SDNS107 – Transient response of a reinforced concrete slab: model with GRILLE_EXCENTRE

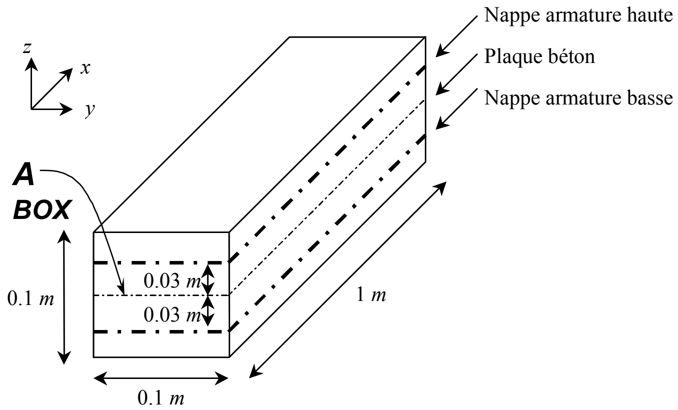
Summarized:

This valid test in transitory linear dynamics the modelization of square reinforced concrete plate using for the concrete a model of plate DKT and for reinforcements elements of grid-membrane GRILLE_EXCENTRE. One checks the frequencies of the eigen modes, the temporal responses in displacement, the reactions, and kinetic energy, for a sinusoidal loading.

1 Problem of reference

1.1 Geometry

the geometry used in this case test is a concrete plate reinforced with thickness $e=0.1\,\mathrm{m}$ and length $l=1\,\mathrm{m}$.



Appear 1.1-a : Studied geometry

the characteristics of the steel three-dimensions functions of the concrete plate reinforced are:

- Higher three-dimensions function: section per linear meter $=0.05\,m^2/ml$; eccentring compared to the average average: $+0.03\,m$,
- Lower Three-dimensions function: section per linear meter $=0.05\,m^2/ml$; eccentring compared to the average average: $-0.03\,m$.

1.2 Properties of the materials

the characteristics material for the multi-layer modelization concrete with steel reinforcements (DKT and GRILLE_EXCENTRE) are summarized in the table which follows.

Modelization	Modulus Young N/m^2	Poisson's ratio	Density kg/m^3
Concrete (plate DKT)	1. 1010	0.0	2500
Steel (GRILLE_EXCENTRE)	1. 1011	0.0	7800

1.3 Boundary conditions and loadings

On the side A ($B0X$) of the plate one embeds displacements $u_x=u_y=u_z=0$, as well as rotations $\theta_x=\theta_y=\theta_z=0$. During computation modal, displacement $u_y=0$ is blocked everywhere on the plate. A linear force is applied to the side $B1X$ (side opposed to $B0X$) in the direction $(0.0,0.0,1,0)$ and depends is worth $F_0=10^6\,N$.

In the case of dynamic computation, a linear force of sinusoidal form is applied to the side $B1X$ in the direction $(0.0,0.0,1.0)$. The frequency of sinusoidal is of $20\,Hz$. The period of the request is of $0,1\,s$.

1.4 Initial conditions

IN an initial state, displacements and the velocities are worth zero everywhere.

2 Reference solution

2.1 Method of calculating

It is possible to calculate the eigenfrequencies of the first and of the second modes of vibration of bending of the plate because it functions like a beam-cantilever.

The frequency f_1 of the first eigen mode is written:

$$f_1=\frac{3,5156}{2\pi L^2}\sqrt{\frac{EI}{\rho}}$$

with L the length of the cantilever (1 m here), EI the product of inertia of bending by the Young's modulus for complete structure and ρ mass of structure per unit of length.

Same way, the frequency f_2 of the second eigen mode is written:

$$f_2=\frac{22,0336}{2\pi L^2}\sqrt{\frac{EI}{\rho}}$$

To compute: f_1 and f_2, one breaks up the parts related on the concrete and reinforcements:

$$(EI)=(EI)_{beton}+(EI)_{acier}$$

where

$$(EI)_{acier}=2E_{acier}(sL_2)e_{exc}^2$$

with E_{acier} the Young modulus of steel, s the section of reinforcements per linear meter and e_{exc} the eccentring of the three-dimensions functions of reinforcements compared to the average average, and:

$$(EI)_{beton}=E_{beton}L_2\frac{e^3}{12}$$

where E_{beton} is the Young modulus of the concrete.

For the mass per unit of length, one breaks up the mass of the concrete and the mass of steel.

By means of the preceding equations, it then becomes possible to calculate the frequency of the eigen modes considered. The results are:

Frequency	**Reference**
First mode of bending	$54{,}67\,Hz$
Second mode of bending	$342{,}64\,Hz$

the center of gravity is located at the center of the cantilever. Its coordinates are thus: $(0.5, 0.05, 0)$. Inertia along the axis of complete structure is there:

$$I_{yy}(G)=\int_V ((x-x_G)^2+(z-z_G)^2)\,\rho\,dv$$

with (x_G, y_G, z_G) the coordinates of the center of gravity, V the volume of structure and ρ its density. By breaking up the elements related to the concrete and those related to steel, it is possible to calculate inertia analytically:

$$I_{yy}(G)=8{,}611\,m^4$$

2.2 Quantities and results of reference

the results of reference are recapitulated in the table which follows.

Quantities	**Reference**
First mode of bending	$54{,}67\,Hz$
Second mode of bending	$342{,}64\,Hz$
Inertia along the axis y	$8{,}611\,m^4$
Moment following z to $t=0{,}1\,s$	$-0.000779\,N/m$
following Displacement z to $t=0{,}09\,s$	$-9480.0\,m$
Displacement following z to $t=0{,}1\,s$	$3720.0\,m$
total Kinetic energy to $t=0{,}1\,s$	$9.895889\,J$

2.3 Uncertainties on the solution

analytical Solutions for the eigen modes.

Comparisons with EUROPLEXUS for the temporal responses in displacement, the reactions, and kinetic energy, for a sinusoidal loading

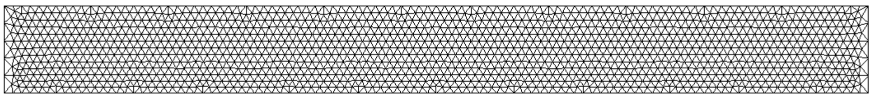
2.4 bibliographical References

[1] HUGHES T.J.R., COHEN Mr., HAROUN, Mr.: “Reduced and selective integration techniques in the finite element analysis of punts”, Nuclear Engineering and Design, vol. 46, p. 203-222 (1978). [R3.07

[2] .03] – shell elements DKT, DST, DKQ, DSQ and Q4g. Modelization

3 A Characteristic

3.1 of the modelization xyB0



Figure

X
B
1
X
B
0
Y
B
1
Y

3.1 - 3.1-a of the modelization A Modelization

: DKTG **Boundary conditions**

: Fixed support

- in, on $B0X$
- $DY=0.0$ the group of the beam. Temporal

integration: Diagram

- : NEWMARK , formulation: DEPLACEMENT , Time step
- : with $1.10^{-3} s$ possible subdivision until. Characteristics $1.10^{-5} s$

3.2 of the mesh Many

nodes: 1536, Number of meshes: elements TRI3 : 2860 , elements SEG2 : 210 .
Meshes are duplicated twice to affect the two grids of reinforcements. Quantities

3.3 tested and results Identification

Reference	Aster	% difference	Frequency
() First Hz mode 54.67	54.582	0.160	Frequency
() Third Hz mode 342.64	338.609	1.176	Position
center of gravity () G 0.05 m	0.05	0. Inertia	
() 8.611 I_{yy} G	8.6038	0.083	For

the transient analysis, one tests in various times (test of NON-regression): the average

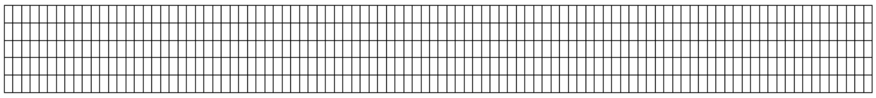
- of vertical displacements of the points of, $B1X$ the resultant
- of the nodal forces applying to, $B1X$ vertical
- nodal reaction to. A

Total kinetic energy is also tested (by comparison with the results provided by a loop Python). Identification

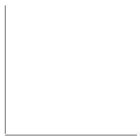
Reference	Aster	% Average	difference
of vertical displacements on (at the sequence number $B1X$ 100) –	7.79 10-4 –	7.7917 10 -4 0.022	vertical
Resultant of the applied forces on (at the sequence number $B1X$ 90) –	9.48 10+3 –	9.4833 10 + 3 0.035	vertical
nodal Reaction on (at the sequence number A 100) 3.72	10+3 3.7139	10 + 3 – 0.161	total
Kinetic energy (at the sequence number 100) 9.89588	9.902	0.062	Modelization

4 B Characteristic

4.1 of the modelization xyB0



Figure



4.1 - 4.1-a of the modelization B

the characteristics of the modelization B are identical to those of the modelization A, only the nature of the elements change (QUAD4 instead of SORTED 3). Temporal

integration: Diagram

- : NEWMARK formulation: DEPLACEMENT , Time step
- : with $1.10^{-3} s$ possible subdivision until. Characteristics $1.10^{-5} s$

4.2 of the mesh Many

nodes: 606, Number of meshes: elements QUA4 : 500 , elements SEG2 : 210 . Meshes are duplicated twice to affect the two grids of reinforcements. Quantities

4.3 tested and results Identification

Reference	Aster	*% difference*	Frequency
() First Hz mode 54.67	54.579	0.166	Frequency
() Third Hz mode 342.64	338.511	1.205	For

the transient analysis, one tests in various times (values compared with the modelization B): the average

of vertical displacements of the points of the resultant $B1X$
of the nodal forces applying to the vertical $B1X$
nodal reaction in ON position also A

tests total kinetic energy (by comparison with the results provided by a loop Python). Identification

Reference	Aster	*% Average*	difference
of vertical displacements on (at the sequence number $B1X$ 100) –	7.79 10-4 –	7.8112 10 -4 0.272	vertical
Resultant of the applied forces on (at the sequence number $B1X$ 90) –	9.48 10+3 –	9.4813 10 + 3 0.014	vertical
nodal Reaction on (at the sequence number A 100) 3.72	10+3 3.7413	10 + 3 0.574	total
Kinetic energy (at the sequence number 100) 9.89588	9.9020	0.06	Summary

5 of the results One finds

a light difference between the solutions obtained by the two computer codes. He is regarded as reasonable and the results are considered to be satisfactory. Even if in this test one wanted to approach as much as possible the modelization Code_Aster of that of the Europlexus code , one is conscious, in particular, of the difference on the level of computation of the mass matrix. The mass matrix implemented in Europlexus follows the method suggested in [1], which is adapted better for a computation software of fast dynamics explicit. Code_Aster uses a more standard approach, explained in [2]. The difference relates to in particular the computation of inertias (component of the mass matrix corresponding to the degrees of freedom of rotation), which are not very important in the case of the slow dynamics. On the other hand, in fast dynamics, it is they which influence more time step criticizes. For example, by neglecting them the mass matrix becomes singular and explicit integration becomes unconditionally unstable.

Various simulations suggested validate the use of modelization GRILLE_EXCENTRE `for` modal computations, explicit dynamics and implicit dynamics.

The values obtained are in agreement with the analytical solutions, when those are available.