# HPLP311 - Murakami 11.17 Fissures in the center of a rectangular thin plate which prevents a uniform heat flux in Summarized isotropic

**medium:**

It is about an isotropic linear thermoelastic computation static.

It is a basic test out of $2D$ plane for a steady thermal loading calculated by finite elements on the same mesh with an isotropic material in mode $II$.

Purpose:

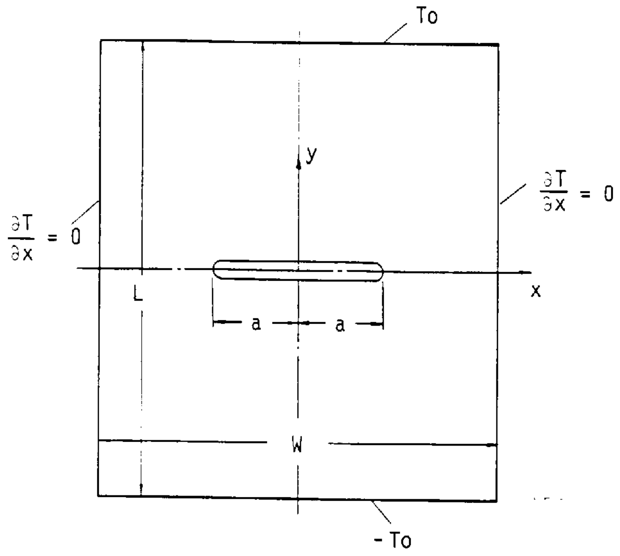
- basic test out of $2D$ plane, for a steady thermal loading calculated by finite elements on the same mesh, with isotropic material, in mode $II$,
- validation of the computation of $K_{II}$,
- variability of $G$ according to topology (sectors, contours) of the radiant mesh. Checking of the invariance of the results in fracture mechanics, at an end of crack, the mesh of the other end of same crack.

The computation is tested on a complete mesh and a half-MAILLAGE. Parameters $L/W$ and $2A/W$ being fixed.

One measures a relative variation on $K_{II}$, the accuracy is nevertheless badly defined.

# 1 Problem of reference

## 1.1 Geometry




Width of the plate: $W=0{,}6\,m$
Length of the plate: $L=0{,}3\,m$
Length of crack: $2\mathrm{a}=0{,}3\,m$

## 1.2 Properties of the material

Notation for thermo-elastic properties:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{bmatrix}S_{11} & S_{12} & 0\\ S_{12} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}+\begin{Bmatrix}\alpha_{11}\\ \alpha_{22}\\ 0\end{Bmatrix}\left(T-T_{ref}\right)$$

$$S_{11}=1/E_x$$
$$S_{22}=1/E_y$$
$$S_{12}=-\nu_x/E_x=-\nu_y/E_y$$
$$S_{66}=1/G_{xy}$$
$$\alpha_{11}=\alpha_x$$
$$\alpha_{22}=\alpha_y$$

One limits oneself to the isotropic material, as well from the thermal point of view as mechanical:

$$E_x=E_y=2.10^5\,MPa$$
$$\nu_x=\nu_y=0{,}3$$
$$\alpha_x=\alpha_y=1{,}2\,10^{-5}\,°C^{-1}$$
$$\lambda_x=\lambda_y=54\,W/m\,°C$$

## 1.3 Boundary conditions and loading

two models are considered:

- half-models $x \geq 0$
- complete mechanical

**Boundary conditions the model:**

- half-model

  $UX=0$ along the axis of symmetry $X=0$
  $UY=0$ at the complete $(W/2,0)$

- model point

  $UX=0$ at the point $(0, L/2)$
  $UY=0$ at the thermal $(-L/2,0)$ points $(L/2,0)$

**and Boundary conditions:**

- half-model

  $T=100°C$ on higher edge $Y=L/2$
  $T=-100°C$ on edge lower $Y=-L/2$
  null flux on the axis of symmetry, free edge $X=W/2$ and edge of model

- crack complete

  $T=100°C$ on higher edge $Y=L/2$
  $T=-100°C$ on edge lower $Y=-L/2$
  flux null on free edges $X=\pm W/2$ and edge of the crack

# 2 Reference solution

## 2.1 Method of calculating used for the complex reference solution

Potential.

## 2.2 Results of reference

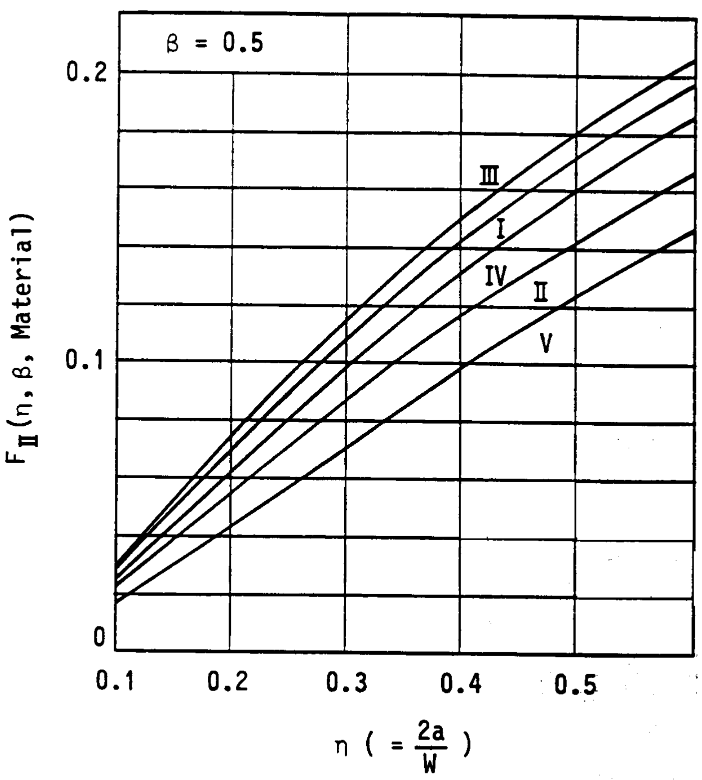
$$\eta = \frac{2a}{W}$$

$$\beta = \frac{L}{W}$$

$$K_{II} = \frac{\alpha_{11} T_0}{S_{11}} \bullet \sqrt{\frac{W}{2}} \bullet F_{II}$$

where the geometrical factor of correction $F_{II}$ is given according to $\eta$ for each material, in the typical case $\beta = 0{,}5$ on the curves below.

The isotropic material being represented by the curve $I$




## 2.3 Uncertainty on the solution

nondefinite Accuracy.

## 2.4 Bibliographical references

1) Y. MURAKAMI: Stress Intensity Factors Handbook, box 11.17, pages 1045-1047. The Society of Materials Science, Japan, Pergamon Near, 1987.

# 3 Modelizations A, B, C, D, E and F

## 3.1 Characteristic of the modelization

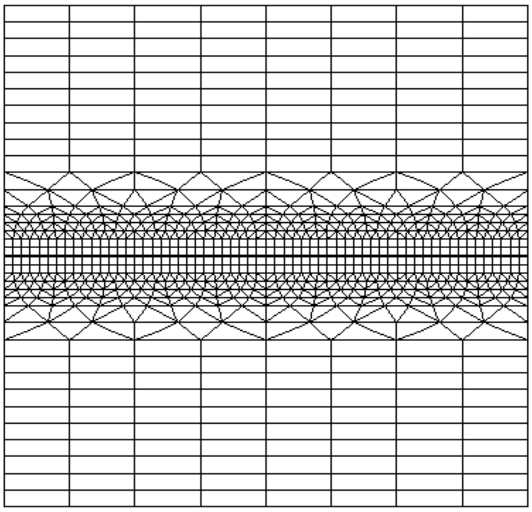
These 6 modelizations correspond to 6 meshes where one varies 3 topological parameters. The table below summarizes the various studied cases:

| | $NS=8 \quad NC=4$ | $NS=4$ , $NC=3$ |
|---|---|---|
| $rt=0,001*a$ | A | B |
| $rt=0,01*a$ | C | D |
| $rt=0,1*a$ | E | F |

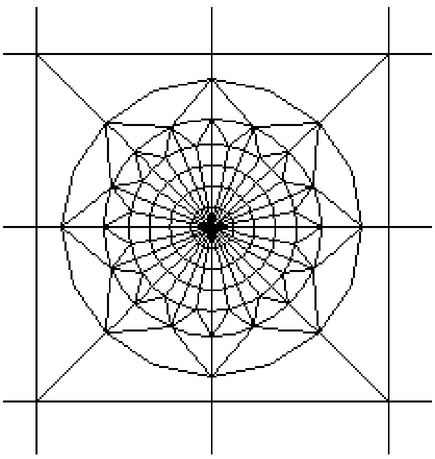
the topological parameters which vary are:

$NS$ : many sectors on 90°
$NC$ : many contours
$rt$ : the radius of greatest contour (with half a: length of crack)

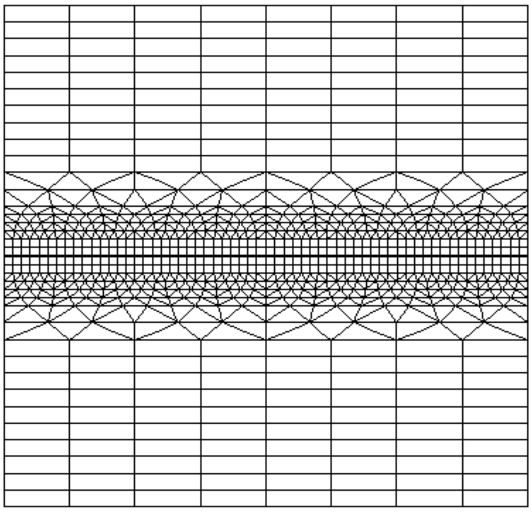
### 3.1.1 Modelizations A and B



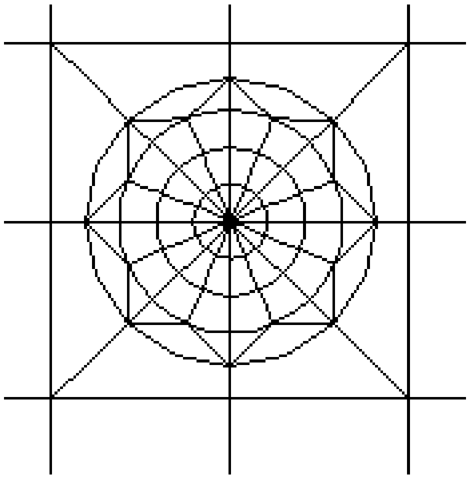
**Half mesh - Modelization A**



**Zoom of the point of crack - Modelization A**

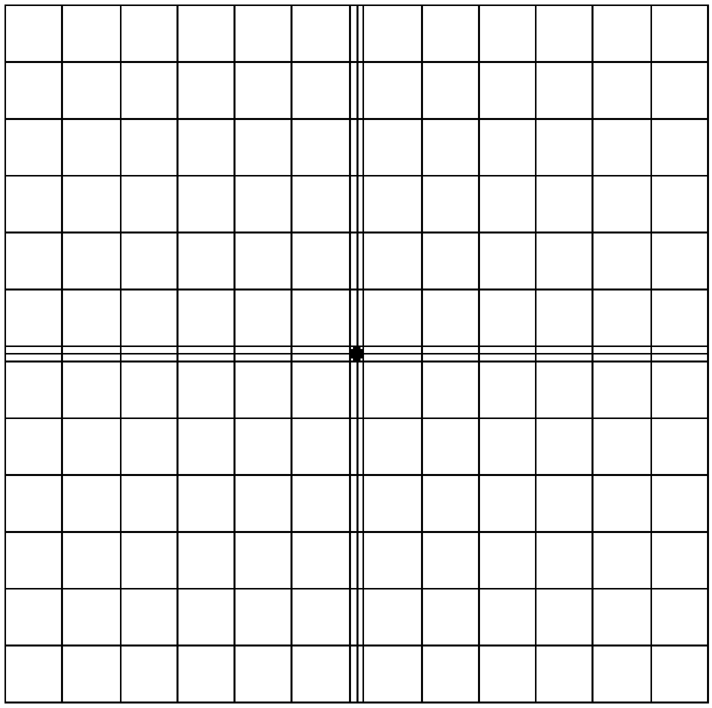


**Half mesh - Modelization B**

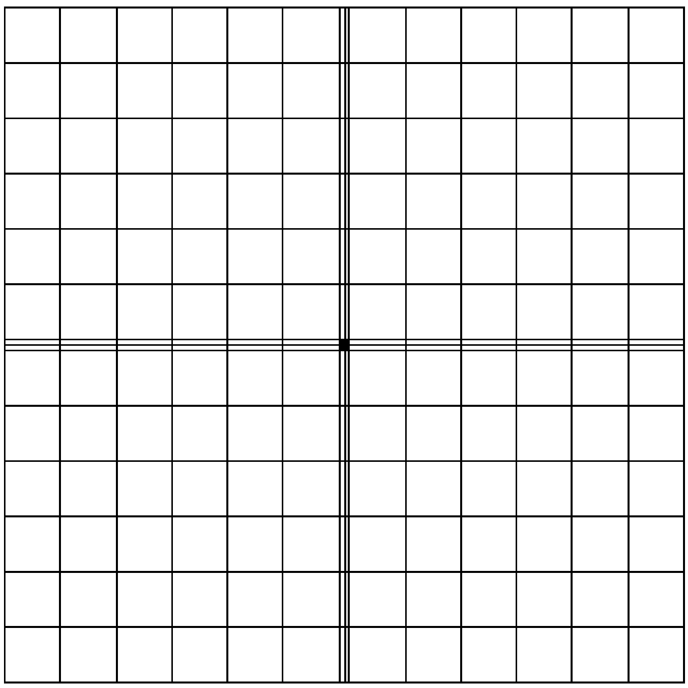


**Zoom of the point of crack - Modelization B**

### 3.1.2 Modelizations C and D

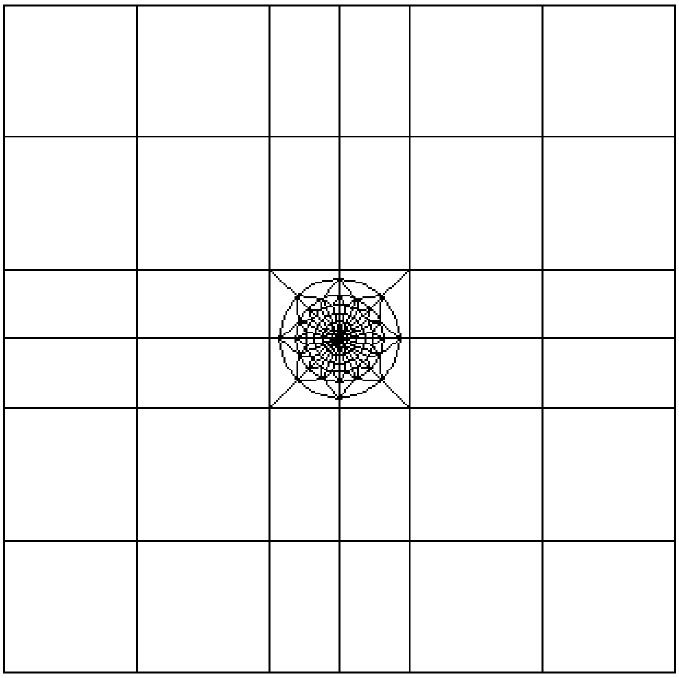


**complete Mesh - Modelization C**

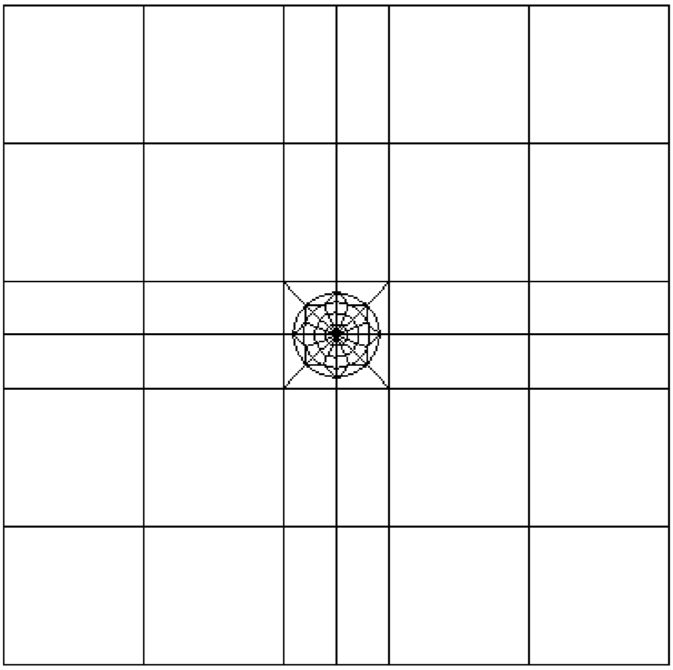


**complete Mesh - Modelization D**

### 3.1.3 Modelizations E and F



**complete Mesh - Modelization E**



**complete Mesh - Modelization F**

### 3.1.4 Definition of radius of contours

For these various cases, we radius define the values of higher and lower, to specify in command CALC_THETA :

**Modelization A**

| | the 1st contour | the 2nd contour | the 3rd contour | the 4th contour |
|---|---|---|---|---|
| $rinf(m)$ | 3,75E-5 | 7,500E-5 | 1,125E-4 | 1,500E-4 |
| $\mathrm{rsup}(m)$ | 7,50E-5 | 1,125E-4 | 1,500E-4 | 1,875E-4 |

**Modelization B**

| | the 1st contour | the 2nd contour | the 3rd contour |
|---|---|---|---|
| $rinf(m)$ | 5,00E-5 | 1,00E-4 | 1,50E-4 |
| $\mathrm{rsup}(m)$ | 1,00E-4 | 1,50E-4 | 2,00E-4 |

**Modelization C**

| | the 1st contour | the 2nd contour | the 3rd contour | the 4th contour |
|---|---|---|---|---|
| $rinf(m)$ | 3,75E-4 | 7,500E-4 | 1,125E-3 | 1,500E-3 |
| $\mathrm{rsup}(m)$ | 7,50E-4 | 1,125E-3 | 1,500E-3 | 1,875E-3 |

**Modelization D**

| | the 1st contour | the 2nd contour | the 3rd contour |
|---|---|---|---|
| $rinf(m)$ | 5,00E-4 | 1,00E-3 | 1,50E-3 |
| $\mathrm{rsup}(m)$ | 1,00E-3 | 1,50E-3 | 2,00E-3 |

**Modelization E**

| | the 1st contour | the 2nd contour | the 3rd contour | the 4th contour |
|---|---|---|---|---|
| $rinf(m)$ | 3,75E-3 | 7,500E-3 | 1,125E-2 | 1,500E-2 |
| $\mathrm{rsup}(m)$ | 7,50E-3 | 1,125E-2 | 1,500E-2 | 1,875E-2 |

**Modelization F**

| | the 1st contour | the 2nd contour | the 3rd contour |
|---|---|---|---|
| $rinf(m)$ | 5,00E-3 | 1,00E-2 | 1,50E-2 |
| $\mathrm{rsup}(m)$ | 1,00E-2 | 1,50E-2 | 2,00E-2 |

## 3.2 Characteristic of the mesh

Half-MAILLAGE; mesh radiating at the right end of crack.

The table below gives the constitution of the studied meshes:

| | $NS=8 \quad NC=4$ | $NS=4$ , $NC=3$ |
|---|---|---|
| $rt=0{,}001*a$ | 3831 nodes,<br>1516 elements,<br>884 `TRI6`,<br>632 `QUA8`. | 3507 nodes,<br>1388 elements,<br>820 `TRI6`,<br>568 `QUA8`. |
| $rt=0{,}01*a$ | 1179 nodes,<br>400 elements,<br>104 `TRI6`,<br>296 `QUA8`. | 855 nodes,<br>272 elements,<br>40 `TRI6`,<br>232 `QUA8`. |
| $rt=0{,}1*a$ | 659 nodes,<br>240 elements,<br>104 `TRI6`,<br>136 `QUA8`. | 335 nodes,<br>112 elements,<br>40 `TRI6`,<br>72 `QUA8`. |

# 4 Results of the modelizations A, B, C, D, E and F

## 4.1 Values tested

| Identification | Reference | *Aster* | % difference |
|---|---|---|---|
| **Diameter crowns external =** $0{,}001*a$ | | | |
| **radiant Mesh** | $NS=8$ | $NC=4$ | **Modelization A** |
| $K_{II}$ , contour n°1 | 2,2347E+7 | 2,2814E7 | 2,09 |
| $K_{II}$ , contour n°2 | 2,2347E+7 | 2,2813E7 | 2,08 |
| $K_{II}$ , contour n°3 | 2,2347E+7 | 2,2814E7 | 2,09 |
| $K_{II}$ , contour n°4 | 2,2347E+7 | 2,2814E7 | 2,09 |
| **radiant Mesh** | $NS=4$ | $NC=3$ | **Modelization B** |
| $K_{II}$ , contour n°1 | 2,2347E+7 | 2,282E7 | 2,10 |
| $K_{II}$ , contour n°2 | 2,2347E+7 | 2,282E7 | 2,10 |
| $K_{II}$ , contour n°3 | 2,2347E+7 | 2,281E7 | 2,09 |
| **Diameter crowns external =** $0{,}01*a$ | | | |
| **radiant Mesh** | $NS=8$ | $NC=4$ | **Modelization C** |
| $K_{II}$ , contour n°1 | 2,2347E+7 | 2,166 $10^{7}$ | 3,058 |
| $K_{II}$ , contour n°2 | 2,2347E+7 | 2,214 $10^{7}$ | 0,919 |
| $K_{II}$ , contour n°3 | 2,2347E+7 | 2,214 $10^{7}$ | 0,919 |
| $K_{II}$ , contour n°4 | 2,2347E+7 | 2,214 107 | 0,919 |
| radiant Mesh | NS= 4 | NC= 3 | Modelization D |
| $K_{II}$ , contour n°1 | 2,2347E+7 | 2,214 107 | 0,919 |
| $K_{II}$ , contour n°2 | 2,2347E+7 | 2,214 107 | 0,919 |
| $K_{II}$ , contour n°3 | 2,2347E+7 | 2,214 107 | 0,919 |
| **Diameter crowns external =** $0{,}1*a$ | | | |
| **radiant Mesh** | $NS=8$ | $NC=4$ | **Modelization E** |
| $K_{II}$ , contour n°1 | 2,2347E+7 | 2,2632 107 | 1,276 |
| $K_{II}$ , contour n°2 | 2,2347E+7 | 2,2572 107 | 1,009 |
| $K_{II}$ , contour n°3 | 2,2347E+7 | 2,2572 107 | 1,008 |
| $K_{II}$ , contour n°4 | 2,2347E+7 | 2,2564 107 | 0,972 |
| **radiant Mesh** | $NS=4$ | $NC=3$ | **Modelization F** |
| $K_{II}$ , contour n°1 | 2,2347E+7 | 2,255E7 | 0,932 |
| $K_{II}$ , contour n°2 | 2,2347E+7 | 2,2568E7 | 0,988 |
| $K_{II}$ , contour n°3 | 2,2347E+7 | 2,2568E7 | 0,987 |

| Identification | Reference | *Aster* | % difference |
|---|---|---|---|
| **Diameter crowns external =** $0{,}001*a$ | | | |
| **radiant Mesh** | $NS=8$ | $NC=4$ | **Modelization A** |
| $G$ , contour n°1 | 2,4969E+3 | 2,5984E+3 | 4,07 |
| $G$ , contour n°2 | 2,4969E+3 | 2,5990E+3 | 4,09 |
| $G$ , contour n°3 | 2,4969E+3 | 2,5992E+3 | 4,10 |
| $G$ , contour n°4 | 2,4969E+3 | 2,5993E+3 | 4,10 |
| **radiant Mesh** | $NS=4$ | $NC=3$ | **Modelization B** |
| $G$ , contour n°1 | 2,4969E+3 | 2,600 $10^3$ | 4,134 |
| $G$ , contour n°2 | 2,4969E+3 | 2,5996 $10^3$ | 4,114 |
| $G$ , contour n°3 | 2,4969E+3 | 2,5996 103 | 4,111 |
| **Diameter crowns external =** $0{,}01*a$ | | | |
| **radiant Mesh** | $NS=8$ | $NC=4$ | **Modelization C** |
| $G$ , contour n°1 | 2,4969E+3 | 2,451 $10^3$ | 1,842 |
| $G$ , contour n°2 | 2,4969E+3 | 2,475 $10^3$ | 0,858 |
| $G$ , contour n°3 | 2,4969E+3 | 2,475 $10^3$ | 0,858 |
| $G$ , contour n°4 | 2,4969E+3 | 2,475 103 | 0,858 |
| **radiant Mesh** | **NS= 4** | **NC= 3** | **Modelization D** |
| $G$ , contour n°1 | 2,4969E+3 | 2,475 103 | 0,858 |
| $G$ , contour n°2 | 2,4969E+3 | 2,475 103 | 0,858 |
| $G$ , contour n°3 | 2,4969E+3 | 2,475 103 | 0,858 |
| **Diameter crowns external =** $0{,}1*a$ | | | |
| **radiant Mesh** | **NS= 8** | **NC= 4** | **Modelization E** |
| $G$ , contour n°1 | 2,4969E+3 | 2,5624E3 | 2,627 |
| $G$ , contour n°2 | 2,4969E+3 | 2,5503E3 | 2,139 |
| $G$ , contour n°3 | 2,4969E+3 | 2,5499E3 | 2,124 |
| $G$ , contour n°4 | 2,4969E+3 | 2,5489 E3 | 2,084 |
| **radiant Mesh** | **NS= 4** | **NC= 3** | **Modelization F** |
| $G$ , contour n°1 | 2,4969E+3 | 2,5470 E3 | 2,006 |
| $G$ , contour n°2 | 2,4969E+3 | 2,5497 E3 | 2,117 |
| $G$ , contour n°3 | 2,4969E+3 | 2,5491 E3 | 2,094 |

## 4.2 Remarks

In the reference, the author supposes that $KI=0$ , but it does not check it a posteriori.

With regard to rate of energy restitution $G$ , if we suppose that $KI=0$ , we draw the value of reference from the formula from IRWIN in plane stresses:

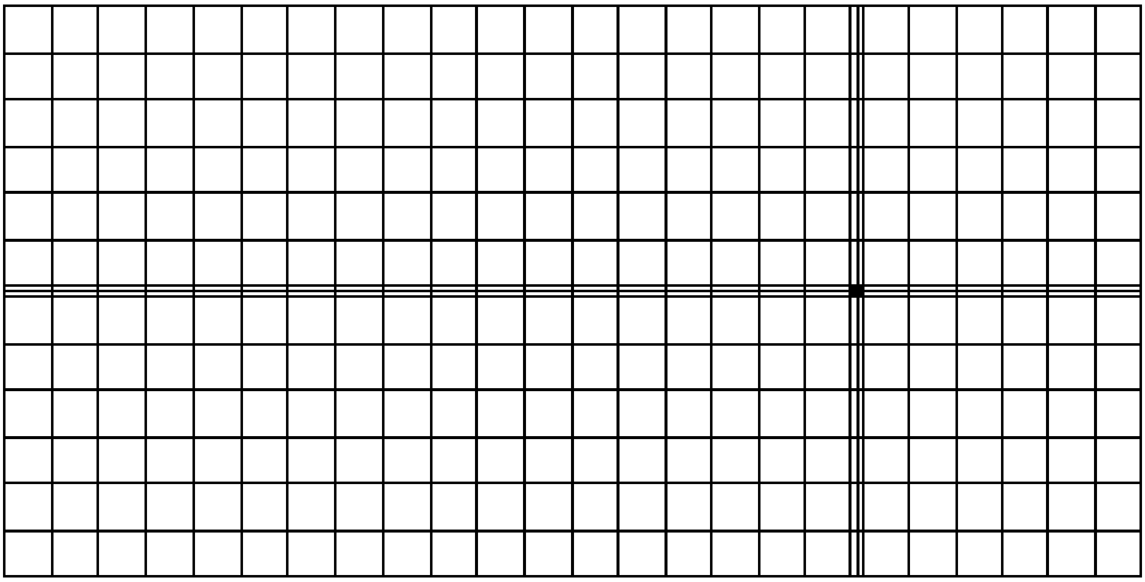
$$G_{ref}=(1/E)*KII^2$$

# 5 Modelization G

## 5.1 Characteristic of the modelization

For this modelization, we use the model complete with the best parameters $NS$, $NC$ and $rt$ calculated in the preceding modelizations. We thus used the following values:

- $NS=8$
- $NC=4$
- $rt=0{,}01*a$ .



**Complete mesh**

## 5.2 Characteristics of the complete

Model mesh, with mesh radiating only at the right end of crack and regular mesh, not refined, at the left end.

The mesh consists of 1718 nodes and 568 elements, including 464 elements `QUA8` and 104 elements `TRI6`.

## 5.3 Quantities tested and results

| Identification | Reference | Aster | % difference |
|---|---|---|---|
| Diameter crowns external = $0,01*a$ | | | |
| radiant Mesh | $NS=8$ | $NC=4$ | |
| $K_{II}$ , contour n°1 | 2,2347E+7 | 2,2640E7 | 1,31 |
| $K_{II}$ , contour n°2 | 2,2347E+7 | 2,2640E7 | 1,31 |
| $K_{II}$ , contour n°3 | 2,2347E+7 | 2,2640E7 | 1,31 |
| $K_{II}$ , contour n°4 | 2,2347E+7 | 2,2641E7 | 1,31 |

| Identification | Reference | Aster | % difference |
|---|---|---|---|
| Diameter crowns external = $0,01*a$ | | | |
| radiant Mesh | $NS=8$ | $NC=4$ | |
| $G$ , contour n°1 | 2,4969E+3 | 2,5620E3 | 2,610 |
| $G$ , contour n°2 | 2,4969E+3 | 2,5626E3 | 2,631 |
| $G$ , contour n°3 | 2,4969E+3 | 2,5627E3 | 2,635 |
| $G$ , contour n°4 | 2,4969E+3 | 2,5628E3 | 2,640 |

# 6 Summary of the results

the differences between the reference solution and the results of *Code_Aster* do not exceed $3\%$ on the coefficients of intensity of stresses and $4\%$ for rate of energy restitution. One checks the invariance of the results compared to various integration contours.