HSNV100 - Thermoplasticity in simple tension

Summarized:

This test treats the thermoplasticity of Von Mises with isotropic hardening on a three-dimensional problem (modelization A into axisymmetric) and two-dimensional (modelization B in plane stresses). The interest of the test is due to the dependence of the elastic limit with the temperature. It also makes it possible to test the orthotropy in thermoelasticity because it applies to an isotropic material then with an isotropic material declared orthotropic.
One tests there also the computation of strain energy.

Two modelizations (C with element PIPE, D with element TUYAU_6M) are added to test thermoplasticity in these elements.

The modelization E allows to test the good taking into account of the variation of the coefficients of behavior VMIS_CINE_LINE with the temperature (axisymmetric).

The modelization F allows to test the computation of thermo-elastic strain energy in the beams (modelization POU_D_T).

The modelization G allows to test the same functionalities as the modelizations A and B, but with a modelization 3D.

The modelizations H and I make it possible to test, in modelization 3D and plane stresses, an initial loading in unelastic strain field. This one is equivalent to a thermal strain.

The modelization J is resulting from the modelization G, and allows to validate the features of SIMU_POINT_MAT in thermoplasticity.

The modelization K is resulting from the modelization A, and allows to validate option AFFE_CHAR_TEMP_R of modelization AXIS_INCO. Even thing for the modelizations L and M but for 3D_INCO.

The modelization N is resulting from the modelization A, and allows to validate option AFFE_CHAR_TEMP_R of modelization AXIS_INCO_UP. Even thing for the modelizations O and P but for 3D_INCO_UP.

The modelization Q is resulting from the modelization A, and allows to validate option AFFE_CHAR_TEMP_R of modelization AXIS_INCO_GD. Even thing for the modelizations R and S but for 3D_INCO_GD.

The modelization T is resulting from the modelization A , and makes it possible to validate option AFFE_CHAR_TEMP_R of modelization AXIS_INCO_GD . Even thing for the modelizations U and V but for 3D_INCO_LOG .

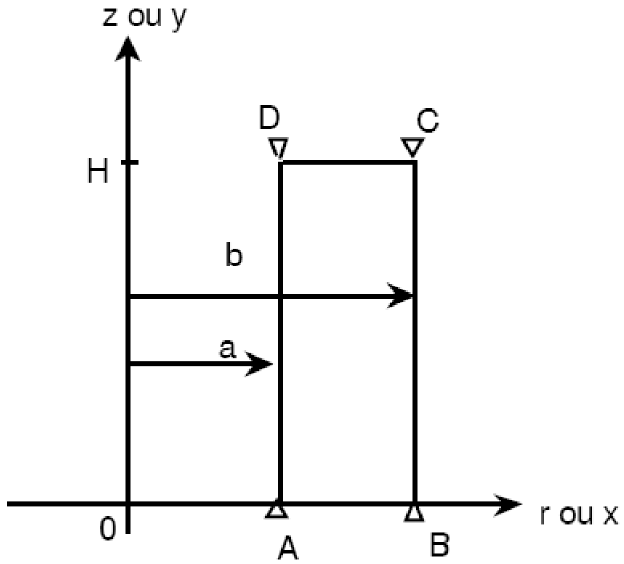
The modelization W is resulting from the modelization A , and makes it possible to validate option AFFE_CHAR_TEMP_R of modelization AXIS_INCO_LUP . Even thing for the modelizations X and Y but for 3D_INCO_LUP .

The solution is analytical.

1 Problem of reference

1.1 Geometry

Rolls axisymmetric (modelizations A and E) or plates rectangular (modelization B) or right pipe (modelizations C and D), or beam (modelization F), or parallelepiped 3D
(modelizations G, H and J).



Geometry of the cylinder (mm) :

- $a=1$
- $b=2$
- $H=4$

1.2 Property of the materials

For all the modelizations:

Young modulus: $E=200000\,MPa$
Tangent modulus: $E_t=50000\,MPa$
Poisson's ratio: $\nu=0.3$
$\sigma_0=400\,MPa$
$s=1.0\,E^{-2}\,°C^{-1}$
Thermal coefficient of thermal expansion: $\alpha=1.0\,E^{-5}\,°C^{-1}$
Voluminal heat: $C^p=0\,J.mm^{-3}.°C^{-1}$
Thermal conductivity: $\lambda=1.0\,E^{-3}\,W.mm^{-1}.°C^{-1}$

For the isotropic material declared orthotropic, it comes:

E_L = E_T = E_N = E

Nu_LT = Nu_LN = Nu_TN = ν

G_LT = G_LN = G_TN = $75000\,MPa$

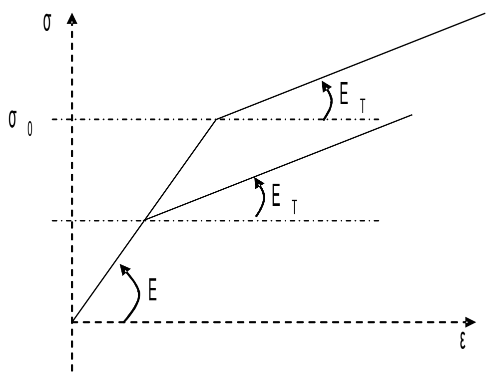
ALPHA_L = ALPHA_T = ALPHA_N = α

For the modelizations of A with G :

$$\sigma_y(T)=\sigma_0(1-s.(T-T_0))$$

For the modélisationet H I :

$$\sigma_y(T)=\sigma_0 \quad (s=0)$$



Appear : Curve of tension of the material

1.3 Boundary conditions and loadings

- Modelizations A , E and K : $uz=0$ on the sides AB and CD (Axis Oz fixes)
- Modelizations B and I : $uy=0$ on the sides AB and CD , $ux=0$ in A
- Modelizations C,D and F : fixed support in A , $Uy=0$ in C
- Modelizations G H , L and M : $uy=0$ on the sides AB and CD , $ux=uz=0$ (node $N3$), $uz=0$ (node $N4$)
- $T(t)=\gamma t+T0$ with: $\gamma=1°C/s$ and $T0=0°C$.
- Modelizations H and I : Strain fields initial: $\varepsilon=\alpha(T-T_0)Id$
- Modelization J : Strain field imposed: $\varepsilon_{yy}=0$

2 Reference solution

2.1 Méthode de calcul used for the analytical reference solution

Solution was determined by F. Voldoire (EDF R & D/AMA):

Axisymmetric case ($2D$)

Fields of displacement $u=U_r(r)e_r$ (blocking in z)

Strain fields: $\varepsilon(u)=\begin{pmatrix} u_r' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{u_r}{r} \end{pmatrix}$ according to $\begin{pmatrix} r \\ z \\ \theta \end{pmatrix}$

Stress fields: $\sigma(u)=\sigma_l\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}(cf. conditions\ aux\ limites)$ according to $\begin{pmatrix} r \\ z \\ \theta \end{pmatrix}$

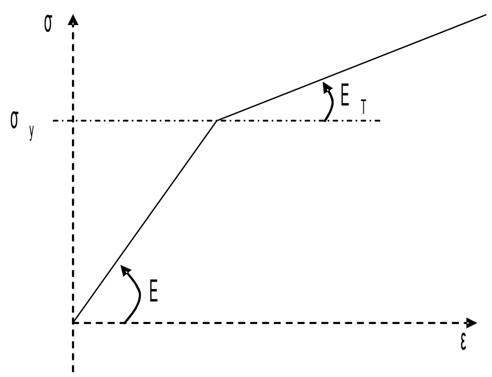
parallelepipedic Case

Fields of displacement $u=U_x(x)e_x+U_y(y)e_y$ (blocking in z)

Strain fields: $\varepsilon(u)=\begin{pmatrix} u_x' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & u_y' \end{pmatrix}$ according to $\begin{pmatrix} x \\ z \\ y \end{pmatrix}$

Stress fields: $\sigma(u)=\sigma\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}(cf. conditions\ aux\ limites)$ according to $\begin{pmatrix} x \\ z \\ y \end{pmatrix}$

the case could be studied in plane stresses and in $3D$.



$$2\nu=\frac{E}{(1+\nu)} \quad K=\frac{E}{(1-2\nu)}$$

The constitutive law is written (scalar local variable p):

$$\begin{cases} \varepsilon=\frac{1}{9K} tr\,\sigma\, Id+\frac{1}{2\mu}\sigma^D+\varepsilon^P+\alpha(T-T^0)\, Id \\ \text{with: } \sigma^D=\sigma-\frac{1}{3}tr(\sigma)\, Id \quad \text{(deviator of the stresses)} \\ \dot{\sigma}^P=\frac{3}{2}\dot{p}\frac{\sigma^D}{\|\sigma_{éq}\|} \text{et} \|\sigma_{éq}\|=\sqrt{\frac{3}{2}\sigma^D.\sigma^D} \\ \dot{p}=0 \quad \text{if} \quad f(\sigma, p)=\|\sigma_{éq}\|-R(p)<0 \\ \dot{p}\geqslant 0 \quad f(\sigma, p)=0 \end{cases}$$

$R(p)$ the function of hardening indicates: $R(p)=\sigma_y+\frac{EE_T}{E-E_T}p$

The tauxpeut $\dot{p}$ to be expressed, when $f(\sigma, p)=0$. Indeed, of $\dot{p}\, f$ identically no one, one draws: $\dot{p}\,\dot{f}+\ddot{p}\, f=0$

Thus, when one is on the criterion, necessarily $\dot{f}=0$. It is - with-to say:

$$\frac{3}{2}\frac{\sigma^D.\dot{\sigma}^D}{\|\sigma_{éq}\|}-\frac{\partial R}{\partial T}.\dot{T}-\frac{\partial R}{\partial p}\dot{p}=0$$
$$\frac{3}{2}\frac{\sigma^D.\dot{\sigma}^D}{\|\sigma_{éq}\|}+\sigma_y^o s\,\dot{T}-\frac{EE_T}{E-E_T}\dot{p}=0$$

From where:

$$\dot{p}=\frac{E-E_T}{EE_T}\left(\frac{3}{2}\frac{\sigma^D\dot{\sigma}^D}{\|\sigma_{éq}\|}+\sigma_y^o.s.\dot{T}\right) \text{ if } \dot{p}\geq 0 \text{, for the field } \|\sigma_{éq}\|=R(p)$$

of stresses being uniaxial, one a:

$$\sigma^D=\frac{\sigma_L}{3}\begin{pmatrix}-1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & -1\end{pmatrix}$$

As follows:

$$\|\sigma_{éq}\|=|\sigma_L|$$

and:

$$\dot{\varepsilon}^P=\frac{\dot{p}}{2}.sgn(\sigma_L).\begin{pmatrix}-1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & -1\end{pmatrix}$$

The behavior model leads to:

$$\begin{cases} \dot{\varepsilon_{rr}}=\dot{\varepsilon_{\theta\theta}}=\dfrac{-\nu}{E}\dot{\sigma}_L-\dfrac{\dot{p}}{2}sgn(\sigma_L)+\alpha\dot{T} \quad (=\dot{\varepsilon}_{xx}=\dot{\varepsilon}_{yy}\text{ pour le cas du parallélépipède}) \\ \dot{\varepsilon_{rr}}=\dot{\varepsilon_{\theta\theta}}=\dfrac{-\nu}{E}\dot{\sigma}_L-\dfrac{\dot{p}}{2}sgn(\sigma_L)+\alpha\dot{T} \end{cases}$$

From where:

$$\begin{cases} \dot{\varepsilon_{rr}}=\dot{\varepsilon_{\theta\theta}}=\dfrac{-3}{2}\alpha\dot{T}+\dfrac{1-2\nu}{2E}\dot{\sigma}_L \\ \dot{p}=sgn(\sigma_L)(-\alpha\dot{T}-\dfrac{\dot{\sigma}_L}{E})=0 \ \text{ si } \ |\sigma_L|<R(p) \\ \dot{p}=Max[0,\dfrac{E-E_T}{EE_T}(\dfrac{3}{2}\dfrac{\sigma^D\dot{\sigma}^D}{\|\sigma_{éq}\|}+\sigma_y^o.s.\dot{T})] \ \text{ sinon} \end{cases}$$

I.e., in the case $|\sigma_L|=R(p)$ (criterion reached):

$$\dot{p}=max\left[0,\frac{E-E_T}{EE_T}\left(\frac{3}{2}\frac{\sigma^D.\dot{\sigma}^D}{\|\sigma_{éq}\|}+\sigma_y^o.s.\dot{T}\right)\right]$$

2.1.1 Elastic phase

At the beginning of the thermal loading, $|\sigma_L|$ being lower than σ_y, $\dot{p}$ is null.

From where: $\dot{\sigma}_L=-E\alpha\dot{T}\,;\dot{\varepsilon_{rr}}=\dot{\varepsilon_{\theta\theta}}=\alpha\dot{T}(1+\nu)$

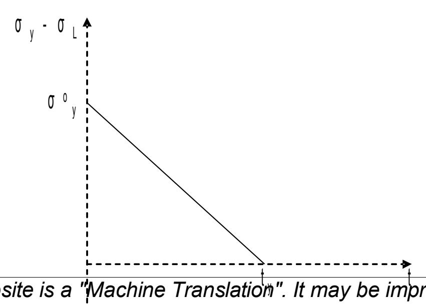
As follows: $\begin{cases}\sigma_L=-E\alpha\theta t\\ \varepsilon_{rr}=\varepsilon_{\theta\theta}=\alpha\theta(1+\nu)t\end{cases}$ (compression $\sigma_L<0$)

This corresponds to the reference solution for the test of orthotropic elasticity

Validity of the elastic solution

the criterion is: $|\sigma_L(t)|-\sigma_y(t)=E=\theta t-\sigma_y^o(1-s\theta t)\leq 0$

The criterion is not crossed for $t=[0,t_y]$, with: $t_y=\dfrac{\sigma_y^o}{\theta(E\alpha+\sigma_y^o s)}$



At time t_y : $\sigma_L(t_y)=\frac{-E\,\alpha\,\sigma_y^o}{E\,\alpha+\sigma_y^o s}$

The density of strain energy is worth: $w(t_y)=\frac{-1}{2}E(\alpha\,\theta)^2$

In the parallelepipedic case one a: $w(t_y)=\frac{-1}{2}E\,(\alpha\theta)^{2.}(x_B-x_A)H$

In the axisymmetric case one a: $w(t_y)=\frac{-1}{2}E\,(\alpha\theta)^2.\frac{(r_B^2-r_A^2)}{2}H$ (for 1 radian)

2.1.2 elastoplastic Phase

$t\geq t_y$. One is on the criterion. Then:

$$\dot{p}=max\left[0,\frac{E-E_T}{EE_T}\left(\dot{\sigma_L}\,sgn(\sigma_L)+\sigma_y^o.s.\dot{T}\right)\right]$$

By admitting that one is “charges some” $\dot{p}\geq 0$, then one eliminates $\dot{p}$ to have:

$$\dot{\sigma_L}=-E_T.\dot{T}\,(\alpha+sgn(\sigma_L).\frac{E-E_T}{EE_T}\sigma_y^o.s)$$

then:

$$\dot{p}=\frac{E-E_T}{E}\dot{T}(-\alpha\,sgn(\sigma_L)+\frac{\sigma_y^o.s}{E})$$

$t=t_y$ $\sigma_L=-E\,\alpha\theta\,t_y<0$; one integrates these statements then for $t\geq t_y(\dot{T}=\theta)$:

$$\begin{cases}\sigma_L(t)=-E_T.\theta(t-t_y)[\alpha-\frac{E-E_T}{EE_T}s\,\sigma_y^o]-\sigma_L(t-y)\\ p(t)=\frac{\sigma_y^o(E-E_T)}{E^2}(\frac{t}{t_y}-1)\end{cases}$$

Maybe, after rearrangement $t\geq t_y$:

$$\begin{cases}\sigma_L(t)=\sigma_y^o(s\,\theta\,t-1+\frac{E_T}{E}(1-\frac{t}{t_y}))\\ p(t)=\frac{\sigma_y^o(E-E_T)}{E^2}(\frac{t}{t_y}-1)\end{cases}$$

Validity of this elastoplastic solution

It should be made sure that $\sigma_L(t)$ remains negative. Knowing that $s\,\theta\,t<1$, and that $t>t_y$, the result preceding one confirms that $\sigma_L(t)<0$. Lastly, it is noticed that:

$$sgn(\sigma_L)\frac{1-2\nu}{2}\dot{p}+\dot{\varepsilon_{rr}}=\alpha(1+\nu)\dot{T}$$

from where, since $\sigma_L(t)<0$:

$$\varepsilon_{rr}(t)=\varepsilon_{\theta\theta}(t)=\alpha\theta(1+\nu)t+\frac{1-2\nu}{2}p(t),\forall\ t\in[t_y,t_{fin}]$$

2.1.3 Cas particulier of the modelizations H and I

the thermal strain is replaced by a given unelastic strain. Like $s=0$,

$\sigma_L=-E\alpha\theta \qquad \varepsilon_{xx}=\varepsilon_{zz}=\alpha\theta(1+\nu)t$

the solution remains elastic as long as $t<t_y=\dfrac{\sigma_0}{\theta E\alpha}=200\,s$

2.2 Results of reference

ε_{rr} Or ε_{xx} , σ_{zz} and p in t_y and beyond:

Elastic phase : for $t<t_y$:

$\sigma_L=-E\alpha\theta t \quad \varepsilon_{rr}=\varepsilon_{\theta\theta}=\alpha\theta(1+\nu)t$ into axisymmetric
$\varepsilon_{xx}=\alpha\theta(1+\nu)t$ in plane stresses

the yield stress is reached in $t_y=\dfrac{\sigma_0}{\theta(E\alpha+\sigma_0 s)}=66.666\,s$ from where $\sigma_L(t_y)=\dfrac{\sigma}{(1+\sigma_0\dfrac{s}{E\alpha})}$

elastoplastic Phase : for $t\geq t_y$

$$\sigma_L(t)=\sigma_0\left(s\theta t-1+\frac{E_t}{E}\left(1-\frac{t}{t_y}\right)\right)$$

$$p(t)=\frac{\sigma_0(E-E_T)}{E^2}\left(\frac{t}{t_y}-1\right)$$

$$\varepsilon_{rr}=\varepsilon_{\theta\theta}=\alpha\theta(1+\nu)t+\frac{1-2\nu}{2}p(t)$$ into axisymmetric

ouen $\varepsilon_{xx}=\varepsilon_{\theta\theta}=\alpha\theta(1+\nu)t+\dfrac{1-2\nu}{2}p(t)$ plane stresses

From where:

elastic phase

$$\left.\begin{array}{l} t_y=66.666\,s \\ \sigma_L(t_y)=-133.333\,MPa \\ \varepsilon_{rr}(t_y)=\varepsilon_{\theta\theta}(t_y)=0.86666E^{-3} \end{array}\right\}\text{phase élastique}$$

$W=4.444\,10-3\,MPa$
$W=0.17778\,MPa.mm^2$ (`PLANE` or `3D`)
$W=0.26666\,Mpa.mm^3.rad-1$ (`axi`)

Then elastoplastic phase :
$à\ t=80\,s$: $\sigma(80)=-100.0\text{MPa}$
$p(80)=0.300\,E^{-3}$

$\varepsilon_{rr}(80)=\varepsilon_{\theta\theta}(80)=1.1\,E^{-3}$

with $t=90\,s$: $\sigma(90)=-75.0\text{MPa}$

$p(90)=0.525E^{-3}$

$\varepsilon_{rr}(90)=\varepsilon_{\theta\theta}(90)=1.275\,E^{-3}$

2.2.1 Cas particulier of the modelizations H and I

WITH $t=66.67\,s$ $\sigma_L(66.67)=-133.33\,MPa$

$\varepsilon_{xx}(66.67)=\varepsilon_{zz}(66.67)=8.667\,E^{-4}$

WITH $t=80\,s$ $\sigma_L(80)=-160\,MPa$

$\varepsilon_{xx}(80)=\varepsilon_{zz}(80)=8.667\,E^{-4}$

To $t=90\,s$ $\sigma_L(90)=-180\,MPa$

$\varepsilon_{xx}(90)=\varepsilon_{zz}(90)=8.667\,E^{-4}$

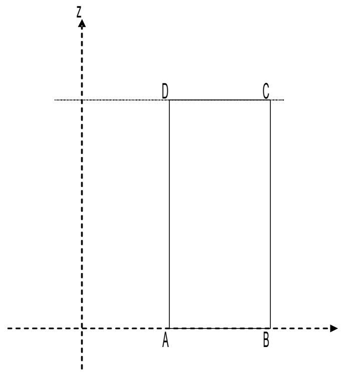
2.3 Uncertainty on the solution

analytical Solution.

3 Modelization A

3.1 Characteristic of modelization

QUAD4 - Axisymmetric



3.2 Characteristics of the mesh

Many nodes: 4
Number of meshes and types: 1 QUAD4, 4 SEG2

3.3 Quantities tested and Variable

results	Urgent (s)	Reference
$\varepsilon_{rr}=\varepsilon_{\theta\theta}$	$t=66.666$	8.6666 10-4
	$t=80$	1.1000 10-3
	$t=90$	1.2750 10-3
p	$t=66.666$	0
	$t=80$	3.0000 10-4
	$t=90$	5.2500 10-4
σ_{zz} (MPa)	$t=66.666$	– 133.333
	$t=80$	– 100.000
	$t=90$	– 75.000
ENEL_ELGA (J)	$t=66.666$	4.444. 10-2
ENER_TOTALE (J)	$t=66.666$	0.2666
ENER_POT (J)	$t=66.666$	0.2666

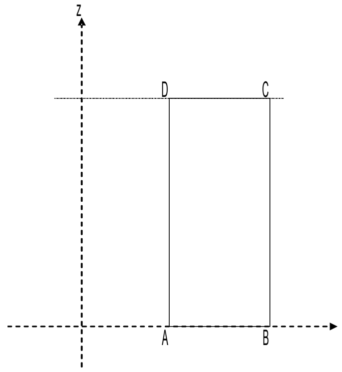
Orthotropy (COMP_ELAS and COMP_INCR)

Variable	Times (s)	Reference
$\varepsilon_{rr}=\varepsilon_{\theta\theta}$	$t=66.666$	8.6666 10-4
	$t=80$	1.04 10-3
	$t=90$	1.17 10-3
σ_{zz} (MPa)	$t=66.666$	– 133.333
	$t=80$	– 160.000
	$t=90$	– 180.000

4 Modelization B

4.1 Characteristic of modelization

QUAD4 - Plane stresses



4.2 Characteristic of the mesh

Many nodes: 4
Number of meshes and types: 1 QUAD4, 4 SEG2

4.3 Quantities tested and Variable

results	Urgent (s)	Reference
$EPXX$	$t=66.666$	8.6666 10-4
	$t=80$	1.1000 10-3
	$t=90$	1.2750 10-3
p	$t=66.666$	0
	$t=80$	3.0000 10-4
	$t=90$	5.2500 10-4
$SIYY\ (MPa)$	$t=66.666$	– 133.333
	$t=80$	– 100.
	$t=90$	– 75.000
ENEL_ELGA (J)	$t=66.666$	4.444. 10-2
ENER_TOTALE (J)	$t=66.666$	0.17777
ENER_POT (J)	$t=66.666$	0.17777

5 Modelization C

5.1 Characteristic of modelization

1 element `PIPE`

5.2 Characteristics of mesh

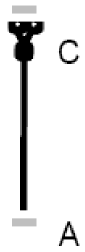
1 element `Urgent`

5.3 PIPE Quantities tested and

Variable	results (s)	Reference
p	$t=66.666$	0
	$t=80$	3. 10-4
	$t=90$	5.25 10-4
σ_{yy} (MPa)	$t=66.666$	– 1.333
	$t=80$	– 100
	$t=90$	– 75

6 Modelization D

6.1 Characteristic of modelization



1 element `PIPE 6M`

6.2 Caractéristiques of mesh

1 element `Urgent`

6.3 PIPE Quantities tested and

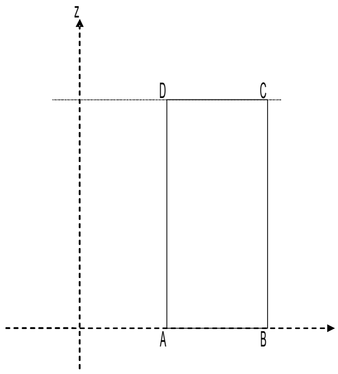
Variable	results (s)	Reference
p	$t=66.666$	0
	$t=80$	3. 10-4
	$t=90$	5.25 10-4
σ_{yy} (MPa)	$t=66.666$	– 1.333
	$t=80$	– 100
	$t=90$	– 75

7 Modelization E

7.1 Characteristic of modelization

QUAD4 - Axisymmetric.

Test of the variation of the coefficients of VMIS_CINE_LINE according to the temperature, in this case E_T (given by D_SIGM_EPSI) varies like: $E_T = 10^5 (1 - 10^{-2}(T - T_0))$. The constant of Prager is worth: $C = \frac{2}{3} \frac{E\,E_T}{E - E_T}$



7.2 Characteristics of the mesh

Many nodes: 4
Number of meshes and types: 1 QUAD4, 4 SEG2

7.3 Notices

One tests the variation of E_T (D_SIGM_EPSI) with the temperature by comparison with behavior VMIS_ECMI_TRAC where C (constant of Prager) varies with the similar temperature of way (not of analytical solution).

7.4 Urgent quantities tested and

results	Variable (s)	Reference (*Aster*) (VMIS_ECMI_TRAC)
$\varepsilon_{rr} = \varepsilon_{\theta\theta}$	$t = 66.666$	8.6666 10-4
	$t = 80$	1.112 10-3
	$t = 90$	1.303 10-3
σ_{zz} (MPa)	$t = 66.666$	– 133.333
	$t = 80$	– 88
	$t = 90$	– 47

7.5 Remark

One gets well the same results with behavior VMIS_CINE_LINE as with behavior VMIS_ECMI_TRAC what validates the taking into account of the temperature in this model.

8 Modelization F

8.1 Characteristic of modelization

1 element POU_D_T

8.2 Characteristics of mesh

1 nets Urgent

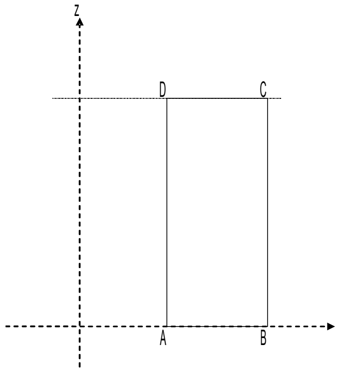
8.3 SEG2 Quantities tested and

Variable	results (s)	Reference
σ_{yy}	$t=66.666$	– 1.333
ENER_POT (J)	$t=66.666$	0.3555

9 Modelization G

9.1 Characteristic of the modelization

$3D, H=1$



9.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 1 `Urgent`

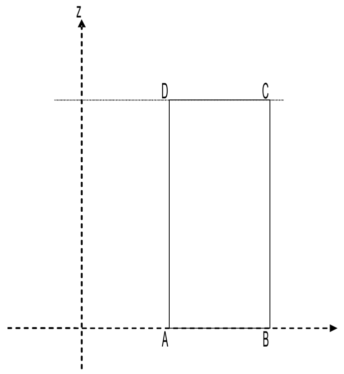
9.3 HEXA8 Quantities tested and

Variable	results (s)	Reference
ε_{xx}	$t=66.666$	8.6666 10-4
	$t=80$	1.1000 10-3
	$t=90$	1.2750 10-3
p	$t=66.666$	0
	$t=80$	3.0000 10-4
	$t=90$	5.2500 10-4
σ_{yy} (MPa)	$t=66.666$	– 133.333
	$t=80$	– 100.
	$t=90$	– 75.000
`ENEL_ELGA`	$t=66.666$	4.444. 10-2
`ENER_TOTALE`	$t=66.666$	4.444. 10^{-2}
`ENER_POT`	$t=66.666$	4.444. 10-2

10 Modelization H

10.1 Characteristic of the modelization

3D , $H=1$



10.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 1 `Urgent`

10.3 HEXA8 Quantities tested and

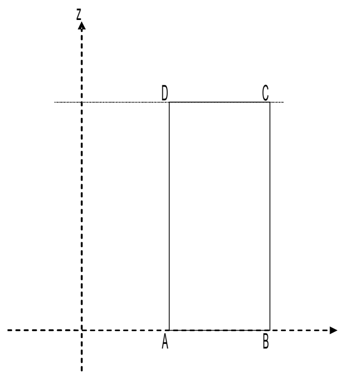
Variable	results (s)	Reference
ε_{xx}	$t=66.666$	8.9930 10-4
	$t=80$	1.0980 10-3
	$t=90$	1.2480 10-3
p	$t=66.666$	0
	$t=80$	2.9400 10-4
	$t=90$	3.9200 10-4
σ_{yy} (MPa)	$t=66.666$	– 100.666
	$t=80$	– 101.2
	$t=90$	– 101.6
`ENEL_ELGA`	$t=66.666$	2.53344 10-2
`EPSP_ELGA`		
ε_{xx}	$t=90$	1.960 10-4
ξ_{yy}	$t=90$	-3.920 10^{-4}
`EPSP_ELNO`		
ε_{xx}	$t=90$	1.960 10-4
ξ_{yy}	$t=90$	-3.920 10-4
`EPME_ELGA`		
ε_{xx}	$t=90$	3.484 10-4
ξ_{yy}	$t=90$	-9.000 10-4
`EPME_ELNO`		
ε_{xx}	$t=90$	3.484 10-4
ξ_{yy}	$t=90$	-9.000 10-4
`EPMG_ELGA`		

ε_{xx}	$t=90$	3.491 10-4
ξ_{yy}	$t=90$	-9.000 10-4
EPMG_ELNO		
ε_{xx}	$t=90$	3.491 10-4
ξ_{yy}	$t=90$	-9.000 10-4
ENER_TOTALE	$t=66.666$	4.17215. 10-2

11 Modelization I

11.1 Characteristic of modelization

QUAD4 - Plane stresses



Many nodes: 4
Number of meshes and types: 1 QUAD4, 4 SEG2

11.2 Quantities tested and Variable

results	Urgent (s)	Reference
p	$t=66.666$	0
	$t=80$	2.940 10-4
	$t=90$	3.9200 10-4
σ_{yy} (MPa)	$t=66.666$	– 100.666
	$t=80$	– 101.2
	$t=90$	– 101.6
ENEL_ELGA	$t=66.666$	2.53344 10-2
EPSP_ELGA		
ε_{xx}	$t=90$	1.960 10-4
ξ_{yy}	$t=90$	-3.920 10^{-4}
EPSP_ELNO		
ε_{xx}	$t=90$	1.959 10-4
ξ_{yy}	$t=90$	-3.9199 10-4
INDIC_ENER	$t=90$	0.00
INDIC_SEUIL	$t=90$	0.00

12 Modelization J

12.1 Characteristic of the modelization

Not of mesh: material point (SIMU_POINT_MAT)

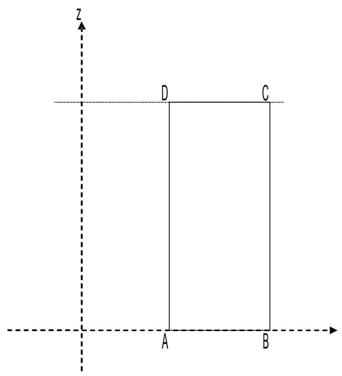
12.2 Quantities tested and Urgent

Variable	results (s)	Reference
ε_{xx}	$t=66.666$	8.6666 10-4
	$t=80$	1.1000 10-3
	$t=90$	1.2750 10-3
p	$t=66.666$	0
	$t=80$	3.0000 10-4
	$t=90$	5.2500 10-4
σ_{yy} (MPa)	$t=66.666$	– 133.333
	$t=80$	– 100.
	$t=90$	– 75.000

13 Modelization K

13.1 Characteristic of modelization

QUAD8 - Axisymmetric



13.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 1 QUAD8, 4 SEG3

13.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=66,666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.1%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.1%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66,666\,s)$	"ANALYTIQUE"	0	0.1%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.1%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.1%
$\sigma_{zz}(t=66,666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.1%
$\sigma_{zz}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.3%
$\sigma_{zz}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.1%

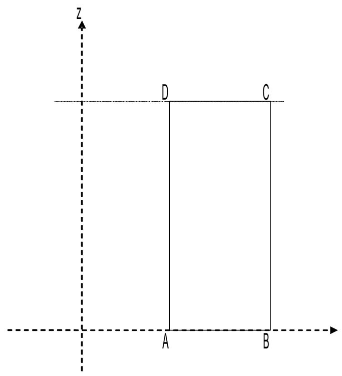
13.4 Remark

One gets well the same results with formulation AXIS_INCO as with formulation AXIS classic.

14 Modelization L

14.1 Characteristic of the modelization

$3\mathrm{D}, H=1$



14.2 Characteristics of the mesh

Many nodes: 20
Number of meshes and types: 1 HEXA20

14.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.1%
$\varepsilon_{xx}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.1%
$\varepsilon_{xx}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66,666\,s)$	"ANALYTIQUE"	0	0.1%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.1%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.1%
$\sigma_{yy}(t=66,666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.1%
$\sigma_{yy}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.1%
$\sigma_{yy}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.1%

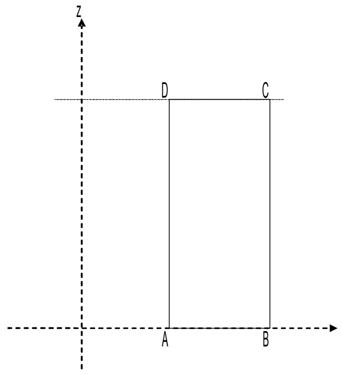
14.4 Remark

One gets well the same results with formulation 3D_INCO as with the formulation 3D classical.

15 Modelization M

15.1 Characteristic of the modelization

$3\mathrm{D}, H=1$



15.2 Characteristics of the mesh

Many nodes: 35
Number of meshes and types: 12 `TETRA10`

15.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.1%
$\varepsilon_{xx}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.1%
$\varepsilon_{xx}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.1%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.1%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.1%
$\sigma_{yy}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.1%
$\sigma_{yy}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.1%
$\sigma_{yy}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.1%

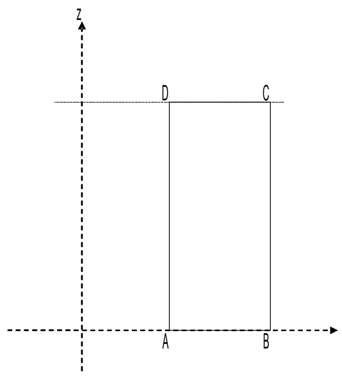
15.4 Remark

One get well the same results with formulation `3D_INCO` as with the formulation `3D` classical.

16 Modelization N

16.1 Characteristic of modelization

QUAD8 - Axisymmetric



16.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 1 QUAD8, 4 SEG3

16.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=66,666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.1%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.1%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66,666\,s)$	"ANALYTIQUE"	0	0.1%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.1%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.3%
$\sigma_{zz}(t=66,666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.1%
$\sigma_{zz}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.1%
$\sigma_{zz}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.1%

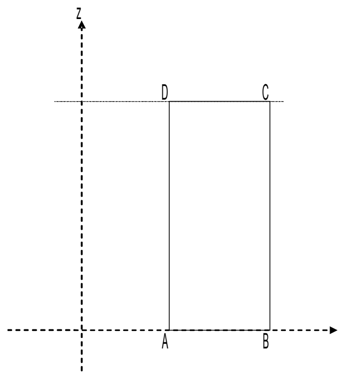
16.4 Remark

One gets well the same results with formulation AXIS_INCO_UP as with formulation AXIS classic.

17 Modelization O

17.1 Characteristic of the modelization

3D , $H=1$



17.2 Characteristics of the mesh

Many nodes: 20
Number of meshes and types: 1 HEXA20

17.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.1%
$\varepsilon_{xx}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.1%
$\varepsilon_{xx}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.1%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.1%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.1%
$\sigma_{yy}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.1%
$\sigma_{yy}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.1%
$\sigma_{yy}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.1%

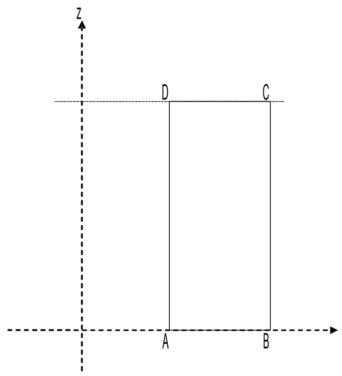
17.4 Remark

One gets well the same results with formulation 3D_INCO_UP as with the formulation 3D classical.

18 Modelization P

18.1 Characteristic of the modelization

$3D, H=1$



18.2 Characteristics of the mesh

Many nodes: 35
Number of meshes and types: 12 TETRA10

18.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.1%
$\varepsilon_{xx}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.1%
$\varepsilon_{xx}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.1%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.1%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.1%
$\sigma_{yy}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.1%
$\sigma_{yy}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.1%
$\sigma_{yy}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.1%

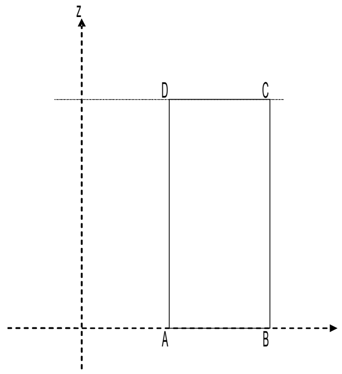
18.4 Remark

One get well the same results with formulation 3D_INCO_UP as with the formulation 3D classical.

19 Modelization Q

19.1 Characteristic of modelization

QUAD8 - Axisymmetric



19.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 1 QUAD8, 4 SEG3

19.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.1%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.15%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.4%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.2%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.5%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.4%
$\sigma_{zz}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.2%
$\sigma_{zz}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.525%
$\sigma_{zz}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.5%

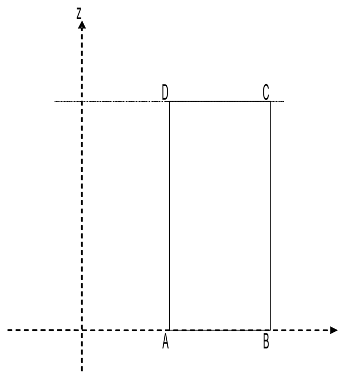
19.4 Remark

One gets well the same results with formulation AXIS_INCO_GD as with formulation AXIS classic.

20 Modelization R

20.1 Characteristic of the modelization

$3\mathrm{D}, H=1$



20.2 Characteristics of the mesh

Many nodes: 20
Number of meshes and types: 1 HEXA20

20.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.1%
$\varepsilon_{xx}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.15%
$\varepsilon_{xx}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.4%
$p(t=66,666\,s)$	"ANALYTIQUE"	0	0.11%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.3%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.4%
$\sigma_{yy}(t=66,666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.1275%
$\sigma_{yy}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.525%
$\sigma_{yy}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.5%

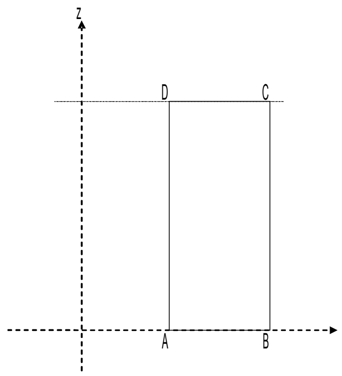
20.4 Remark

One gets well the same results with formulation 3D_INCO_GD as with the formulation 3D classical.

21 Modelization S

21.1 Characteristic of the modelization

3D, $H=1$



21.2 Characteristics of the mesh

Many nodes: 35
Number of meshes and types: 12 TETRA10

21.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.1%
$\varepsilon_{xx}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.15%
$\varepsilon_{xx}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.4%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.2%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.5%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.35%
$\sigma_{yy}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.2%
$\sigma_{yy}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.525%
$\sigma_{yy}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.5%

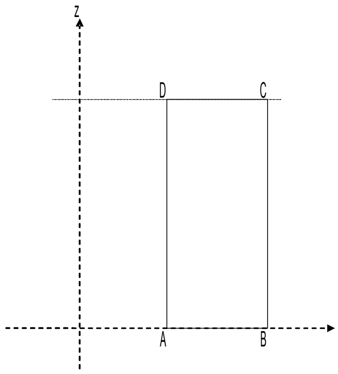
21.4 Remark

One get well the same results with formulation 3D_INCO_GD as with the formulation 3D classical.

22 Modelization T

22.1 Characteristic of modelization

QUAD8 - Axisymmetric



22.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 1 QUAD8, 4 SEG3

22.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.05%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=80\text{s})$	"ANALYTIQUE"	1.1 10-3	0.1%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=90\text{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.2%
$p(t=80\text{s})$	"ANALYTIQUE"	3 10-4	1 10-4%
$p(t=90\text{s})$	"ANALYTIQUE"	5.25 10-4	1 10-4%
$\sigma_{zz}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.2%
$\sigma_{zz}(t=80\text{s})$	"ANALYTIQUE"	-100 MPa	0.25%
$\sigma_{zz}(t=90\text{s})$	"ANALYTIQUE"	-75 MPa	0.3%

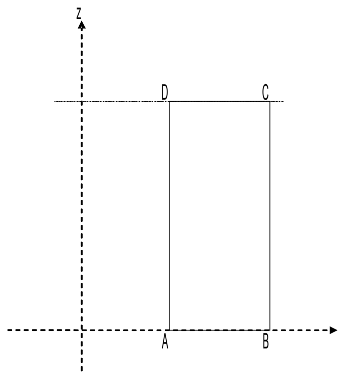
22.4 Remark

One gets well the same results with formulation AXIS_INCO_LOG as with formulation AXIS classic.

23 Modelization U

23.1 Characteristic of the modelization

3D, $H=1$



23.2 Characteristic of the mesh

Many nodes: 20
Number of meshes and types: 1 HEXA20

23.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.05%
$\varepsilon_{xx}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.06%
$\varepsilon_{xx}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.11%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.05%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.01%
$\sigma_{yy}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.2%
$\sigma_{yy}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.25%
$\sigma_{yy}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.3%

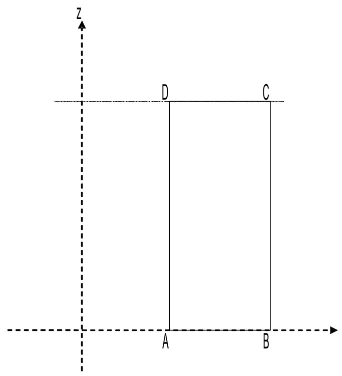
23.4 Remark

One gets well the same results with formulation 3D_INCO_LOG as with the formulation 3D classical.

24 Modelization V

24.1 Characteristic of the modelization

$3\mathrm{D}, H=1$



24.2 Characteristic of the mesh

Many nodes: 35
Number of meshes and types: 12 TETRA10

24.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.05%
$\varepsilon_{xx}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.06%
$\varepsilon_{xx}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66,666\,s)$	"ANALYTIQUE"	0	0.2%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.005%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.001%
$\sigma_{yy}(t=66,666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.2%
$\sigma_{yy}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.25%
$\sigma_{yy}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.3%

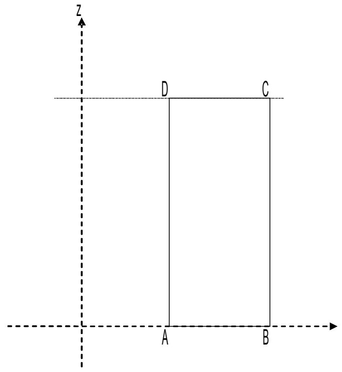
24.4 Remark

One gets well the same results with formulation 3D_INCO_LOG as with the formulation 3D classical.

25 Modelization W

25.1 Characteristic of modelization

QUAD8 - Axisymmetric



25.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 1 QUAD8, 4 SEG3

25.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.05%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.1%
$\varepsilon_{rr}=\varepsilon_{\theta\theta}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.2%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	1 10-4%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	1 10-4%
$\sigma_{zz}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.2%
$\sigma_{zz}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.25%
$\sigma_{zz}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.3%

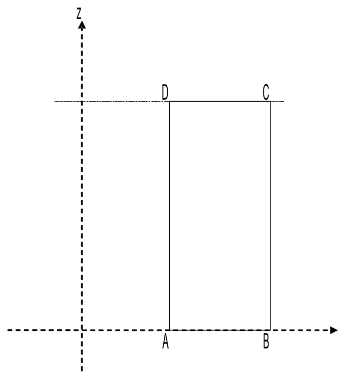
25.4 Remark

One gets well the same results with formulation AXIS_INCO_LUP as with formulation AXIS classic.

26 Modelization X

26.1 Characteristic of the modelization

3D, $H=1$



26.2 Characteristic of the mesh

Many nodes: 20
Number of meshes and types: 1 HEXA20

26.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.05%
$\varepsilon_{xx}(t=80\mathrm{s})$	"ANALYTIQUE"	1.1 10-3	0.06%
$\varepsilon_{xx}(t=90\mathrm{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.11%
$p(t=80\mathrm{s})$	"ANALYTIQUE"	3 10-4	0.05%
$p(t=90\mathrm{s})$	"ANALYTIQUE"	5.25 10-4	0.01%
$\sigma_{yy}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.2%
$\sigma_{yy}(t=80\mathrm{s})$	"ANALYTIQUE"	-100 MPa	0.25%
$\sigma_{yy}(t=90\mathrm{s})$	"ANALYTIQUE"	-75 MPa	0.3%

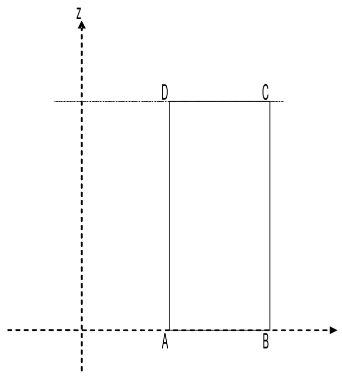
26.4 Remark

One gets well the same results with formulation 3D_INCO_LUP as with the formulation 3D classical.

27 Modelization Y

27.1 Characteristic of the modelization

3D, $H=1$



27.2 Characteristic of the mesh

Many nodes: 35
Number of meshes and types: 12 TETRA10

27.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66{,}666\,s)$	"ANALYTIQUE"	8.6666 10-4	0.05%
$\varepsilon_{xx}(t=80\text{s})$	"ANALYTIQUE"	1.1 10-3	0.06%
$\varepsilon_{xx}(t=90\text{s})$	"ANALYTIQUE"	1.275 10-3	0.1%
$p(t=66{,}666\,s)$	"ANALYTIQUE"	0	0.2%
$p(t=80\text{s})$	"ANALYTIQUE"	3 10-4	0.005%
$p(t=90\text{s})$	"ANALYTIQUE"	5.25 10-4	0.001%
$\sigma_{yy}(t=66{,}666\,s)$	"ANALYTIQUE"	-133.333 MPa	0.2%
$\sigma_{yy}(t=80\text{s})$	"ANALYTIQUE"	-100 MPa	0.25%
$\sigma_{yy}(t=90\text{s})$	"ANALYTIQUE"	-75 MPa	0.3%

27.4 Remark

One gets well the same results with formulation 3D_INCO_LUP as with the formulation 3D classical.

28 Summary of the results

the results are satisfactory and validate the behaviors thermo-elastic and thermoplastic of Von Mises with isotropic and kinematical hardening linear. The finite elements used are the elements $2D$ (quadrilateral in plane stresses or axisymmetric) $3D$, the elements incompressible (3D_INCO, 3D_INCO_GD, 3D_INCO_UP, AXIS_INCO, AXIS_INCO_UP and AXIS_INCO_GD) and the elements PIPE.

One notes in particular a good modelization of the variation of the elastic limit and constant of Prager with the temperature.

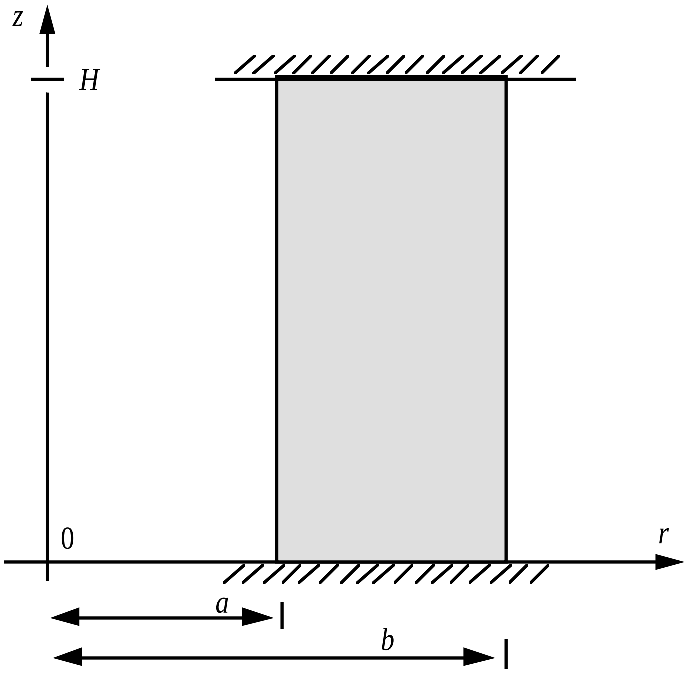
29 Appendix

29.1 Presentation

the problem studied is such that the solution is uniform in space, without any external force given, so as to test only the processing of the behavior model.

Following solid thus is considered:

- height H,
- axisymmetric (of radius a and b),
- or parallelepipedic (thickness $b-a$).



It is placed between two lubricated rigid plates.

The material is thermoelastoplastic homogeneous (see hereafter) with isotropic hardening and criterion of Von Mises.

One supposes the uniform temperature spaces some, and increasing.

29.2 Kinematics, axisymmetric

29.2.1 equilibrium Case (2D)

Fields of displacement: $u=u_r(r)e_r$ (blocking in z)

strain fields: $\varepsilon(u)=\begin{pmatrix} u_r' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{u_r}{r} \end{pmatrix}$ (according to $\begin{pmatrix} r \\ z \\ \theta \end{pmatrix}$)

strain fields: $\sigma(u)=\sigma_L\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (cf boundary conditions) (according to $\begin{pmatrix} r \\ z \\ \theta \end{pmatrix}$)

29.2.2 parallelepipedic Case

Fields of displacement: $u=u_x(x)e_x+u_y(y)e_y$ (blocking in z)

strain fields: $\varepsilon(u)=\begin{pmatrix} u_x' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & u_y' \end{pmatrix}$ (according to $\begin{pmatrix} x \\ z \\ y \end{pmatrix}$)

strain fields: $\sigma(u)=\sigma_L\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (cf boundary conditions) (according to $\begin{pmatrix} x \\ z \\ y \end{pmatrix}$)

the case could be studied in D_PLAN and 3D.

29.3 Behavior model

Hardening isotropic, linear (constant tangent E_T modulus).

Criterion of Von Mises.

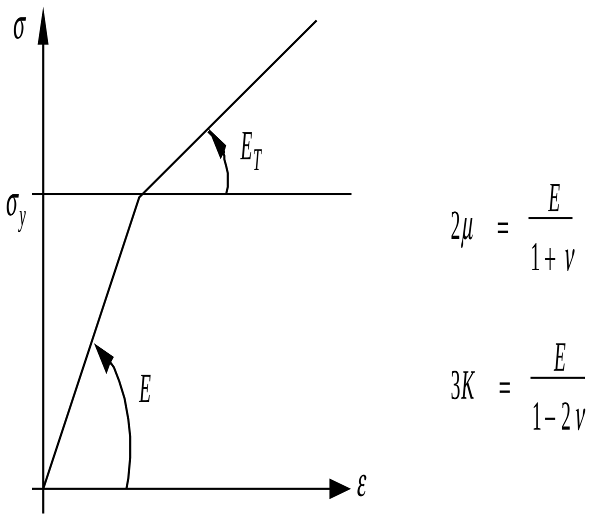
Elastic coefficients, E and ν, as well as the tangent modulus E_T are invariants according to the temperature.

The elastic limit σ_y varies according to the temperature T:

$$\sigma_y(T)=\sigma_y^o(1-s(T-T^o))$$

(for the temperature range studied, σ_y is positive!).

The thermal coefficient of thermal expansion α is constant.



$$2\mu=\frac{E}{1+\nu} \quad 3\mathrm{K}=\frac{E}{1-2\nu}$$

The constitutive law is written (scalar local variable P):

$$\varepsilon=\frac{1}{9\mathrm{K}}tr\,\sigma\,Id+\frac{1}{2\mu}\sigma^D+\varepsilon^p=\alpha(T-T^o)\,Id$$

with: $\sigma^D=\sigma-\frac{1}{3}tr\,\sigma\,Id$ (deviator of the stresses)

$\dot{\sigma}^p=\frac{3}{2}\dot{p}\frac{\sigma^D}{\|\sigma_{éq}\|}$, with $\|\sigma_{éq}\|=\sqrt{\frac{3}{2}}\sqrt{\sigma^D\sigma^D}$

$\dot{p}=0$ if $f(\sigma,p)=\|\sigma_{éq}\|-R(p)<0$

$\dot{p}\geq 0 \quad f(\sigma,p)=0$

$R(p)$ the function of hardening indicates:

$$R(p)=\sigma_y+\frac{EE_T}{E-E_T}p$$

Rate $\dot{p}$ can be expressed, when $f(\sigma,p)=0$. Indeed, of $\dot{p}f$ identically no one, one draws: $\dot{p}\dot{f}+\ddot{p}f=0$. Thus, when one is on the criterion $f=0$, necessarily $\dot{f}=0$. It is - with-to say:

$$\frac{3}{2}\frac{\sigma^D\dot{\sigma}^D}{\|\sigma_{éq}\|}-R_{,T}\dot{T}-R_{,p}\dot{p}=0$$
$$\Leftrightarrow\ \frac{3}{2}\frac{\sigma^D\dot{\sigma}^D}{\|\sigma_{éq}\|}-\sigma_y^o s\dot{T}-\frac{EE_T}{E-E_T}\dot{p}=0$$

From where:

$$\dot{p}=\frac{E-E_T}{EE_T}\left(\frac{3}{2}\frac{\sigma^D\dot{\sigma}^D}{\|\sigma_{éq}\|}+\sigma_y^o s\dot{T}\right)\ \text{if}\ \dot{p}\geq 0\text{, for}\ \|\sigma_{éq}\|=R(p)$$

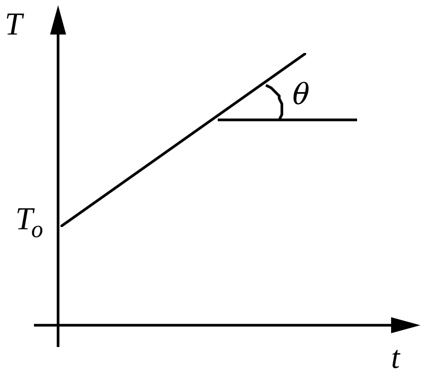
(criterion reached, in "load")

29.4 thermal Loading

uniform Temperature spaces some

$$T(t)=\theta t+T_o\quad,\quad\theta>0$$
$$t\in\left[0,t_{fin}\right]\quad;\quad\text{avec}\ t_{fin}<\frac{1}{s\theta}$$

$$T(t)=\theta t+T_o\quad,\quad\theta>0$$
$$t\in[0,t_{fin}]\quad\text{with}\ t_{fin}<\frac{1}{s\theta}$$



virgin initial State: $\sigma_L=0$
$p=0$

29.5 Solution

the stress field being uniaxial, one a:

$$\sigma^D = \frac{\sigma_L}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

As follows:

$$\|\sigma_{éq}\| = |\sigma_L|$$

and:

$$\dot{\varepsilon}^p = \frac{\dot{p}}{2} sgn(\sigma_L) \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

The behavior model leads to

$$\dot{\varepsilon}_{rr} = \dot{\varepsilon}_{\theta\theta} = \frac{-\nu}{E} \dot{\sigma}_L - \frac{\dot{p}}{2} sgn(\sigma_L) + \alpha \dot{T} \quad (\dot{\varepsilon}_{xx} = \dot{\varepsilon}_{yy} \text{ for the case of the parallelepiped})$$

$$\dot{\varepsilon}_{zz} = 0 = \frac{1}{E} \dot{\sigma}_L + \dot{p}\, sgn(\sigma_L) + \alpha \dot{T}$$

From where:

$$\dot{\varepsilon}_{rr} = \dot{\varepsilon}_{\theta\theta} = \frac{3}{2} \alpha \dot{T} + \frac{1-2\nu}{2E} \dot{\sigma}_L$$

$$\dot{p} = sgn(\sigma_L)(-\alpha \dot{T} - \frac{\dot{\sigma}_L}{E}) = 0 \qquad \text{if} \quad |\sigma_L| < R(p)$$

$$\dot{p} = Max[0; \frac{E-E_T}{EE_T}(\frac{3}{2}\frac{\sigma^D \sigma^D}{\|\sigma_{éq}\|} + \sigma_y^o s \dot{T})] \qquad \text{not}$$

I.e., in the case $|\sigma_L| = R(p)$ (criterion reached)

$$\dot{p} = Max[0; \frac{E-E_T}{EE_T}(sgn(\sigma_L)\dot{\sigma}_L + \sigma_y^o s \dot{T})]$$

29.5.1 elastic Phase

At the beginning of the thermal loading, $|\sigma_L|$ being lower than σ_y, $\dot{p}$ is null.

From where:

$$\dot{\sigma_L} = -E\alpha\dot{T}$$
$$\dot{\varepsilon_{rr}} = \dot{\varepsilon_{\theta\theta}} = \alpha\dot{T}(1+\nu)$$

As follows:

$$\sigma_L = -E\alpha\theta t \qquad \text{(compression } \sigma_L < 0 \text{)}$$
$$\varepsilon_{rr} = \varepsilon_{\theta\theta} = \alpha\theta(1+\nu)t$$

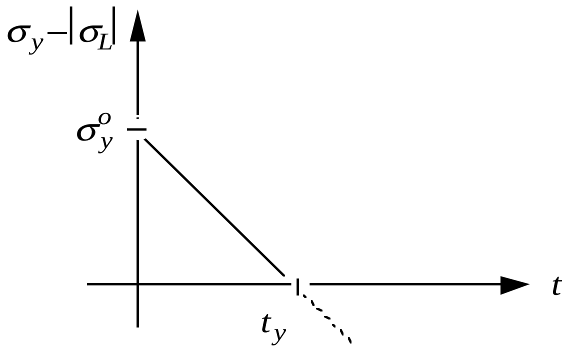
Validity of the elastic solution

the criterion is:

$$|\sigma_L(t)| - \sigma_y(t) = E = \theta t - \sigma_y^o(1 - s\theta t) \le 0$$

The criterion is not crossed for $t = [0, t_y]$, with:

$$t_y = \frac{\sigma_y^o}{\theta(E\alpha + \sigma_y^o s)}$$



At time t_y:

$$\sigma_L(t_y) = \frac{-E\alpha\sigma_y^o}{E\alpha + \sigma_L^o s}$$

29.5.2 Elastoplastic phase

$t \geq t_y$. One is on the criterion. Then:

$$\dot{p} = Max[0; \frac{E-E_T}{EE_T}(\dot{\sigma}_L sgn(\sigma_L) + \sigma_y^o s\dot{T})]$$

By admitting that one is "charges some" $(\dot{p}>0)$, then one eliminates $\dot{p}$ to have

$$\dot{\sigma_L} = -E_T\dot{T}(\alpha + sgn(\sigma_L)\frac{E-E_T}{EE_T}s\sigma_y^o)$$

then:

$$\dot{p} = \frac{E-E_T}{E}\dot{T}(-\alpha\, sgn(\sigma_L) + \frac{s\sigma_y^o}{E})$$

with $t=t_y$ $\sigma_L = -E\alpha\theta t_y < 0$; one integrates these statements then for $t \geq t_y(\dot{T}=\theta)$:

$$\sigma_L(t) = -E_T\theta(t-t_y)[\alpha - \frac{E-E_T}{EE_T}s\sigma_y^o] - \sigma_L(t_y)$$
$$p(t) = \frac{E-E_T}{E^2}\theta[\alpha E + s\sigma_y^o](t-t_y)$$

Maybe, after rearrangement $(t>0\mathrm{t}_y)$:

$$\sigma_L(t) = \sigma_y^o(s\theta t - 1 + \frac{E_T}{E}(1-\frac{t}{t_y}))$$
$$p(t) = \frac{\sigma_y^o(E-E_T)}{E^2}(\frac{t}{t_y}-1)$$

Validity of this elastoplastic solution

It should be made sure that $\sigma_L(t)$ remains negative. Knowing that $s\theta t<1$, and that $t>t_y$, the result preceding one confirms that $\sigma_L(t)<0$.

Lastly, it is noticed that:

$$sgn(\sigma_L)\frac{1-2\nu}{2}\dot{p} + \dot{\varepsilon_{rr}} = \alpha(1+\nu)\dot{T}$$

from where:

$$\varepsilon_{rr}(t) = \varepsilon_{\theta\theta}(t) = \alpha\theta(1+\nu)t + \frac{1-2\nu}{2}p(t) \quad , \quad \forall t \in [t_y, t_{fin}]$$

(since $\sigma_L(t)<0$).

29.6 Numerical application

$E=200000\,MPa$; $\nu=0.3$; $\alpha=10.0\,E-5\,°C^{-1}$; $\theta=1.0\,s^{-1}$
$\sigma_y^o=400\,MPa$; $T^o=0\,°C$; $s=10.0\,E-2\,°C^{-1}$; $t_{fin}<100\,s$
$E_T=50000\,MPa$;

From where one obtains in elastic phase:

$$t_y=66.6666\,s$$
$$\sigma_L(t_y)=-133.333\,MPa$$
$$\varepsilon_{rr}(t_y)=\varepsilon_{\theta\theta}(t_y)=0.866666\,E-3$$

Then, elastoplastic phase:

with $t=80\text{s}$: $\sigma_L(80)=-100\,MPa$
$p(80)=0.30\,E-3$
$\varepsilon_{rr}(80)=\varepsilon_{\theta\theta}(80)=1.100\,E-3$

with $t=90\text{s}$: $\sigma_L(90)=-75\,MPa$
$p(90)=0.525\,E-3$
$\varepsilon_{rr}(90)=\varepsilon_{\theta\theta}(90)=1.275\,E-3$