HPLA310 - Biblio_49 Fissures radial external in a circular bar subjected to a Summarized thermal

shock:

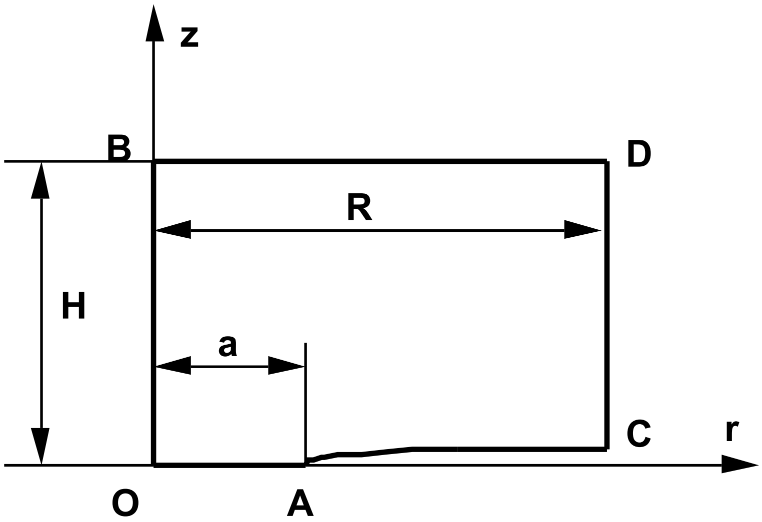
This test is resulting from the validation independent of version 3 in fracture mechanics.

It is about a basic static test into axisymmetric under non stationary thermal loading. The behavior of structure is thermo-elastic linear isotropic.

It understands only one axisymmetric modelization.

1 Problem of reference

1.1 Geometry



Fissures annular external in a semi-infinite cylindrical bar

One will take $a/R=0,5$ and $H/R\geq 5$.

$a=1\,m$
$R=2\,m$
$H=10\,m$

1.2 Properties of the material

the material is thermo-elastic linear isotropic.

Young modulus	$E=2\text{E}11\,Pa$
linear	$\nu=0,3$
Poisson's ratio Coefficient of thermal expansion	$\alpha=1\text{E-}5\,C^{\circ -1}$
thermal Conductivity	$\lambda=50\,W/m.C^{\circ}$
thermal Diffusivity	$\kappa=\lambda/\rho C_p=0,5\,m^2/s$
thermal Coefficient of heat exchange	$h=250\,W/m^2C^{\circ}$

One will choose h such as $Bi=hR/\lambda=10.$

1.3 Boundary conditions and loading

mechanical Boundary conditions

$UX=U_r=0$ on the axis of revolution $r=0$
$UY=U_z=0$ on the ligament $0\leq r\leq a$

resulting Thrust load null with higher edge; one will translate this boundary condition by a set of $(n-1)$ linear relations $UY(1)=UY(2)=...=UY(n)$ between longitudinal displacements of n the nodes of higher edge (free axial thermal expansion, conservation of the flatness of the cross section of the bar).

Conditions of unilateral contact on the lip of crack in order to manage the closing of this one.

Thermal boundary conditions

Heat flux no one on the axis of revolution AB (by symmetry)
Heat flux no one on the ligament OA (by symmetry) and on crack AC .

Flux of convection $\frac{\partial T}{\partial r}=h(T_{ext}-T)$ to edge $r=R$, T_{ext} indicating the temperature of the external medium.

Thermal loading

the temperature of the external medium undergoes an instantaneous level $T_{ext}=T0*H(t)$ where $H(t)$ is the function level-unit of Heaviside. Taking into account the boundary conditions the temperature does not vary according to z . One will take $T_0=100°C$ in order to obtain the closing of the lip, in the vicinity of the skin of the part, the beginning of the thermal shock.

1.4 Mechanical

initial conditions Initial conditions

Displacements, strains and stresses null in all points.

Thermal initial conditions

initial Temperature null in any point.

2 Reference solution

2.1 Method of calculating used for the reference solution

Field of temperature: exact analytical computation.

Thermomechanical computation: thermo-elastic stress field in the bar not fissured given by an exact analytical statement

displacement of the lips of crack calculated from functions of influence determined numerically by finite elements

factor of intensity of the stresses calculated starting from the surface stresses released along crack, by means of the functions weight of unlimited solid for a distribution of pressure on the lips constant by interval along the radius.

2.2 Results of reference

Number of Fourier: $Fo=\frac{\kappa t}{R^2}$ (adimensional time)

Number of Biot: $Bi=\frac{hR}{\lambda}$ (coefficient of heat exchange without dimension)

Statement of the temperature according to R and T:

$$T = T_0 \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{Bi J_0(\mu_n r / R)}{(\mu_n^2 + Bi^2) J_0(\mu_n)} \bullet \exp(-\mu_n^2 Fo) \right\}$$

où

$$Bi J_0(\mu_n) = \mu_n J_1(\mu_n)$$

the eigenvalues μ_n are the solutions of the equation above in which J_0 and J_1 are the functions of Bessel of first species of order 0 and 1.

The tables below summarize the values of the temperatures ($°C$) for three radius particular and three numbers of Fourier:

$F0=0{,}001$

	Ref. (10000 terms)	Aster	% variation
$r=0$	3,9968E-12	2,06844E-18	-
$r=1$	2,2204E-13	8,94917E-12	-
$r=2$	2,79689E+1	2,80013E+1	0,116

$F0=0{,}4$

	ref. (900 terms)	Aster	% variation
$r=0$	1,6230E-1	1,78593E-1	10,04
$r=1$	6,2391E+0	6,2555E+0	0,262
$r=2$	7,7365E+1	7,73319E+1	– 0,044

$F0=1$

	ref. (900 terms)	Aster	% variation
$r=0$	9,8644E+1	9,8637E+1	0,006
$r=1$	9,9018E+1	9,9013E+1	– 0,005
$r=2$	9,9835E+1	9,9834E+1	– 0,001

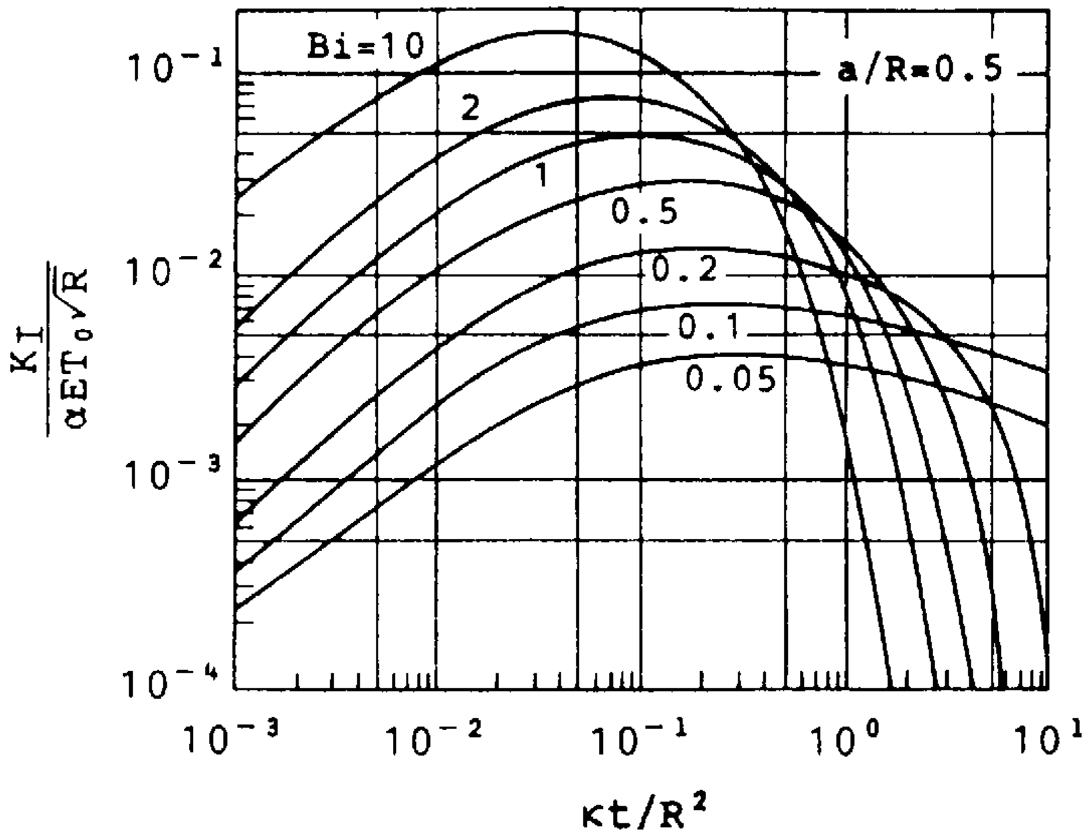
Statement of axial stress in the bar not fissured according to r and of t :

$$\sigma_{zz} = \frac{2\alpha E T_0}{(1-\nu)} \sum_{n=1}^{\infty} \left\{ \frac{Bi}{\left(\mu_n^2 + Bi^2\right) J_0\left(\mu_n\right)} \exp\left(-\mu_n^2 Fo\right) \right\} \left\{ J_0\left(\mu_n \frac{r}{R}\right) - \frac{2Bi}{\mu_n^2} J_0\left(\mu_n\right) \right\}$$

The table below summarizes the values of the stresses $\sigma_{zz}(Pa)$ for $r=a$ (crack tip) and three numbers of Fourier:

	Ref. (900 terms)	Aster	% variation
$F0=0{,}001$	4,584029E+6	4,58508E+6	0,017
$F0=0{,}4$	6,397099E+7	6,38746E+7	– 0,151
$F0=1$	8,200300E+5	8,23974E+5	0,481

Factor of intensity of the stresses (adimensional) according to the number of Fourier



2.3 Uncertainty on the solution

Lower than 5% .

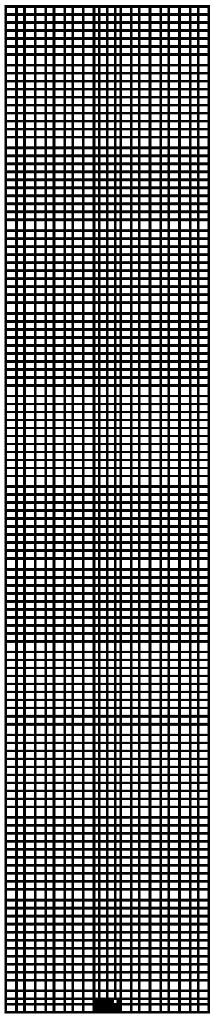
2.4 Bibliographical references

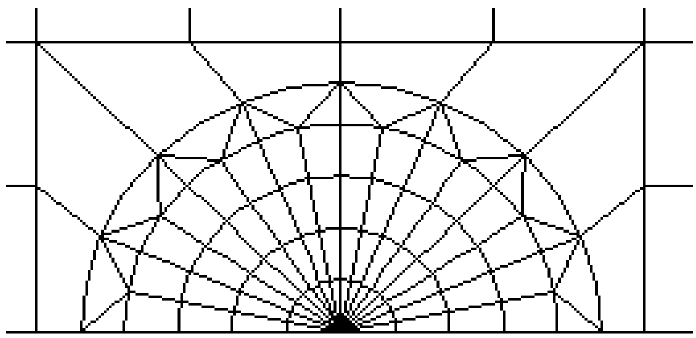
1.J.M. ZHOU, T. TAKASE and Y. IMAI: Opening and closing behavior of year external circular ace due to axisymmetrical heating. Engng.Fract.Mechs., 47, n°4, 559-568, 1994. Modelization A

3 Characteristic

3.1 of the non stationary modelization The computation

thermal precedes mechanical computation. Two computations are done using the same mesh to avoid the phenomena of lissage. Complete mesh





Zoom on the point **of crack Characteristics**

3.2 of the mesh The mesh

consists of 8651 nodes and 2772 elements, including 2732 elements QUA8 and 40 elements TRI6. The radial density

of the mesh is determined by successive tests in order to reduce to the difference between 1% the theoretical solution and the numerical solution, as well from the thermal point of view as thermomechanical, in the case of the bar not fissured. The height of the half

- models is built-in arbitrarily with 5 times the radius. It is supposed a priori R that the effect of the limitation of size of the mesh in the direction on the factor of Z intensity of the stresses is lower than. An indeformable block 1%

, located under the lip, was with a grid in order to manage the contact without friction induced by the closing of the lip. Values tested

3.3 and results of the modelization A Identification Reference

	Aster	**difference**	***2,3E***	$\%$ **+2 2,9449E+2**
$G(Fo=0{,}001)(J.m^2)$		* 30.7,0	E+6 8,0438	E6 **
$K_I(Fo=0{,}001)(Pa.m^{0,5})$		14.1,0	E+4 1,25016	E+
$G(Fo=0{,}04)(J.m^2)$		4* 19.4,8	E+7 5,24175	E7
$K_I(Fo=0{,}04)(Pa.m^{0,5})$		** 9.1,0	1,2104864*	15.4,8
$G(Fo=1)(J.m^2)$			E+5 5,1579	E+5
$K_I(Fo=1)(Pa.m^{0,5})$		** 7 *	In the axisymmetric	calculation case

, to obtain the total rate of refund, it is necessary to divide the rate of refund obtained with ASTER by (cf Documentation of reference $R_{fissure}=a$ [R7.02.01] - page18). ** Values obtained

with the formula of IRWIN in plane strains, by supposing that, and while taking $K_{II}=0$ calculated by ASTER G , which does not allow the automatic computation of into axisymmetric K_I . Remarks To compute:

3.4 ,

one uses G_{ref} the formulas of IRWIN in plane strains: , The raised maximum change

$$G_{ref}=\frac{1-\nu^2}{E}(K_I^2+K_{II}^2) \qquad K_{II}=0$$

is of on (), of on 30% (). G $Fo=1$ The maximum $14\,\%$ relative K_I $Fo=1$

variation on the temperature in crack tip, compared to the analytical solution (added on 900 terms), is lower A. the maximum relative 1%

variation on in the bar σ_{zz} before cracking, compared to the analytical solution added on 900 terms, with the site of the later crack tip, is lower than. With ASTER, in $0{,}5\,\%$ axisymmetric mode, the stress field obtained is following form: SIXX SIYY SIZZ SIXYs

and the forced associated are: SIRR SIZZ SITT SIRZ

To
compute:

the values of reference, we use the curve in (page 5). The accuracy $\log/\log$ with the reading of the values not being very good, we can estimate that the results on rate of energy restitution are not too G far away from the reference. It should be noted

that rate of energy restitution is invariable on G contours of computation. Summary of the results

4 With regard to

the bar not fissured, the Aster results in temperature and stress are very close to the reference (less maximum for 1% the temperature and less maximum for $0{,}5\%$ the stresses). On the other hand, for rate of energy restitution, the Aster results are far away from the reference since we raise a maximum change of for, with 30% an accuracy $Fo=0{,}001$ announced from on the reference solution 5% .