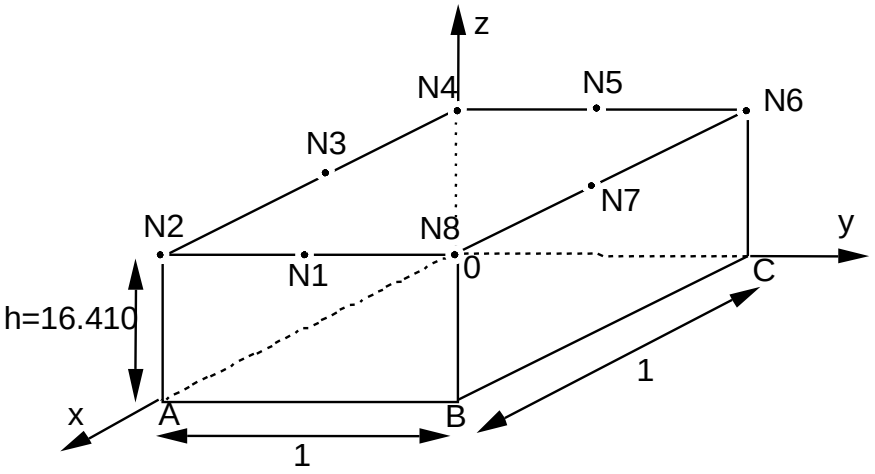
HPLV101 - Homogenization of a Summarized homogeneous

material:

This test tests, in a commonplace situation where the material is homogeneous, the resolution of the thermal and mechanical problems steady, with loadings corresponding to a variation in temperature and an imposed strain, close to those corresponding to the elementary problems of the method of periodic homogenization.

1 Problem of reference

1.1 Geometry



	x	y	z
N4	0	0	16.410
N8	1.	1.	16.410
N3	0.5	0	16.410

1.2 Material properties

Modelization A

$E = 1.0\,MPa$
$\nu = 0.3$
$k = 1.0\,W/(m.°C)$
$C_p = 0\,J/(°C.m^3)$

Modelization B

$E_L = 1.0\,MPa$
$E_T = 0.9\,MPa$
$E_N = 0.8\,MPa$
$\nu_{LT} = 0.1$
$\nu_{LN} = 0.25$
$\nu_{TN} = 0.3333333$
$k = 1.0\,W/(m.°C)$
$C_p = 0\,J/(°C.m^3)$

1.3 Boundary conditions and loadings

- Mechanics 3D:

Plane $z=0$:	$dz=0$	for the membrane loading;
	$dx=0$, $dy=0$	for the loading of cross-bending
$y=0$ $y=1$:	$dy=0$	
Planes $x=0$ $x=1$:	$dx=0$	
The node is outside the field of definition with a right profile of the EXCLU type node: O	$dz=0$	(for the only loading of bending)

Loading:

$$\text{déformation membranaire:}\quad \boldsymbol{E} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{flexion uniforme imposée:}\quad \boldsymbol{E} = \begin{pmatrix} \frac{z}{z_0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$z_0 = 1\text{m}$$

- Mechanics 2D, plane stresses:
  Center: $x=0$ $dx=0$ (these conditions do not correspond to the application of the method of homogenization).
  The node is outside the field of definition with a right profile of the EXCLU type node: O $dy=0$

  Loading: thermal strain $\boldsymbol{E} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ uniforme imposée

- 3D and 2D:
  Plane $x=0$ $temp=0$ (this condition does not correspond to the application of the method of homogenization).

  Loading: imposed $\mathbf{G} = (-1,0,0)$ uniform gradient.

2 Reference solution

2.1 Method of calculating used for the reference solution

- In thermal: the steady thermal problem is solved:

$$\int_\Omega \nabla T.\mathbf{K}.\nabla\theta = \int_\Omega \mathbf{G}.\mathbf{K}.\nabla\theta \quad , \quad \forall\theta \in \mathrm{V} \quad , \quad \text{avec } \mathbf{G} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Note:

The boundary conditions chosen here are not those necessary to the method of homogenization: one would find indeed $T = 0$ everywhere.

The solution is then (checking the conditions defined in [§1.3]): $T(x,y,z) = -x$

Potential energy is then with the equilibrium: $\mathrm{W}^{th} = -\frac{1}{2}\int_\Omega \nabla T.\mathbf{K}.\nabla T = -\frac{1}{2}|\Omega|$ ici

- In mechanics: one solves the problem of elastostatic:

$$\int_\Omega \varepsilon(\mathbf{u}).\mathbf{A}.\varepsilon(\mathbf{v}) = \int_\Omega \boldsymbol{E}.\mathbf{A}.\varepsilon(\mathbf{v}) \quad , \quad \forall \mathbf{v}\, \varepsilon\, \mathrm{W}$$

for the cases:

loading 3D membrane	loading 3D of bending	loading 2D plane stresses
$\boldsymbol{E} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\boldsymbol{E} = \begin{pmatrix} \frac{z}{z_0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\boldsymbol{E} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

the solutions are:

- in 3D, membrane **loading** and isotropic elasticity: $\mathbf{u}(x,y,z) = \left(0,0,-\frac{\nu z}{(1-\nu)}\right)$

potential energy with the equilibrium is:

$$\mathrm{W}^{pot} = -\frac{1}{2}\int_\Omega \varepsilon(\mathbf{u}).\mathbf{A}.\varepsilon(\mathbf{u}) = -\left(\frac{\nu}{1-\nu}\right)^2.\frac{|\Omega|}{2}(\lambda+2\mu)$$

- in 3D, membrane **loading** and orthotropic elasticity: $\mathbf{u}(x,y,z) = (0,0,-\beta z)$ with $\beta = \frac{C_{13}}{C_{33}}$ is

$\beta = \dfrac{\frac{E_N}{E_L}(\nu_{LN} + \nu_{LT}\nu_{TN})}{1 - \frac{E_N}{E_T}\nu_{TN}^{\,2}}$ because the local coordinate system is not confused with the total reference (nautical angles being worth all 90°).

$$\mathrm{W}^{pot} = -\frac{1}{2}\int_\Omega (\mathbf{u}).\mathbf{A}.\varepsilon(\mathbf{u}) = -\beta^2.\frac{|\Omega|}{2}C_{33}$$

where $C_{33}=\frac{1-\nu_{NT}\nu_{TN}}{E_T E_N \Delta}$ and $\Delta=\frac{1-\nu_{LT}\nu_{TL}-\nu_{LN}\nu_{NL}-\nu_{TN}\nu_{NT}-2\nu_{LT}\nu_{TN}\nu_{NL}}{E_L E_T E_N}$

- in 3D, loading of **bending** : $\mathbf{u}(x,y,z)=\left(0,0,\frac{+\nu z^2}{2z_0(1-\nu)}\right)$;

$$W^{pot}=-\left(\frac{\nu}{1-\nu}\right)^2.\frac{|\Omega|}{2}(\lambda+2\mu).\frac{h^2}{3z_0^2}$$

- in 3D, loading **of bending** and orthotropic elasticity: $\mathbf{u}(x,y,z)=\left(0,0,-\beta\frac{z}{L}\right)$ with $\beta=\frac{C_{13}}{C_{33}}$ is $\beta=\frac{\frac{E_N}{E_L}(\nu_{LN}+\nu_{LT}\nu_{TN})}{1-\frac{E_N}{E_T}\nu_{TN}^2}$ because the local coordinate system is not confused with the total reference (nautical angles being worth all 90°).

$$W^{pot}=-\frac{1}{2}\int_\Omega \varepsilon(\mathbf{u}).\mathbf{A}.\varepsilon(\mathbf{u})=-\beta^2.\frac{|\Omega|}{2}C_{33}\frac{h^2}{3z_0^2}$$

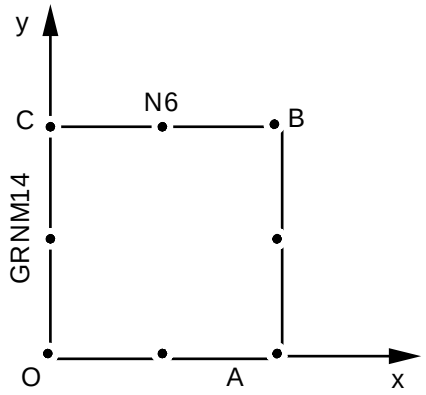
where $C_{33}=\frac{1-\nu_{NT}\nu_{TN}}{E_T E_N \Delta}$ and $\Delta=\frac{1-\nu_{LT}\nu_{TL}-\nu_{LN}\nu_{NL}-\nu_{TN}\nu_{NT}-2\nu_{LT}\nu_{TN}\nu_{NL}}{E_L E_T E_N}$

- in 2D, plane loading: $\mathbf{u}(x,y)=(-x,\ 0)$;

$$W^{pot}=\frac{-|\Omega|}{2(1-\nu^2)}$$

3 Modelization A

3.1 Characteristic of the modelization



Boundary conditions and loading:

Thermal:
```
GROUP_NO: GRNM14: TEMP: 0.0
PRE_GRAD_TEMP: FLUX_X: - 1.0
```

Mechanics:
(plane stresses)
```
GROUP_NO: GRNM14: DX: 0.0

THE NODE IS OUTSIDE THE FIELD OF DEFINITION WITH A
RIGHT PROFILE OF THE EXCLU TYPE NODE: O
DY: 0.0
PRE_EPSI: EPXX: - 1.0
```

3.2 Characteristics of the mesh

Many nodes: 8

Number of meshes and types: 1 QUAD8

3.3 Values tested

Not	Quantity	Reference
	CMP	
A	*TEMP*	– 1.0000
A	*DX*	– 1.0000
N6	*DX*	– 0.5000

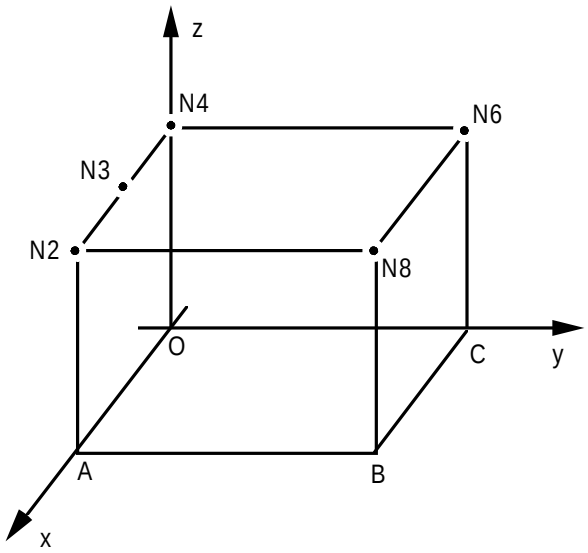
Mesh	Potential energy with the Thermal	equilibrium
M1	Reference	– 0.500000000
M1	Mechanics	– 0.549450550

3.4 Remarks

Code_Aster provides the value of strain energy, equal contrary to potential energy to the equilibrium (elastic case).

4 Modelization B

4.1 Characteristic of the modelization



Name of meshes of the sides:	*ZEGAL0*	*YEGAL0*	*YEGAL1*	*XEGAL0*	*XEGAL1*
Summits:	*B C O A*	*O A N2 N4*	*B C N6 N8*	*C O N4 N6*	*A B N8 N2*

Boundary conditions:

Thermal:
```
ZERO: DEFI_CONSTANTE (VALE: 0.0)  ;
FCT1:   DEFI_FONCTION   (Nom_para:   "Z",   VALE:   (0.0
0.0.1.0.1.0));
GROUP_NO: XEGAL0: TEMP: 0.0
PRE_GRAD_TEMP: FLUX_X: -1.0
```

Mechanics:
```
GROUP_NO:  YEGALO: DY = 0.0
            XEGAL1: DX = 0.0
            YEGAL1: DY = 0.0
            XEGALO: Membrane DZ =
```

0.0 Case:
```
GROUP_NO:  ZEGALO: DZ = 0.0
PRE_EPSI: EPXX: -1.0
```

Case bending:
```
GROUP_NO:  ZEGALO: DX = ZERO,  DY = ZERO
THE NODE IS OUTSIDE THE FIELD OF DEFINITION WITH A RIGHT
PROFILE     OF     THE     EXCLU     TYPE     NODE:     0
DZ = ZERO
PRE_EPSI:                       EPXX: FCT1
```

4.2 Characteristic of the mesh

Many nodes: 20
Number of meshes and types: 1 HEXA20

4.3 Values tested

In isotropic elasticity

Case	Thermal	Quantity	Not
Reference	*temp*	*N8*	– 1.000000
	temp	*N3*	– 0.500000
Mechanics membrane	*dz*	*N4*	– 7.03285714
	dz	*N8*	– 7.03285714
Mechanics bending	*dz*	*N4*	57.70459285
	dz	*N8*	57.70459285

Nets	Potential energy with the Thermal	equilibrium
M1	Reference	– 8.20500
M1	Mechanical	
	Membrane	– 2.0287088
	Bending	– 1.8210238 102

In orthotropic elasticity

Case	Thermal	Quantity	Not
Reference	*temp*	*N8*	– 1.000000
	temp	*N3*	– 0.500000
Mechanics membrane	*dz*	*N4*	– 6.63044894
	dz	*N8*	– 6.63044894
Mechanics bending	*dz*	*N4*	54.40283358
	dz	*N8*	54.40283358

Nets	Potential energy with the Thermal	equilibrium
M1	Reference	– 8.20500

5 Summary of the results

the results are exact with errors rounding close, since the sought solutions are part of space of the finite elements selected for the modelization.