28973	– 1.14% Kinetic energy		(J) Value
max on [0.1, 0.33] S	Rigid body: 4.1192 With retiming	: 4.6021 4.74225 15.1% 3.04% Values	of times	of collision

, Type of **reference “ANALYTIQUE”: Identification `energy (J)` Reference**

(S)	Aster relative	Error % Urgent	*1st*	collision (S)
Value max on [0.0, 0.33] S	Rigid body: 0.0544098 0.054400	– 0.018% Time	2nd collision	(S)
Value max on [0.10, 0.33]	S Rigid body: 0.13574 With	retiming: 0.13820 0.14154 5.05% 2.42% Remarks	Appear	5.6-a : Evolution

5.6 of L



“kinetic energy () of the block (continuous method in J displacements). Summary of the results

6 the results got with

the Code_Aster, in dynamics with *diagram D”* temporal integration implicit, expressed on displacements, are about in conformity with those expected by comparison with the analytical solution: error lower than 12% over times of collision (on all five modelled collision). One gets results contained in the beach of the two possible analytical models: that of the rigid body without any rebound and that adjusted by taking account of an elastic rebound, from where a stronger coefficient of restitution of shock. One of the contact notes

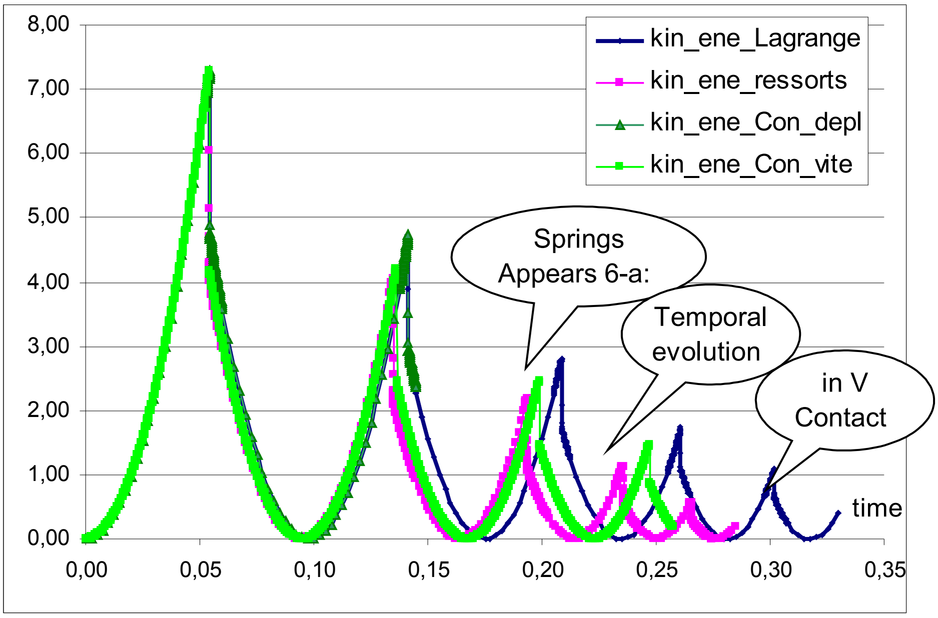
differences in prediction between the methods of processing selected. The method of penalization (by discrete springs of shock), which functions only by also envisaging damping located, cause a drop in the coefficient of restitution of shock, therefore approach plus the values of times of collision of the analytical model of rigid body without rebound. These values are very close

to those obtained by a software dedicated, LMGC90, developed at the University of Montpellier, resting on one - diagram of temporal integration θ of velocity and proposing two methods to treat the contact (concerned, of velocity), which give here the both same predictions. On the other hand, with the “

exact” methods of contact of Code_Aster – Lagrange and continues – associated with a diagram of implicit temporal integration of family Newmark-HHT, expressed on displacements, one rather gets results governed by the analytical solution adjusted by taking account of the rebound. Moreover, these two methods of Lagrange contact and continuous give times of collision (and energies) almost identical. One notes however on this test with small number of d.o.f. which the “continuous” method is more expensive in System time than the method “Lagrange”. Lagrangian method of contact

of Code_Aster formulated in displacement *with* a temporal integration using one - diagram in displacements provides θ results very close to those obtained with LMGC90 and the analytical solution in rigid body: in energies, times d'impact, reactions and percussions. This confirms the superiority – allowed largely in the literature – of a diagram such as - diagram to express the condition θ of contact of Signorini-Moreau. The performances in TEMPS CPU are satisfactory in comparison with the integration methods temporal of the Newmark family, because it is permissible to choose time step the coarser without degrading the solution excessively. On Linux platform, with

the “exact” methods of contact – Lagrange and continues – certain results are slightly different from those obtained on platforms Bull and Alphaserver: about % over energies, times of collision and the velocities starting from the 3rd rebound. The reactions are more stable. The method by discrete springs of shock seems less sensitive to the choice of the object computer, just as it - diagram of temporal integration θ . Contact Lagrange Contact continues



(time in S) of the kinetic energy (in) of the block according to the methods J on Bull machine, with a diagram of temporal integration implicit in displacement of the type Newmark-HHT (Lagrange method; method of penalization by springs; continuous method formulated in displacement), or with one - diagram of temporal integration θ implicit in displacement (method of contact of the Lagrange type).