HPLP101 - Plate fissured in thermoelasticity (plane stresses)

Summarized:

This test is resulting from the validation independent of *Code_Aster* in fracture mechanics (reference resulting from Murakami: Mura11-17). It makes it possible to validate the operators of fracture mechanics for a two-dimensional problem (assumption of the plane stresses) in isotropic linear thermoelasticity.

This test understands a modelization in plane stresses in which are calculated:

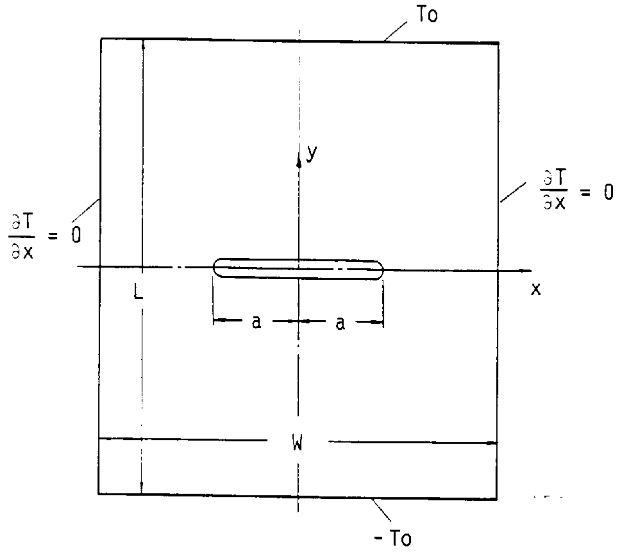
- rate of energy restitution G (classical computation by the method theta),
- coefficients of intensity of stresses K_I and K_{II} .

These two computations are carried out on 6 different integration contours.

The interest of the test is to compare the values of G and K_{II} compared to the reference solution and to test the invariance of computations compared to various integration contours.

1 Problem of reference

1.1 Geometry



Width of the plate: $W=0.6\,m$
Length of the plate: $L=0.3\,m$
Length of crack: $2\mathrm{a}=0.3\,m$

1.2 Properties of the material

Notation for thermo-elastic properties:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{bmatrix}S_{11} & S_{12} & 0\\ S_{12} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}+\begin{Bmatrix}\alpha_{11}\\ \alpha_{22}\\ 0\end{Bmatrix}\cdot(T-T_{ref})$$

$$S_{11}=1/E_x$$
$$S_{22}=1/E_y$$
$$S_{12}=-\nu_x/E_x=-\nu_y/E_y$$
$$S_{66}=1/G_{xy}$$
$$\alpha_{11}=\alpha_x$$
$$\alpha_{22}=\alpha_y$$

One limits oneself to the isotropic material, as well from the thermal point of view as mechanical:

$$E_x=E_y=2.10^5\,MPa$$
$$\nu_x=\nu_y=0.3$$
$$\alpha_x=\alpha_y=1.2\,10^{-5}\,°C^{-1}$$
$$\lambda_x=\lambda_y=54.\,W/m\,°C$$

1.3 Boundary conditions and loading

two models are considered:

- half-models $x=0$
- complete mechanical

Boundary conditions the model:

- half-model

  $UX=0$ along the axis of symmetry $X=0$
  $UY=0$ at the point ($W/2.$)
- models complete

  $UX=0$ at the point $(0, L/2.)$
  $UY=0$ at the thermal $(-L/2.)$ points $(L/2.)$

and Boundary conditions:

- half-model

  $T=100°C$ on higher edge $Y=L/2$.
  $T=-100°C$ on lower edge $Y=-L/2$.
  null flux on the axis of symmetry, free edge $X=W/2.$ and edge of model
- crack complete

  $T=100°C$ on higher edge $Y=L/2$.
  $T=-100°C$ on lower edge $Y=-L/2$.
  null flux on free edges $X=\pm W/2.$ and edge of the crack

2 Reference solution

2.1 Method of calculating used for the complex reference solution

Potential [bib1].

2.2 Results of reference

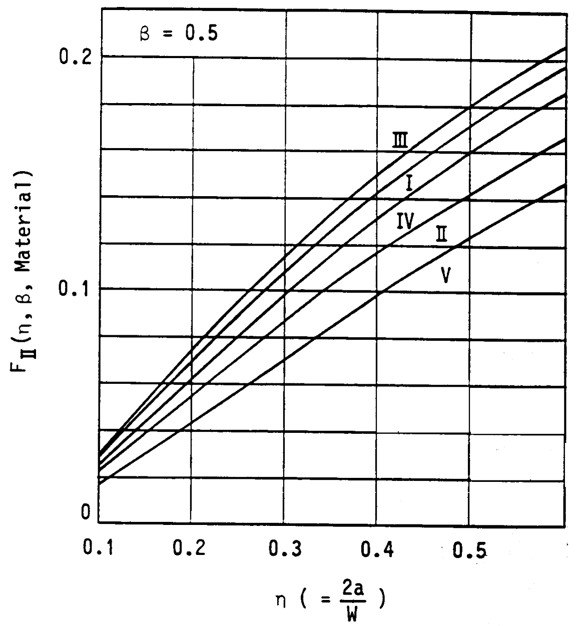
$$\eta=\frac{2a}{W}$$

$$\beta=\frac{L}{W}$$

$$K_{II}=\frac{\alpha_{11}T_0}{S_{11}}\cdot\sqrt{\frac{W}{2}}\cdot F_{II}$$

where the geometrical factor of correction F_{II} is given according to η for each material, in the typical case $\beta=0.5$ on the curves below.

The isotropic material being represented by the curve I



2.3 Uncertainty on the solution

nondefinite Accuracy.

2.4 Bibliographical references

1) Y. MURAKAMI: Stress Intensity Factors Handbook, box 11.17, pages 1045-1047. The Society of Materials Science, Japan, Pergamon Near, 1987.

3 Modelization A

3.1 Characteristic of the modelization

For this modelization, the 3 topological parameters of the block crack are:

- NS : many sectors on $90°$
- NC : many contours
- rt : the radius of greatest contour (with half a: length of crack)

$$NS=8$$
$$NC=4$$
$$rt=0{,}001\times a$$

the values of radius higher and lower, to specify in command `CALC_THETA` are:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
Rinf	3,75E-5	7,500E-5	1,125E-4	1,500E-4	1,875E-4	2,250E-4
Rsup	7,50E-5	1,125E-4	1,500E-4	1,875E-4	2,250E-4	3,000E-4

3.2 Characteristic of the mesh

Half-MAILLAGE; mesh radiating at the right end of crack.

3831 nodes,
1516 elements,
884 TRI6,
632 QUA8.

3.3 Quantities tested and results of the modelization A

Identification	**Reference**	***Aster***	**% difference**
K_{II} , contour n°1	2,2347E+7	2,2814E+7	2,09
K_{II} , crowns n°2	2,2347E+7	2,2813E+7	2,08
K_{II} , crowns n°3	2,2347E+7	2,2814E+7	2,09
K_{II} , crowns n°4	2,2347E+7	2,2814E+7	2,09
K_{II} , crowns n°5	2,2347E+7	2,2817E+7	2,10
K_{II} , crowns n°6	2,2347E+7	2,2818E+7	2,11
G , crowns n°1	2,4969E+3	2,5984E+3	4,07
G , crowns n°2	2,4969E+3	2,5990E+3	4,09
G , crowns n°3	2,4969E+3	2,5992E+3	4,10
G , crowns n°4	2,4969E+3	2,5993E+3	4,10
G , crowns n°5	2,4969E+3	2,6013E+3	4,18
G , crowns n°6	2,4969E+3	2,5985E+3	4,07

3.4 Remarks

In the reference, the author supposes that $K_I=0$, but it does not check it a posteriori. With the sights of the deformed shapes resulting from Code_Aster, the coefficient K_I is different from zero, but there remains very weak compared to K_{II} (the crack slips more than it does not open).

With regard to rate of energy restitution G, if we suppose that $K_I=0$, we draw the value of reference from the formula from IRWIN in plane stresses:

$$G_{ref}=(1/E)\times K_{II}^2$$

4 Summary of the results

the differences between the reference solution and the results of *Code_Aster* do not exceed 2% on the coefficients of intensity of stresses and 4% for rate of energy restitution. One checks the invariance of the results compared to various integration contours.