HPLA100 - Heavy thermo-elastic the purpose of hollow roll in uniform rotation

Summarized

This test is testing the second members corresponding to the effects of gravity, a thermal thermal expansion and acceleration due to a uniform rotation. For the modelizations C and D (shell $3\mathrm{D}$), a chained thermoelastic computation and a thermoelastoplastic computation without plastic evolution were carried out.

One has the results for the modelizations:

$2\mathrm{D}$ axisymmetric: axisymmetric isoparametric finite elements on meshes QUAD8,

axisymmetric shells: axisymmetric isoparametric finite elements on meshes SEG3 (linear mesh of the meridian section),

shells $3\mathrm{D}$: finite elements MEC3QU9H, MEC3TR7H on meshes QUAD9 and TRIA7, respectively,

plates DKT : finite elements planes DKQ, DKT on meshes QUAD4 and TRIA3, respectively. One tests also the orthotropic shells of DEFI_COMPOSITE for two layers of the same isotropic material and key word ELAS_COQU of DEFI_MATERIAU for the assignment of characteristics of homogenized plates.

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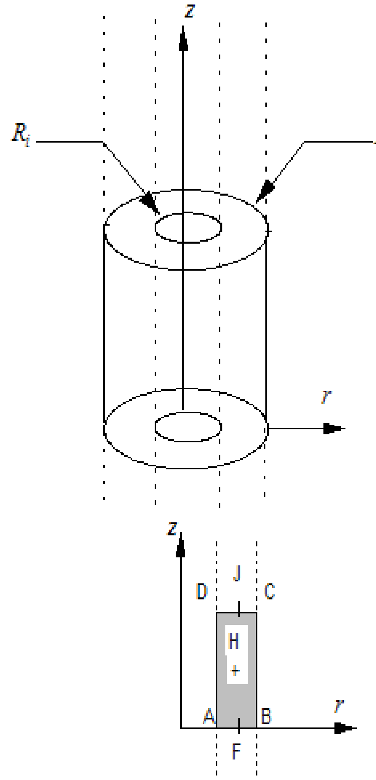
1 Problem of Geometrical

1.1 reference



Geometry of the cylinder (m) :

Interior radius	$R_i = 19.5$
Radius external	$R_e = 20.5$
Point F	$R = 20.0$
Thickness	$h = 1.0$
Height	$L = 10.0$

1.2 Properties of the material

the material is homogeneous isotropic, thermo-elastic linear, the initial state is virgin.

Young modulus	$E = 2.0\,E-5\,N.mm^{-2}$
voluminal	$\nu = 0.3$
Poisson's ratio Density	$\rho = 8.0\,E^{-6}\,kg.mm^{-3}$
Coefficient of thermal expansion	$\alpha = 1.0\,E^{-5}\,{}^\circ C^{-1}$

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1.3 Boundary conditions and loadings

imposed Displacement:

- $\Omega=1.0\,s^{-1}$ according to the axis OZ

imposed Loading:

- gravity, $g=10.0\,m.s^{-2}$ according to the axis OZ
- tensile force on the upper face: $-160.0\,E^{-4}\,N$ equivalent with a distributed force on CD $-8.0\,E^{-4}\,N.mm^{-1}$

thermal Thermal expansion: $T(\rho)-T_{ref}(\rho)=\frac{(T_s+T_i)}{2}+\frac{(T_s-T_i).(r-R)}{h}$ with:

- case 1: $T_s=0.5\,°C, T_i=-0.5\,°C, T_{ref}=0.0\,°C$
- case 2: $T_s=0.1\,°C, T_i=0.1\,°C, T_{ref}=0.0\,°C$

These fields of temperature are calculated with THER_LINEAIRE, using a steady computation on the same mesh, but with a PLANE model in order to have a solution closely connected in the thickness.

The boundary conditions in displacement (and rotation) being different according to the modelization considered, they will be described later on (in the paragraphs relating to the modelizations).

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2 Reference solution

2.1 Méthode de calcul used for the reference solution

the méthode de calcul used for the reference solution was determined by F. Voldoire (EDF R & D/AMA) and is presented in the appendix.

The analytical results of reference are:

displacements and rotations,
axial stress, forces generalized (in theory of shells),

in skins internal and external on the sections AB and CD.

In 2D axisymmetric, the complete solutions given in appendix are such as $\varepsilon_{rz}=0$ oùetsont r z the directions radial and axial of the cylinder, respectively. For the loadings of uniform rotation and thermal thermal expansion, the boundary conditions are selected so that the solutions do not depend on z (one has in particular $\varepsilon_{zz}=0$).

For the shells, with equivalent boundary conditions, rotation θ_θ around the axis orthoradial is null for the loadings of uniform rotation and thermal thermal expansion, which is not the case of the loading of gravity where rotation is constant (the cylinder is formatted conical then). On the other hand in all the cases, the transverse distortion is null; thus the theories of Coils - Kirchhoff and those of Hencky-Mindlin provide the same reference solution.

2.2 Reference variable

DX : displacement according to the axis OX
DY : displacement according to the axis OY
DZ : displacement according to the axis OZ
NXX : normal force according to the axis OX
NYY : normal force according to the axis OY
MXX : moment around the axis OX
MYY : moment around the axis OY
$SIXX$: stress according to the axis OX
$SIYY$: stress according to the axis OY.

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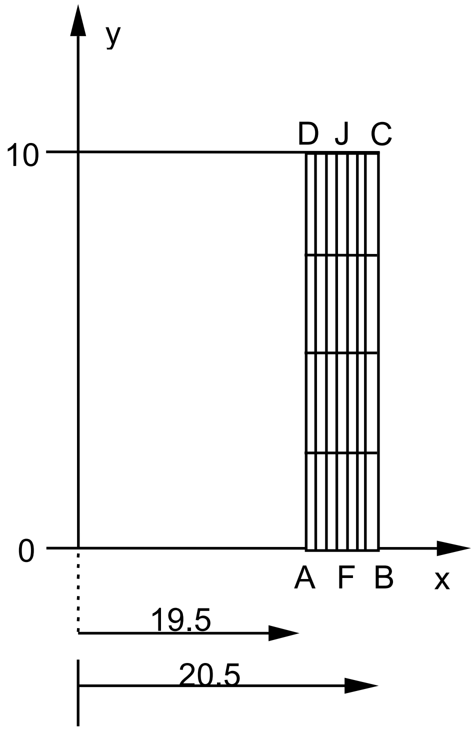
3 Modelization A

3.1 Characteristic of the modelization

Finite elements 2D axisymmetric



the discretized geometry is represented above:

Bord	*group _ no*
BC	*BC*
DA	*DA*
AB	*BAS*
CD	*HAUT*

3.2 Characteristics of the mesh

The mesh is regular: 4 elements in the height, 8 in the thickness.

Many nodes: 121

Number of meshes and type: 32 QUAD8

3.3 Boundary conditions in Gravity

3.3.1 displacement

displacement DY are blocked at the point F alone.

3.3.2 Rotation

displacement DY is blocked on the sides $[AB]$ (GROUP_NO = "LOW") and on $[CD]$ (GROUP_NO = "HIGH").

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3.3.3 Thermal case of thermal expansion n°1

displacement DY is blocked on all structure.

3.3.4 Thermal case of thermal expansion n°2

displacement DY is blocked on the sides $[AB]$ (GROUP_NO = "LOW") and $[CD]$ (GROUP_NO = "HIGH").

3.4 Results of the modelization A

Identification	Node (Mesh)	Value tested	Reference	
Gravity	$N78$	DX (mm)	– 2.34000	10-8
	$N120$	DY (mm)	– 1.185	10-9
	$N13$	DY (mm)	1.2150	10-9
	$N78(M13)$	$SIYY$ (MPa)	8.0000	10-4
uniform Rotation - centrifugal force	$N120$	DX (mm)	– 2.94240	10-7
	$N13$	DX (mm)	2.88010	10-7
	$N120(M1)$	$SIYY$ (MPa)	9.94880	10-4
	$N13(M32)$	$SIYY$ (MPa)	9.26310	10-4
Thermal expansion cases 1	$N120$	DX (mm)	1.056145	10-6
	$N13$	DX (mm)	1.110317	10-6
	$N120(M1)$	$SIYY$ (MPa)	1.4321427	
Thermal expansion cases 2	$N120$	DX (mm)	2.53500	10-5
	$N120(M1)$	$SIYY$ (MPa)	– 2.00000	10-1

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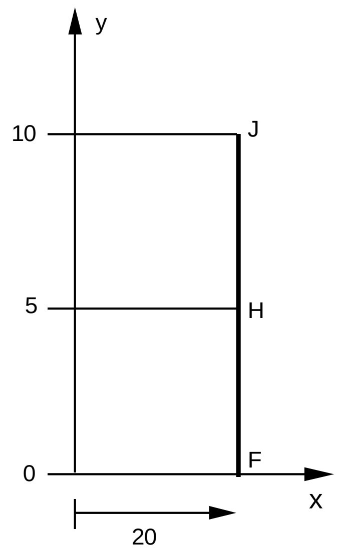
4 Modelization B

4.1 Characteristic of the axisymmetric

modelization Shell elements



the discretized geometry is represented above. One chooses the theory of shells of Coils - Kirchhoff (for that one takes a transverse shear coefficient of 106). One neglects the correction of metric in the thickness. The thickness is of $1\,mm$.

node	GROUP_NO
J	$GRNO13$
H	$GRNO14$
F	$GRNO6$

4.2 Characteristics of the mesh

Many nodes: 21

Number of meshes and type: 10 SEG3

4.3 Boundary conditions in displacement and rotation

4.3.1 Gravity

displacement DY are blocked at the point F alone ($GRNO6$).

4.3.2 Rotation

displacement DY is blocked at the point F ($GRNO6$) at the point J ($GRNO13$).
Rotation around the axis Z is null in these two points.

4.3.3 Thermal case of thermal expansion n°1

displacement DY as well as rotation around the axis Z is blocked on all structure (GROUP_NO = "GRNO15").

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4.3.4 Thermal case of thermal expansion n°2

displacement DY is blocked at the point F (`GROUP_NO = "GRNO06"`) and at the point J (`GROUP_NO = "GRNO13"`). Rotation around the axis Z is null on the same nodes groups.

4.4 Results of the modelization B

Identification	Node (Mesh)	Value tested	Reference
Gravity	J	$DX(mm)$	– 2.40000 10-8
	H	$DY(mm)$	5.00000 10-9
	H	DRZ	2.40000 10-9
	$J(M10)$	$NXX(N)$	8.00000 10-4
	$J(M10)$ Skin interns	$SIXX(MPa)$	8.00000 10-4
Rotation - centrifugal force	F	$DX(mm)$	2.91200 10-7
	$F(M1)$	$NXX(N)$	9.60000 10-4
	$F(M1)$ Skin interns	$SIXX(MPa)$	9.60000 10-4
Thermal expansion cases 1	$F(M1)$	$MXX(N.mm)$	– 2.38095 10-1
	$F(M1)$ Skin interns	$SIXX(Mpa)$	1.428571
Thermal expansion cases 2	F	$DX(mm)$	26.0000 10-6
	$F(M1)$	$NXX(N)$	– 2.00000 10-1
	$F(M1)$ internal Skin	$SIXX(MPa)$	– 2.00000 10-1

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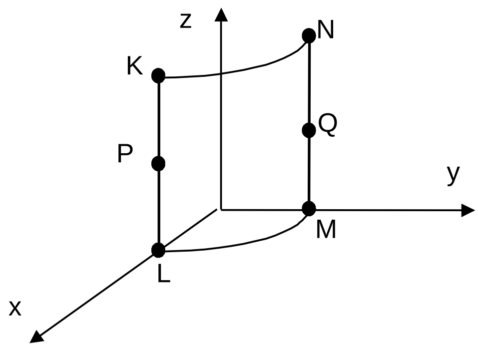
5 Modelization C

5.1 Characteristic of the modelization

Modelization of a quarter of cylinder.

Shell elements 3D: QUAD9



the discretized geometry is represented above.

point	*nœud*
K	$NO72$
L	$NO1$
M	$NO33$
N	$NO39$
P	$NO186$
Q	$NO190$

The factor of correction of shears A_{CIS} is worth $5/6$ (theory of shells of Reissner).

5.2 Characteristics of the mesh

Many external nodes: 121

Number of meshes and types: 32 QUAD9 + 8 SEG3

5.3 Boundary conditions in displacement and rotation

5.3.1 Gravity

displacement DZ is blocked on the nodes group LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .

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5.3.2 Rotation

displacement DZ as well as rotations around the axes X and Y are blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

5.3.3 Thermal case of thermal expansion n°1

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

5.3.4 Thermal case of thermal expansion n°2

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

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5.4 Results of the modelization C

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX\ (mm)$	– 2.40000 10-8
	N	$DY\ (mm)$	– 2.40000 10-8
	P	$DZ\ (mm)$	5.00000 10-9
	Q	$DZ\ (mm)$	5.00000 10-9
	P	DRY	2.40000 10-9
	Q	DRX	2.40000 10-9
	$K(M4)$	$NYY\ (N)$	8.00000 10-4
	$N(M32)$	$NYY\ (N)$	8.00000 10-4
	$K(M4)$ Skin interns	$SIYY\ (MPa)$	8.00000 10-4
	$N(M32)$ Skin interns	$SIYY\ (MPa)$	8.00000 10-4
Rotation – centrifugal force	L	$DX\ (mm)$	2.91200 10-7
	M	$DY\ (mm)$	2.91200 10-7
	$L(M1)$	$NYY\ (N)$	9.60000 10-4
	$M(M29)$	$NYY\ (N)$	9.60000 10-4
	$L(M1)$ Skin interns	$SIYY\ (MPa)$	9.84600 10-4
	$M(M29)$ Skin interns	$SIYY\ (MPa)$	9.84600 10-4
Thermal expansion cases 1	$L(M1)$	$MYY\ (N.mm)$	– 2.38095 10-1
	$M(M29)$	$MYY\ (N.mm)$	– 2.38095 10-1
	$L(M1)$ Skin interns	$SIYY\ (MPa)$	1.428571
	$M(M29)$ Skin interns	$SIYY\ (MPa)$	1.428571
Thermal expansion cases 2	L	$DX\ (mm)$	25.9946 10-6
	M	$DY\ (mm)$	25.9946 10-6
	$L(M1)$	$NYY\ (N)$	– 2.00000 10-1
	$M(M29)$	$NYY\ (N)$	– 2.00000 10-1
	$L(M1)$ internal Skin	$SIYY\ (MPa)$	– 1.97800 10-1
	$M(M29)$ internal Skin	$SIYY\ (MPa)$	– 2.97800 10-1

5.5 Remarks

satisfactory and identical Results for computations with `MECA_STATIQUE` and `STAT_NON_LINE`.

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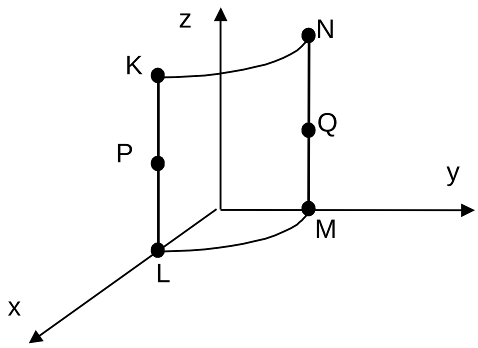
6 Modelization D

6.1 Characteristic of the modelization

Modelization of a quarter of cylinder.

Shell elements 3D: `TRIA7`



the discretized geometry is represented above.

point	*nœud*
K	*NO72*
L	*NO1*
M	*NO33*
N	*NO39*
P	*NO186*
Q	*NO190*

The factor of correction of shears A_{CIS} is worth $5/6$ (theory of Reissner shells).

6.2 Characteristics of the mesh

Many external nodes: 153

Number of meshes and types: 64 `TRIA7` + 8 `SEG3`

6.3 Boundary conditions in displacement and rotation

6.3.1 Gravity

displacement DZ is blocked on the nodes group LM.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.

6.3.2 Rotation

displacement DZ as well as rotations around the axes X and Y are blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.

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Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

6.3.3 Thermal case of thermal expansion n°1

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

6.3.4 Thermal case of thermal expansion n°2

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

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6.4 Values tested

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	– 2.40000 10-8
	N	$DY(mm)$	– 2.40000 10-8
	P	$DZ(mm)$	5.00000 10-9
	Q	$DZ(mm)$	5.00000 10-9
	P	$-DRY$	2.40000 10-9
	Q	DRX	2.40000 10-9
	$K(M60)$	$NYY(N)$	8.00000 10-4
	$N(M56)$	$NYY(N)$	8.00000 10-4
	$K(M60)$ Skin interns	$SIYY(MPa)$	8.00000 10-4
	$N(M56)$ Skin interns	$SIYY(MPa)$	8.00000 10-4
Rotation – centrifugal force	L	$DX(mm)$	2.91200 10-7
	M	$DY(mm)$	2.91200 10-7
	$L(M25)$	$NYY(N)$	9.60000 10-4
	$M(M53)$	$NYY(N)$	9.60000 10-4
	$L(M25)$ Skin interns	$SIYY(MPa)$	9.84600 10-4
	$M(M53)$ Skin interns	$SIYY(MPa)$	9.84600 10-4
Thermal expansion cases 1	$L(M25)$	$MYY(N.mm)$	– 2.38095 10-1
	$M(M53)$	$MYY(N.mm)$	– 2.38095 10-1
	$L(M25)$ Skin interns	$SIYY(MPa)$	1.428571
	$M(M53)$ Skin interns	$SIYY(MPa)$	1.428571
Thermal expansion cases 2	L	$DX(mm)$	26.0000 10-6
	M	$DY(mm)$	26.0000 10-6
	$L(M25)$	$NYY(N)$	– 2.00000 10-1
	$M(M53)$	$NYY(N)$	– 2.00000 10-1
	$L(M25)$ internal Skin	$SIYY(MPa)$	– 1.97800 10-1
	$M(M53)$ internal Skin	$SIYY(MPa)$	– 1.97800 10-1

6.5 Remarks

satisfactory and identical Results for computations with `MECA_STATIQUE` and `STAT_NON_LINE`.

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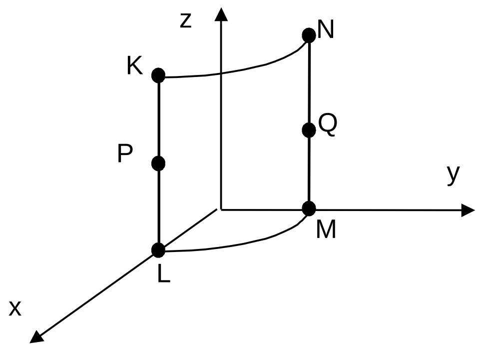
7 Modelization E

7.1 Characteristic of the modelization

Modelization of a quarter of cylinder.

Shell elements DKQ : QUAD4



the discretized geometry is represented above.

not	node
K	$NO160$
L	$NO203$
M	$NO11$
N	$NO1$
P	$NO226$
Q	$NO6$

7.2 Characteristics of the mesh

Many nodes: 231

Number of meshes and types: 200 QUAD4 + 80 SEG2

7.3 Boundary conditions in displacement and rotation

7.3.1 Gravity

displacement DZ is blocked on the nodes group LM.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.

7.3.2 Thermal case of thermal expansion n°1

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.

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Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

7.3.3 Thermal case of thermal expansion n°2

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

7.4 Results of the modelization E

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	– 2.40000 10-8
	N	$DY(mm)$	– 2.40000 10-8
	P	$DZ(mm)$	5.00000 10-9
	Q	$DZ(mm)$	5.00000 10-9
	P	$-DRY$	2.40000 10-9
	Q	DRX	2.40000 10-9
	$K(181)$	$NYY(N)$	8.00000 10-4
	$N(M200)$	$NYY(N)$	8.00000 10-4
	$K(M181)$ Skin interns	$SIYY(MPa)$	8.00000 10-4
	$N(M200)$ Skin interns	$SIYY(MPa)$	8.00000 10-4
Thermal expansion cases 1	$L(M1)$	$MYY(N.mm)$	– 2.38095 10-1
	$M(M20)$	$MYY(N.mm)$	– 2.38095 10-1
	$L(M1)$ Skin interns	$SIYY(MPa)$	1.428571
	$M(M20)$ Skin interns	$SIYY(MPa)$	1.428571
Thermal expansion cases 2	L	$DX(mm)$	26.0000 10-6
	M	$DY(mm)$	26.0000 10-6
	$L(M1)$	$NYY(N)$	– 2.00000 10-1
	$M(M20)$	$NYY(N)$	– 2.00000 10-1
	$L(M1)$ internal Skin	$SIYY(MPa)$	– 2.00000 10-1
	$M(M20)$ internal Skin	$SIYY(MPa)$	– 2.00000 10-1

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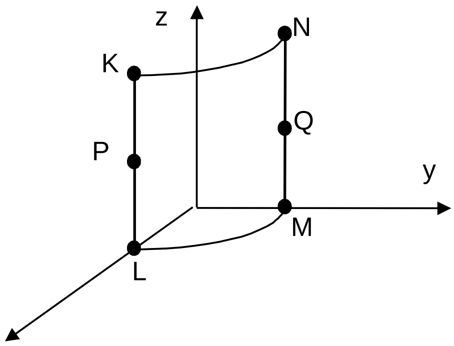
8 Modelization F

8.1 Characteristic of the modelization

Modelization of a quarter of cylinder.

Shell elements DKT : TRIA3



the discretized geometry is represented above.

not	node
K	$NO1$
L	$NO11$
M	$NO161$
N	$NO227$
P	$NO6$
Q	$NO215$

8.2 Characteristics of the mesh

Many nodes: 231

Number of meshes and type: 400 TRIA3 + 80 SEG2

8.3 Boundary conditions in displacement and rotation

8.3.1 Gravity

displacement DZ is blocked on the nodes group LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .

8.3.2 Thermal case of thermal expansion n°1

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .

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Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

8.3.3 Thermal case of thermal expansion n°2

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

8.4 Results of the modelization F

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	– 2.40000 10-8
	N	$DY(mm)$	– 2.40000 10-8
	P	$DZ(mm)$	5.00000 10-9
	Q	$DZ(mm)$	5.00000 10-9
	$-DRX$	$-DRY$	2.40000 10-9
	DRX	DRX	2.40000 10-9
	$K(M362)$	$NYY(N)$	8.00000 10-4
	$N(M400)$	$NYY(N)$	8.00000 10-4
	$L(M362)$ Skin interns	$SIYY(MPa)$	8.00000 10-4
	$N(M400)$ Skin interns	$SIYY(MPa)$	8.00000 10-4
Thermal expansion cases 1	$L(M1)$	$MYY(N.mm)$	– 2.38095 10-1
	$M(M39)$	$MYY(N.mm)$	– 2.38095 10-1
	$L(M1)$ Skin interns	$SIXX(MPa)$	1.428571
	$M(M39)$ Skin interns	$SIXX(MPa)$	1.428571
Thermal expansion cases 2	L	$DX(mm)$	26.0000 10-6
	M	$DY(mm)$	26.0000 10-6
	$L(M1)$	$NYY(N)$	– 2.00000 10-1
	$M(M39)$	$NYY(N)$	– 2.00000 10-1
	$L(M1)$ internal Skin	$SIYY(MPa)$	– 2.00000 10-1
	$M(M39)$ internal Skin	$SIYY(MPa)$	– 2.00000 10-1

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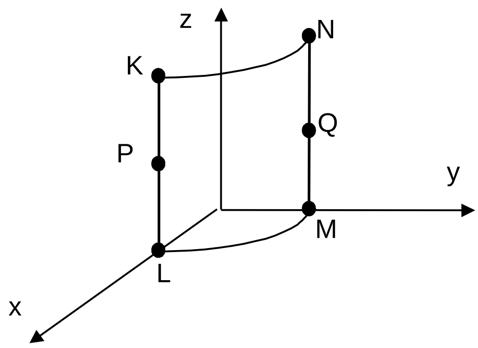
9 Modelization G

9.1 Characteristic of the modelization

Modelization of a quarter of cylinder.

Shell elements DKT : TRIA3. The modelization uses a double-layered plate whose characteristics of orthotropy are those of the material defined in $\S 1.2$.



The discretized geometry is represented above.

not	node
K	$NO1$
L	$NO11$
M	$NO161$
N	$NO227$
P	$NO6$
Q	$NO215$

9.2 Characteristics of the mesh

Many nodes: 231

Number of meshes and types: 400 TRIA3 + 80 SEG2

9.3 Boundary conditions in displacement and rotation

9.3.1 Gravity

displacement DZ is blocked on the nodes group LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .

9.3.2 Thermal case of thermal expansion n°1

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .

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Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

9.3.3 Thermal case of thermal expansion n°2

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

9.4 Results of the modelization G

Identification	Node (mesh)	Value tested	Reference
Gravity	K	DX	– 2.40000 10-8
	N	DY	– 2.40000 10-8
	P	DZ	5.0 10-9
	Q	DZ	5.0 10-9
	P	$-DRY$	2.40000 10-9
	Q	DRX	2.40000 10-9
	$K(M362)$	NYY	8.00000 10-4
	$N(M400)$	NYY	8.00000 10-4
	$K(M362)$	$SIYY$	8.00000 10-4
	$N(M400)$	$SIYY$	8.00000 10-4
Thermal expansion cases 1	$L(M1)$	MYY	– 2.38095 10-1
	$M(M39)$	MYY	– 2.38095 10-1
	$L(M1)$, skin interns	$SIXX$	1.428571
	$L(M1)$, external skin	$SIXX$	– 1.428571
	$M(M39)$, skin interns	$SIXX$	1.428571
	$M(M39)$, external skin	$SIXX$	– 1.428571
Thermal expansion cases 2	L	DX	26.0 10-6
	M	DY	26.0 10-6
	$L(M1)$	NYY	– 2.00000 10-1
	$M(M39)$	NYY	– 2.00000 10-1
	$L(M1)$	$SIYY$	– 2.00000 10-1
	$M(M39)$	$SIYY$	– 2.00000 10-1

10 Modelization H

10.1 Characteristic of the modelization

Modelization of a quarter of cylinder.

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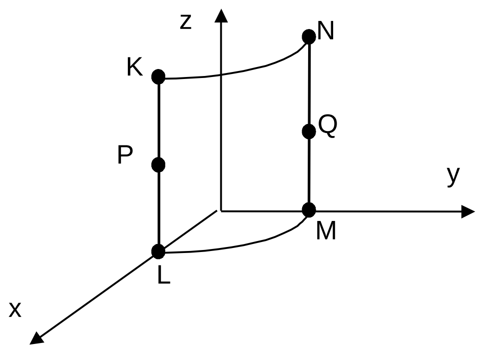
Shell elements DKQ : QUAD4. The modelization uses a double-layered plate whose characteristics of orthotropy are those of the material defined in $\S 1.2$.



The discretized geometry is represented above.

not	node
K	$NO160$
L	$NO203$
M	$NO11$
N	$NO1$
P	$NO226$
Q	$NO6$

10.2 Characteristics of the mesh

Many external nodes: 231

Number of meshes and types: 200 QUAD4 + 80 SEG2

10.3 Boundary conditions in displacement and rotation

10.3.1 Gravity

displacement DZ is blocked on the nodes group LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .

10.3.2 Thermal case of thermal expansion n°1

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

10.3.3 Thermal case of thermal expansion n°2

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displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

10.4 Results of the modelization H

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	– 2.40000 10-8
	N	$DY(mm)$	– 2.40000 10-8
	P	$DZ(mm)$	5.00000 10-9
	Q	$DZ(mm)$	5.00000 10-9
	P	$-DRX$	2.40000 10-9
	Q	DRX	2.40000 10-9
	$K(M181)$	$NYY(N)$	8.00000 10-4
	$N(M200)$	$NYY(N)$	8.00000 10-4
	$K(M181)$	$SIYY(Pa)$	8.00000 10-4
	$N(M200)$	$SIYY(Pa)$	8.00000 10-4
Thermal expansion cases 1	$L(M1)$	$MYY(N.mm)$	– 2.38095 10-1
	$M(M20)$	$MYY(N.mm)$	– 2.38095 10-1
	$L(M1)$, skin interns	$SIYY(Pa)$	1.428571
	$L(M1)$, external skin	$SIYY(Pa)$	– 1.428571
	$M(M20)$, skin interns	$SIYY(Pa)$	1.428571
	$M(M20)$, external skin	$SIYY(Pa)$	– 1.428571
Thermal expansion cases 2	L	$DX(mm)$	26.0000 10-6
	M	$DY(mm)$	26.0000 10-6
	$L(M1)$	$NYY(N)$	– 2.00000 10-1
	$M(M20)$	$NYY(N)$	– 2.00000 10-1
	$L(M1)$	$SIYY(Pa)$	– 2.00000 10-1
	$M(M20)$	$SIYY(Pa)$	– 2.00000 10-1

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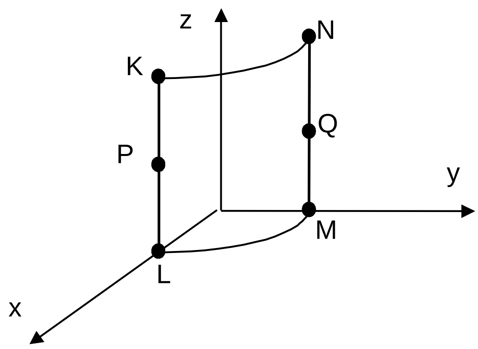
11 Modelization I

11.1 Characteristic of the modelization

Modelization of a quarter of cylinder.

Shell elements `DKQ` : `QUAD4`. Assignment of characteristics of homogenized plates corresponding to the material of $\S 1.2$.



The discretized geometry is represented above.

not	node
K	$NO160$
L	$NO203$
M	$NO11$
N	$NO1$
P	$NO226$
Q	$NO6$

11.2 Characteristics of the mesh

Many external nodes: 231

Number of meshes and types: 200 `QUAD4` + 80 `SEG2`

11.3 Boundary conditions in displacement and rotation

11.3.1 Gravity

displacement DZ is blocked on the nodes group LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .

11.3.2 Thermal case of thermal expansion n°1

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .

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Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

11.3.3 Thermal case of thermal expansion n°2

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

11.4 Results of the modelization I

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	– 2.40000 10-8
	N	$DY(mm)$	– 2.40000 10-8
	P	$DZ(mm)$	5.00000 10-9
	Q	$DZ(mm)$	5.00000 10-9
	P	$-DRX$	2.40000 10-9
	Q	DRX	2.40000 10-9
	$K(M181)$	$NYY(N)$	8.00000 10-4
	$N(M200)$	$NYY(N)$	8.00000 10-4
Thermal expansion cases 1	$L(M1)$	$MYY(N.mm)$	– 2.38095 10-1
	$M(M20)$	$MYY(N.mm)$	– 2.38095 10-1
Thermal expansion cases 2	L	$DX(mm)$	26.0000 10-6
	M	$DY(mm)$	26.0000 10-6
	$L(M1)$	$NYY(N)$	– 2.00000 10-1
	$M(M20)$	$NYY(N)$	– 2.00000 10-1

11.5 Remarks

the local stresses are not calculated since the characteristics material of the plate are supposed to come from a computation of homogenization.

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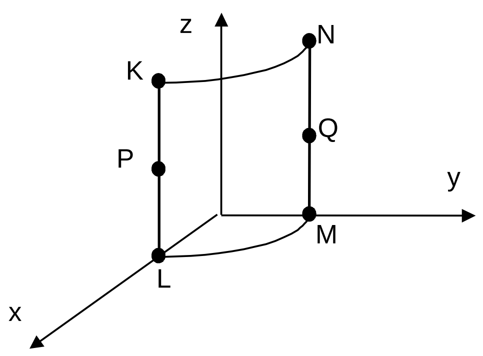
12 Modelization J

12.1 Characteristic of the modelization

Modelization of a quarter of cylinder.

Shell elements DKT : TRIA3. Assignment of characteristics of homogenized plates corresponding to the material of $\S 1.2$.



The discretized geometry is represented above.

not	node
K	$NO1$
L	$NO11$
M	$NO161$
N	$NO227$
P	$NO6$
Q	$NO215$

12.2 Characteristics of the mesh

Many nodes: 231

Number of meshes and types: 400 TRIA3 + 80 SEG2

12.3 Boundary conditions in displacement and rotation

12.3.1 Gravity

displacement DZ is blocked on the nodes group LM .
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN .

12.3.2 Thermal case of thermal expansion n°1

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL .

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Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

12.3.3 Thermal case of thermal expansion n°2

displacement DZ as well as rotations around the axes X and Y is blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

12.4 Results of the modelization J

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	– 2.40000 10-8
	N	$DY(mm)$	– 2.40000 10-8
	P	$DZ(mm)$	5.0 10-9
	Q	$DZ(mm)$	5.0 10-9
	P	$-DRY$	2.40000 10-9
	Q	DRX	2.40000 10-9
	$K(M362)$	$NYY(N)$	8.00000 10-4
	$N(M400)$	$NYY(N)$	8.00000 10-4
Thermal expansion cases 1	$L(M1)$	$MYY(N.mm)$	– 2.38095 10-1
	$M(M39)$	$MYY(N.mm)$	– 2.38095 10-1
Thermal expansion cases 2	L	$DX(mm)$	26.0 10-6
	M	$DY(mm)$	26.0 10-6
	$L(M1)$	$NYY(N)$	– 2.00000 10-1
	$M(M39)$	$NYY(N)$	– 2.00000 10-1

12.5 Remarks

the local stresses are not calculated since the characteristics material of the plate are supposed to come from a computation of homogenization.

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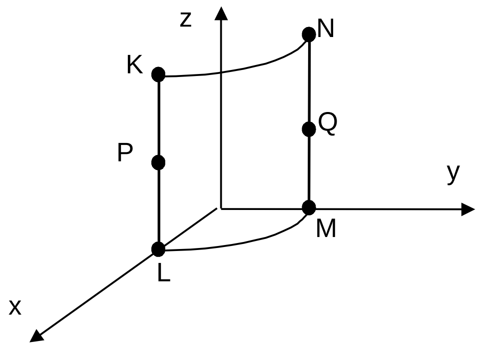
13 Modelization K

13.1 Characteristic of the modelization

Identical to the modelization E with elements DKQG (modelization DKTG).
The purpose of this modelization is testing the taking into account of the temperature for elements DKQG.

Shell elements DKQG : QUAD4



the discretized geometry is represented above.

not	node
K	$NO160$
L	$NO203$
M	$NO11$
N	$NO1$
P	$NO226$
Q	$NO6$

13.2 Characteristics of the mesh

Many nodes: 231

Number of meshes and types: 200 QUAD4 + 80 SEG2

13.3 Boundary conditions in displacement and rotation

13.3.1 Gravity

formula DZ is blocked on the nodes group LM.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.

13.3.2 Thermal case of thermal expansion n°1

displacement DZ as well as rotations around the axes X and Y are blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.

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Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

13.3.3 Thermal case of thermal expansion n°2

displacement DZ as well as rotations around the axes X and Y are blocked on the nodes groups $KNSANSKN$ and $LMSANSLM$.
Displacement DY as well as rotations around the axes X and Z are blocked on the nodes group KL.
Displacement DX as well as rotations around the axes Y and Z are blocked on the nodes group MN.
Displacement DZ as well as rotation around the axis Y are blocked on the nodes group $KETL$.
Displacement DZ as well as rotation around the axis X are blocked on the nodes group $METN$.

13.4 Results of the modelization K

Identification	Node (mesh)	Value tested	Reference
Gravity	K	$DX(mm)$	– 2.40000 10-8
	N	$DY(mm)$	– 2.40000 10-8
	P	$DZ(mm)$	5.00000 10-9
	Q	$DZ(mm)$	5.00000 10-9
	P	$-DRY$	2.40000 10-9
	Q	DRX	2.40000 10-9
	$K(181)$	$NYY(N)$	8.00000 10-4
	$N(M200)$	$NYY(N)$	8.00000 10-4
Thermal expansion cases 1	$L(M1)$	$MYY(N.mm)$	– 2.38095 10-1
	$M(M20)$	$MYY(N.mm)$	– 2.38095 10-1
Thermal expansion cases 2	L	$DX(mm)$	26.0000 10-6
	M	$DY(mm)$	26.0000 10-6
	$L(M1)$	$NYY(N)$	– 2.00000 10-1
	$M(M20)$	$NYY(N)$	– 2.00000 10-1

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14 Summary of the results

the very good results got for the modelizations A and B are explained by the fact why the reference solutions belong to the space generated by the selected finite elements. Only remain the numerical rounding errors.

Satisfactory results were got for the modelizations of plates and shells in space C, D, E, F, G, H, I, J and K. For these last, a chained thermoelastic computation was carried out. The results with modelization DKT (E, F, G, H, I, J) show that the elements quadrangle have a better behavior than the elements triangle. It is necessary to have a sufficiently fine discretization with these plane elements in order to be able to correctly model the circular geometry of the cylindrical shell. Indeed, to discretize the geometry of the cylinder by plane or parabolic facets is not in conformity and induces a parasitic bending which decreases with the smoothness of mesh. Thus a multiplication amongst elements by two on the height of structure makes fall the maximum error relative of 5,48% (case presented here) to 2,8%. The results with modelization COQUE_3D (C and D) are very good except for gravity with the element triangle.
Computations with modelization DKTG (K) give the same results as with modelization DKT.

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15 Appendix

15.1 uniform Loading of rotation around OZ

15.1.1 Model 2D axisymmetric

the density of centrifugal force is: $\rho\,\Omega^2 r\, e_r$.

The following boundary conditions are considered:

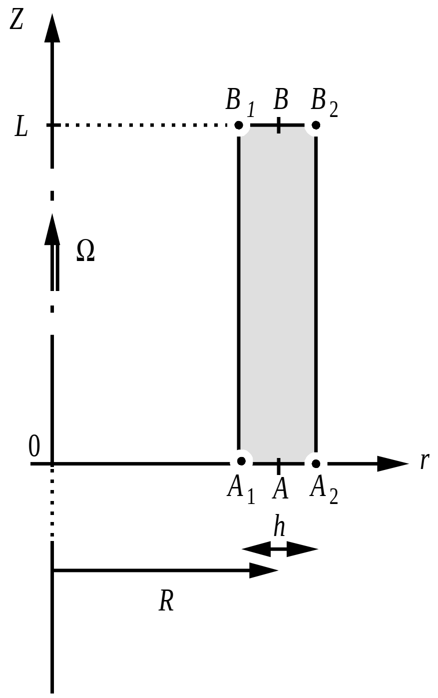
$u_z(r,z)=0$ in $z=0$ and $z=L$

One applies displacement in the form:

$$u_r=u(r)$$
$$u_z=u_\theta=0$$

As follows:

$$\varepsilon_{rr}=u'\,;\qquad \varepsilon_{\theta\theta}=\frac{u}{r}\,;\qquad \varepsilon_{zz}=\varepsilon_{rz}=\varepsilon_{\theta z}=\varepsilon_{r\theta}=0$$



Geometry of the hollow roll:

$$R=20\,mm$$
$$h=1\,mm$$

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The elastic stresses are expressed:

$$\sigma_{rr}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)u'+\nu\frac{u}{r}]$$
$$\sigma_{\theta\theta}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{u}{r}+\nu u']$$
$$\sigma_{zz}=\frac{\nu E}{(1+\nu)(1-2\nu)}[\frac{u}{r}+u']$$

The balance equation radial is written:

$$(r\sigma_{rr})_{,r}-\sigma_{\theta\theta}=-\rho\Omega^2 r^2$$

As follows:

$$(\frac{(ru)'}{r})'=\frac{-(1+\nu)(1-2\nu)}{(1-\nu)E}\rho\Omega^2 r \qquad éq\,1.1-1$$

Note:

$$\frac{u}{r}+u'=\frac{(ru)'}{r}$$

From where the general solution:

$$u(r)=\frac{-(1+\nu)(1-2\nu)}{(1-\nu)E}\rho\Omega^2\frac{r^3}{8}+Ar+\frac{B}{r} \qquad éq\,1.1-2$$

The stresses are then:

$$\sigma_{rr}(r)=\frac{-3-2\nu}{1-\nu}\rho\Omega^2\frac{r^2}{8}+\frac{E}{(1+\nu)(1-2\nu)}(A-(1-2\nu)\frac{B}{r^2})$$
$$\sigma_{\theta\theta}(r)=\frac{-1+2\nu}{1-\nu}\rho\Omega^2\frac{r^2}{8}+\frac{E}{(1+\nu)(1-2\nu)}(A-(1-2\nu)\frac{B}{r^2}) \qquad éq\,1.1-3$$
$$\sigma_{zz}(r)=\frac{-\nu}{1-\nu}\rho\Omega^2\frac{r^2}{2}+\frac{2\nu E}{(1+\nu)(1-2\nu)}A$$

The boundary conditions in stresses are:

$\sigma_{rr}=0$ in $r=R\pm\frac{h}{2}$

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One notes:

$$x=\frac{h}{2\mathrm{R}}$$

One obtains thanks to $[\acute{e}q\,1.1-3]$:

$$B=\frac{(3-2\nu)(1+\nu)}{8(1-\nu)E}\rho\,\Omega^2 R^4(1-x^2)^2$$

then:

$$A=\frac{(3-2\nu)(1+\nu)(1-2\nu)}{4(1-\nu)E}\rho\,\Omega^2 R^2(1-x^2)$$

Numerical application:

$$\rho=8.10-6\ kg/mm^3$$
$$\Omega=1\ s^{-1}$$
$$E=2.105\,N/mm^2$$
$$\nu=0.3$$

From where: $A=7.13588.10^{-9}\,mm^2$
$B=3.561258.10^{-6}\ mm^2$

Note:

$$\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}\rho\frac{\Omega^2}{8}=3.714286\ E-12\ mm^2$$
$$\frac{\nu}{1-\nu}\rho\frac{\Omega^2}{2}=1.714286\ E-6\ MPa.mm^2$$

As follows:

in intern skin: $\begin{cases}u_r=2.9424\,E^{-7}\,mm\\ \sigma_{zz}=0.99488\,E^{-3}\,Mpa\end{cases}$

in external skin: $\begin{cases}u_r=2.8801\,E^{-7}\,mm\\ \sigma_{zz}=0.92631\,E^{-3}\,Mpa\end{cases}$

15.1.2 Model axisymmetric shell

the centrifugal force is equivalent to **a distributed pressure** :

$$p=\rho\,\Omega^2 hR(1+\frac{h^2}{12\mathrm{R}^2})$$

The solution is membranous, the normal equilibrium is written:

$$N_{\theta\theta}=pR$$

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The membrane strain is: $E_{\theta\theta}=\frac{w}{R}$, whereas $E_{\theta\theta}=0=K_{\theta\theta}=K_{zz}$.

In elasticity:

$$N_{\theta\theta}=\frac{Eh}{1-\nu^2}E_{\theta\theta}\ ;\ N_{zz}=\nu N_{\theta\theta}\ ;\ M_{\alpha\beta}=0$$

From where the solution (deflection and circumferential normal force):

$$w=\frac{(1-\nu^2)\rho\Omega^2}{E}R^3(1+\frac{h^2}{12\mathrm{R}^2})\ ;\ N_{\theta\theta}=\rho\Omega^2R^2h(1+\frac{h^2}{12\mathrm{R}^2})$$

Axial stress is worth:

$$\sigma_{zz}=\nu\rho\Omega^2R^2(1+\frac{h^2}{12\mathrm{R}^2})$$ (constant in the thickness)

If one does not take account of the correction of metric, it is necessary to remove the term $(1+\frac{h^2}{12\mathrm{R}^2})$ in the preceding statements.

Numerical application (without correction of metric) **:**

$$p=1,600000.10-4\,MPa$$
$$w=2,912000.10-7\,mm$$
$$N_{zz}=0,96000.10-3\,N/mm$$
$$\sigma_{zz}=0,96000.10-3\,MPa$$

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15.2 Loading of gravity

15.2.1 2D Models axisymmetric

the density of force is: $-\rho\, g\, e_z$ (vertical gravity).

The following boundary conditions are considered:

$U_z(r, z)=0$ in $r=R$ and $z=0$ (circle of bearing)

with the uniform tension: $\sigma_{zz}(r, z)=\rho\, g\, L$ in $z=L$, balancing the weight.

One applies the elastic solution of the type:

$$\sigma=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

so that:

$$\varepsilon_{rr}=\varepsilon_{\theta\theta}=-\nu\, \varepsilon_{zz}=-\nu\, u_{z,z}=-\nu \frac{\sigma_{zz}}{E} \;;\; \varepsilon_{rz}=0=\varepsilon_{r\theta}=\varepsilon_{\theta z}$$

One observes as follows:

$$u_{r,r}=\frac{u_r}{r} \;\Leftrightarrow\; u_r(r, z)=-\nu A'(z) r$$

Then:

$$\begin{aligned} -\nu A'(z)=\varepsilon_{rr}=-\nu\, \varepsilon_{zz} \;&\Leftrightarrow\; u_{z,z}(r, z)=A'(z) \\ &\Leftrightarrow\; u_r(r, z)=A(z)+B \end{aligned}$$

From $\varepsilon_{rz}=0$, one draws:

$$B'(r)-\nu r A''(z)=0$$

that is to say:

$$A''(z)=cste=\alpha \quad ; \quad B'(r)=\alpha\, \nu\, r$$

Boundary conditions in force, one obtains:

$$A(z)=\frac{\rho\, g\, z^2}{2E}+\beta \quad ; \quad B(r)=\nu \frac{\rho\, g\, r^2}{2E}$$

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Lastly, β checks: $\beta=-\nu\,\rho\,g\,\frac{R^2}{2E}$

As follows:

$$u_r(r,z)=\frac{-\nu\,\rho\,g\,z\,r}{E} \quad ; \quad u_z(r,z)=\frac{\rho\,g}{2E}(z^2+\nu(r^2-R^2)) \qquad éq\,2.1-1$$

$$\sigma_{zz}(r,z)=\rho\,g\,z$$

Numerical application

$$g=10\ N/kg$$
$$\rho=8.10^{-6}\ kg/mm^3$$
$$R=20\ mm$$
$$L=10\ mm$$
$$E=2.10^5\ N/mm^2$$
$$\nu=0.3$$
$$h=1\ mm$$

in intern skin: $\begin{cases} u_r(L)=-2.34000\,E^{-8}\,mm \\ \sigma_{zz}(L)=8.0000\,E^{-4}\,Mpa \\ u_z(O)=-1.185000\,E^{-9}\,mm \end{cases}$

in external skin: $\begin{cases} u_r(L)=-2.46000\,E^{-8}\,mm \\ \sigma_{zz}(L)=8.0000\,E^{-4}\,Mpa \\ u_z(O)=1.215000\,E^{-9}\,mm \end{cases}$

15.2.2 Model axisymmetric shell

a vertical tension is exerted in $z=L$:

$$F=\rho\,g\,h\,L$$

Gravity leads to a vertical force:

$$f=-\rho\,g\,h\,e_z$$

The boundary condition on the circle of bearing is: $u_z(z)=0$ in $z=0$

the solution is membranous, the vertical equilibrium is written:

$$N_{zz,z}=\rho\,g\,h$$

Moreover: $N_{\theta\theta}=0$. In elasticity, one deduces then:

$$E_{\theta\theta}=\frac{w}{R}=\frac{-\nu\,N_{zz}}{Eh}=\frac{-\nu\,\rho\,g\,z}{E}$$

$$E_{zz}=u_{z,z}=\frac{N_{zz}}{Eh}\Rightarrow u_z(z)=\frac{\rho\,g}{2E}z^2$$

Axial stress is:

$\sigma_{zz}=\rho\,g\,z$ (constant in the thickness)

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numerical Application:

$$F=8.10^{-4}\,N.mm^{-1}$$
$$w(L)=-2.4000.10^{-8}\,mm$$
$$N_{zz}(L)=8.0000.10^{-4}\,N.mm^{-1}$$
$$\sigma_{zz}(L)=8.0000.10^{-4}\,N.mm^{-1}$$

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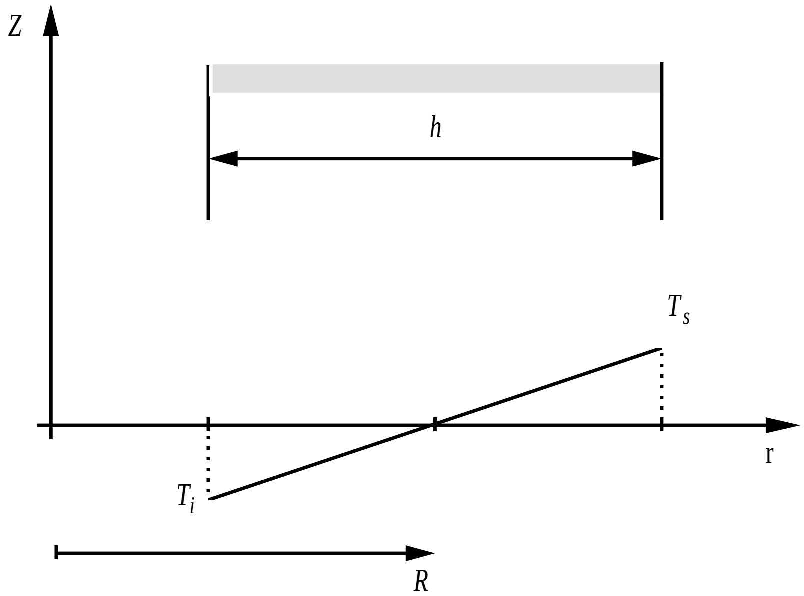
15.3 Thermomechanical loading

15.3.1 Models 2D axisymmetric

$$T(r)-T_{réf}(r)=\frac{T_s+T_i}{2}+\frac{(T_s-T_i)}{h}(r-R) \qquad éq\,3.1.-1$$



One applies displacement in the form:

$$u_r=u(r) \quad ; \quad u_z=u_\theta=0$$

with the suitable boundary conditions. Thus, the elastic stresses are expressed:

$$\sigma_{rr}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)u'+\nu\frac{u}{r}]-\frac{\alpha E}{1-2\nu}(T-T_{réf})$$
$$\sigma_{\theta\theta}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{u}{r}+\nu u']-\frac{\alpha E}{1-2\nu}(T-T_{réf}) \qquad éq\,1.1-3$$
$$\sigma_{zz}=\frac{\nu E}{(1+\nu)(1-2\nu)}[\frac{u}{r}+u']-\frac{\alpha E}{1-2\nu}(T-T_{réf})$$

The balance equation radial $(r,\sigma_{rr})_{,r}-\sigma_{\theta\theta}=0$ gives:

$$\left(\frac{(ru)'}{r}\right)'=\frac{\alpha(1+\nu')}{(1-\nu)}(T-T_{réf})' \qquad éq\,3.1-2$$

From where the general solution:

$$u(r)=\frac{\alpha(1+\nu)}{(1-\nu)}\frac{(T_s-T_i)}{h}\frac{r^2}{3}+Ar+\frac{B}{r} \qquad éq\,3.1-3$$

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The stresses are then:

$$\sigma_{rr}(r)=\frac{\alpha E(T_s-T_i)}{h}\left(\frac{R}{1-2\nu}-\frac{r}{3(1-\nu)}\right)-\frac{\alpha E}{1-2\nu}(T_s-T_i)+\frac{E}{(1+\nu)(1-2\nu)}\left(A-(1-2\nu)\frac{B}{r^2}\right)$$

$$\sigma_{\theta\theta}(r)=\frac{\alpha E(T_s-T_i)}{h}\left(\frac{R}{1-2\nu}-\frac{2r}{3(1-\nu)}\right)-\frac{\alpha E}{1-2\nu}(T_s-T_i)+\frac{E}{(1+\nu)(1-2\nu)}\left(A-(1-2\nu)\frac{B}{r^2}\right)$$

$$\sigma_{zz}(r)=\frac{\alpha E(T_s-T_i)}{h}\left(\frac{R}{1-2\nu}-\frac{r}{3(1-\nu)}\right)-\frac{\alpha E}{1-2\nu}(T_s-T_i)+\frac{2\nu E}{(1+\nu)(1-2\nu)}A \qquad éq\,3.1-4$$

The boundary conditions in forces are: in $r=R\pm\frac{h}{2}$, $\sigma_{rr}=0$. One notes: $x=\frac{h}{2R}$. One obtains thanks to $[éq\,3.1-4]$:

$$B=\frac{\alpha(T_s-T_i)}{6h(1-\nu)}(1+\nu)R^3(1-x^2)^2$$

then:

$$A=\alpha(1+\nu)\left[\frac{-(T_s-T_i)R}{6h(1-\nu)}(3-(1-2\nu)x^2)+\frac{(T_s+T_i)}{2}\right]$$

Numerical application:

$$F=8.10^{-4}\,N.mm^{-1}$$
$$w(L)=-2.4000.10^{-8}\,mm$$
$$N_{zz}(L)=8.0000.10^{-4}\,N.mm^{-1}$$
$$\sigma_{zz}(L)=8.0000.10^{-4}\,N.mm^{-1}$$

From where: $A=-0.18569881.10^{-3}\,mm^2$
$B=0.02473096\,mm^2$

Note:

$$\frac{\alpha(1+\nu)}{1-\nu}\frac{T_s-T_i}{3h}=0.61904762\,E-5$$

in intern skin: $\begin{cases}u_r=1.056145\,E^{-6}\,mm\\ \sigma_{zz}=1.4321427\,Mpa\end{cases}$

in external skin: $\begin{cases}u_r=1.110317\,E^{-6}\,mm\\ \sigma_{zz}=-1.4250001\,Mpa\end{cases}$

$$\sigma_{zz}=1.449319\,Mpa$$

$$ou$$

$$\sigma_{zz}=1.428671\,Mpa(\text{ sans correction métrique })$$

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If one takes $T_s = T_i = 0.1°C$:

$$A = 0{,}00130000 . 10^{-3}$$
$$B = 0{,}0\ mm^2$$

As follows:

in intern skin: $\begin{cases} u_r = 25.350000\, E^{-6} mm \\ \sigma_{zz} = -0.200000\, Mpa \end{cases}$

in external skin: $\begin{cases} u_r = 26.650000\, E^{-6} mm \\ \sigma_{zz} = -0.200000\, Mpa \end{cases}$

15.3.2 Model axisymmetric shell

For the field temperature in the thickness given by $[éq\,3.1-1]$, one obtains the following statement of the constitutive law:

$$\begin{aligned} N_{\theta\theta} &= \frac{Eh}{1-\nu^2}(E_{\theta\theta} + \nu E_{zz}) - \frac{\alpha E h}{1-\nu}\left[\frac{T_s+T_i}{2} + \frac{T_s-T_i}{12}\frac{h}{R}\right] \\ N_{zz} &= \frac{Eh}{1-\nu^2}(\nu E_{\theta\theta} + E_{zz}) - \frac{\alpha E h}{1-\nu}\left[\frac{T_s+T_i}{2} + \frac{T_s-T_i}{12}\frac{h}{R}\right] \end{aligned} \qquad éq\,3.2-1$$

and:

$$\begin{aligned} M_{\theta\theta} &= \frac{Eh^3}{12(1-\nu^2)}(K_{\theta\theta} + \nu K_{zz}) - \frac{\alpha E h^2}{12(1-\nu)}\left[\frac{T_s+T_i}{2}\frac{h}{R} + T_s - T_i\right] \\ M_{zz} &= \frac{Eh^3}{12(1-\nu^2)}(\nu K_{\theta\theta} + K_{zz}) - \frac{\alpha E h^2}{12(1-\nu)}\left[\frac{T_s+T_i}{2}\frac{h}{R} + T_s - T_i\right] \end{aligned} \qquad éq\,3.2-2$$

According to these statements, the thermal terms $\frac{h}{R}$ are to be in the case of neglected if one does not consider the correction of metric in the thickness, i.e. the usual models.

In our situation:

$$E_{\theta\theta} = \frac{w}{R}$$
$$E_{zz} = 0$$
$$K_{\theta\theta} = K_{zz} = 0$$

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The normal equilibrium with the shell is written:

$$N_{\theta\theta}=0$$

from where the deflection:

$$w=\alpha(1+\nu)[\frac{T_s+T_i}{2}+\frac{T_s+T_i}{12}\frac{h}{R}]R$$

and:

$$N_{zz}=\alpha Eh[\frac{T_s+T_i}{2}+\frac{T_s+T_i}{12}\frac{h}{R}]$$

$$M_{zz}=\frac{-\alpha Eh^2}{12(1-\nu)}[(T_s-T_i)+\frac{T_s+T_i}{2}\frac{h}{R}]$$

As the second member of thermal expansion does not take account of the correction of metric, the terms in h/R above are neglected.

Numerical application

$$R=20\ mm$$
$$h=1\ mm$$
$$\alpha=10^{-5}\,°C^{-1}$$
$$T_s=T_i=0.5\,°C$$
$$\nu=0.3$$
$$E=2.10^5\,N/mm^2$$

From where:

$$M_{zz}=-0.2380952\,N$$

in intern skin:

$$\sigma_{zz}=1.449319\,Mpa$$
$$ou$$
$$\sigma_{zz}=1.428671\,Mpa(\text{ sans correction métrique })$$

If one takes $T_s=T_i=0,1\,°C$:

$$w=26.00000.10^{-6}\,mm$$
$$N_{zz}=-0.2\,N.mm^{-1}$$
$$M_{zz}=-0.001190476\,N$$

in intern skin:

$$\sigma_{zz}=-0.2122466\,Mpa$$
$$ou$$
$$\sigma_{zz}=-0.200000\,Mpa(\text{ sans correction métrique })$$

** the stresses in the thickness with correction of metric are given by:*

$$\sigma_{zz}(x_3)=\frac{N_{zz}-\frac{M_{zz}}{R}}{h(1-\frac{h^2}{12R^2})}+(M_{zz}-N_{zz}\frac{h^2}{12R})\frac{12x_3}{h^3(1-\frac{h^2}{12R^2})}$$

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