

HPLP100 - Computation of the rate of refund of the energy of a plate fissured in thermoelasticity

Summarized

It acts of a test in thermoelasticity for a two-dimensional problem. A fissured rectangular plate is considered and one places oneself on the assumption of the plane strains.

In the **modelization A**, the rate of refund of energy is calculated in postprocessing by two different methods:

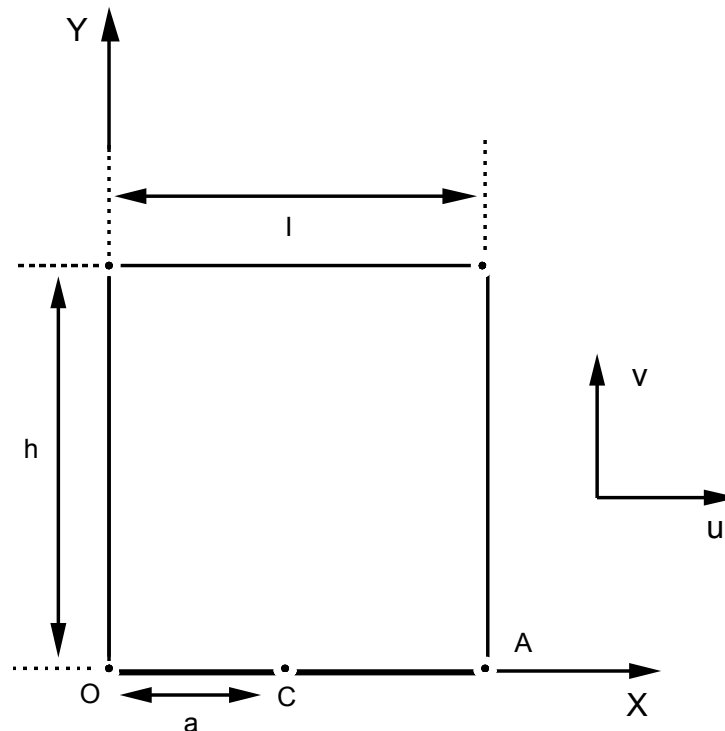
- classical computation by the method theta,
- computation by the formula of IRWIN starting from the coefficients of intensity of stresses KI and KII .

These two computations are carried out on 4 different integration contours. Their interest is to compare the values of G and from $G(IRWIN)$ ratio with the reference solution and to test the invariance of computations compared to various integration contours.

1 Problem of reference

1.1 Geometry

It acts of a fissured rectangular plate (one represents only the quarter of structure):



Appear 1.1-a: Fissured rectangular plate

dimensions of this plate are the following ones:

Half-height of the plate: $h = 200.0 \text{ mm}$

Half-width of the plate: $I = 100.0 \text{ mm}$

Half-length of crack: $a = 50.0 \text{ mm}$

1.2 Properties of the material

thermal Properties: $C_p = 0.$

$\lambda = 1.0 \text{ W / m}^\circ\text{C}$

Mechanical properties: $E = 200000 \text{ MPa}$

$\nu = 0.3$

$\alpha = 5.10^{-6} / ^\circ\text{C}$

We are on the assumption of the plane strains

1.3 Boundary conditions and loadings

- Temperature imposed in $X=0.$: $T = -100.0^\circ\text{C}$
- Temperature imposed in $X=100$: $T = +100.0^\circ\text{C}$
- Displacement for $a < X < I$ $Y=0.$: $v=0.$
- Displacement for $0 < X < I$ $Y=H$: $v=0.$

- Displacement for $X=0$. $Y=H$: $u=0$.

2 Reference solution

2.1 Méthode de calcul used for the reference solution

the reference solution is resulting from WILSON and YU [bib1]:

$$K_I = \frac{E \alpha T_0}{1-\nu} F \sqrt{\pi a} \quad F = 0.154$$

a en mm
 E en N/mm²

$$K_I = 92.0291$$

In plane strains, the formula of IRWIN gives: $G = \frac{(1-\nu^2)}{E} (K_I^2 + K_{II}^2)$

that is to say numerically: $G = 3.8535 \cdot 10^{-1}$

2.2 Results of reference

the results of reference are those resulting from the reference solution from WILSON and YU [bib1]:

$$G = 3.8535 \cdot 10^{-1}$$
$$K_I = 92.0291$$
$$K_{II} = 0.$$

2.3 Bibliographical references

- 1) The Uses of J-Integrals in thermal stress ace problems - International Newspaper of Fracture (1979) WILSON and YU.
- 2) Qualification complementary to codes INCA/MAYA in linear thermoelasticity. Note technical DRE/STRE/LMA 84/598

3 Modelization A

3.1 Characteristic of the modelization

There are 4 contours defined by the command `CALC_THETA` :

Crown 1:	$R_{inf} = 10.$	$R_{sup} = 40.$
Crown 2:	$R_{inf} = 15.$	$R_{sup} = 45.$
Crown 3:	$R_{inf} = 5.$	$R_{sup} = 47.$
Crown 4:	$R_{inf} = 3.$	$R_{sup} = 48.$

The crack tip is defined by `DEFI_FOND_FISS`, and for each contour one carries out:

- a computation of G classic (option `CALC_G` of `CALC_G`),
- a computation from G the formula of IRWIN starting from the coefficients of intensity of stresses K_I and K_{II} (option `CALC_K_G` of `CALC_G`).

3.2 Characteristics of the mesh

Many nodes: 853

Number of meshes and types: 359 meshes `TRIA6` and 27 meshes `QUAD8`

3.3 Values tested and results of the modelization A

the values tested are those of G obtained by the classical method and that of G_{IRWIN} obtained by the formula of IRWIN starting from the coefficients of intensity of stresses:

Identification	Reference	Tolerance
Crowns 1 G	$3.8535 \cdot 10^{-1}$	8.00%
Contour 1 G_{IRWIN}	$3.8535 \cdot 10^{-1}$	8.00%
Contour 2 G	$3.8535 \cdot 10^{-1}$	8.00%
Contour 2 G_{IRWIN}	$3.8535 \cdot 10^{-1}$	8.00%
Contour 3 G	$3.8535 \cdot 10^{-1}$	8.00%
Contour 3 G_{IRWIN}	$3.8535 \cdot 10^{-1}$	8.00%
Contour 4 G	$3.8535 \cdot 10^{-1}$	8.00%
Contour 4 G_{IRWIN}	$3.8535 \cdot 10^{-1}$	8.00%

3.4 Remarks

the numerical values are stable compared to various integration contours and almost identical for the two methods of calculating. Nevertheless the variation with the values of reference is about 6 with 7% , which seems high.

4 Summaries of the results

At the time of **the first modelization**, the variation with the values of reference is from 6 to 7%. The validation independent of the batch fracture mechanics should bring brief replies on the validity of G in thermoelasticity.