

## HSNV101 - decoupled Thermoelasticity and metallurgy in simple tension

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### Summarized:

One treats the determination of the mechanical evolution of a cylindrical bar subjected to known and  $T(t)$  uniform evolutions  $Z(t)$  thermal and metallurgical (the metallurgical transformation is of bainitic type).

The elements used are axisymmetric elements and the behavior model is the plasticity of von Mises with linear isotropic hardening (for the modelization B, one also takes account of the plasticity of transformation).

The yield stress and the slope of curve of tension depend on the temperature and the metallurgical composition.

The coefficient of thermal expansion  $\alpha$  depends on the metallurgical composition.

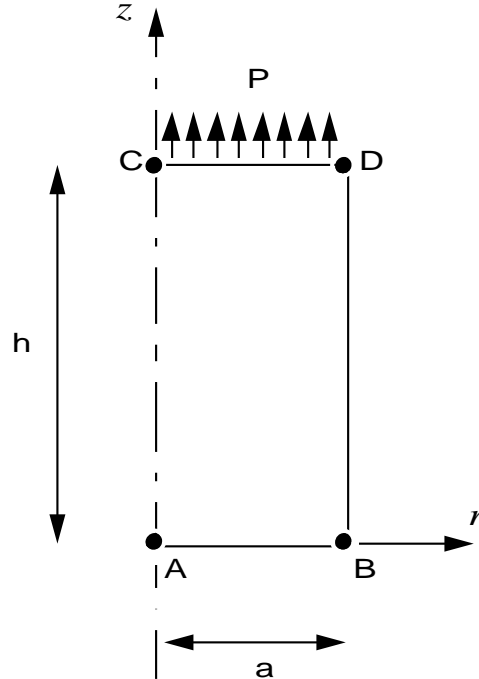
The metallurgical transformations take place with  $\dot{\varepsilon}^p = 0$  (it is in the sense that the test **uncouples** plasticity from transformation of classical plasticity).

The results provided by Code\_Aster are very satisfactory with errors lower than 2 % .

## 1 Problem of reference

### 1.1 Geometry

Rayon :  $a = 0.05 \text{ m}$   
Hauteur :  $h = 0.2 \text{ m}$



### 1.2 Properties of the materials

$E = 200000 \cdot 10^6 \text{ Pa}$	$\sigma_y^{aust} = \sigma_o^{aust} + s^{aust}(T - T^o)$	notons $H(t) = \frac{\alpha(t) \cdot E(t)}{E(t) - \alpha(t)}$
$\nu = 0.3$	$\sigma_o^{aust} = 400 \cdot 10^6 \text{ Pa}$	$H^{aust} = H_o^{aust} + \lambda^{aust}(T - T^o)$
$\alpha_{fbm} = 15 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1}$	$s^{aust} = 0.5 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1}$	$H_o^{aust} = 1250 \cdot 10^6 \text{ Pa}$
$\alpha_{aust} = 23.5 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1}$	$\sigma_y^{fbm} = \sigma_o^{fbm} + s^{fbm}(T - T^o)$	$\lambda^{aust} = -5 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1}$
$\epsilon_{ref_{fbm}} = 2.52 \cdot 10^{-3}$	$\sigma_o^{fbm} = 530 \cdot 10^6 \text{ Pa}$	$H^{fbm} = H_o^{fbm} + \lambda^{fbm}(T - T^o)$
$T^{ref} = 900^\circ\text{C}$	$s^{fbm} = 0.5 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1}$	$H_o^{fbm} = -50 \cdot 10^6 \text{ Pa}$
$cp = 2\,000\,000 \text{ J} \cdot \text{m}^{-3} \cdot ^\circ\text{C}^{-1}$	$\lambda = 9999.9 \text{ W} \cdot \text{m}^{-1} \cdot ^\circ\text{C}^{-1}$	$\lambda^{fbm} = -5 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1}$
		$k^m = 1 \cdot 10^{-10} \text{ Pa}^{-1}$

- \**aust* = characteristic relating to the austenitic phase
- \**fbm* = characteristic relating to the phases ferritic, bainitic and martensitic
- $\alpha_{fbm}$  = thermal coefficient of thermal expansion of the phases ferritic, bainitic and martensitic
- $\alpha_{aust}$  = coefficient of thermal expansion of the austenitic phase
- $\epsilon_{ref_{fbm}}$  = strain of the phases ferritic, bainitic and martensitic with the reference temperature, austenite being regarded as not deformed with this temperature: translated the difference in compactness between cubic crystallographic structures with centered sides (austenite) and cubic centered (ferrite).

TRC to model a metallurgical evolution of bainitic type, on all structure, of the form:

$$Z_{fbm} = \begin{cases} 0. & \text{si } t \leq \tau_1 & \tau_1 = 60 \text{ s} \\ \frac{t - \tau_1}{\tau_2 - \tau_1} & \text{si } \tau_1 \leq t < \tau_2 & \tau_2 = 112 \text{ s} \\ 1. & \text{si } t \geq \tau_2 \end{cases}$$

Model of plasticity of transformation:  $\dot{\varepsilon}^{pt} = K^{fbm} F(Z_{fbm}) \dot{Z}_{fbm}$

$$\text{with } F(Z_{fbm}) = Z_{fbm} (2 - Z_{fbm})$$

## 1.3 Boundary conditions and loadings

$u_z = 0$  on the side  $AB$  (condition of symmetry).

tension imposed on the side  $CD$

$$p(t) = \begin{cases} p_o t & \text{pour } t \leq \tau_1 & p_o = 6 \cdot 10^6 \text{ Pa} \\ 360 \cdot 10^6 \text{ Pa} & \text{pour } t \geq \tau_1 & \tau_1 = 60 \text{ s} \end{cases}$$

$$T = T^o + \mu t, \quad \mu = -5 \text{ } ^\circ\text{C} \cdot \text{s}^{-1} \text{ on all structure.}$$

## 1.4 Initial conditions

$$T^o = 900^\circ\text{C} = T^{ref}$$

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

Before transformation, elastic solution for  $t < \tau_1$ .

$$\sigma(t) = p_o t \quad \varepsilon_{zz}(t) = \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) = \frac{\sigma(t)}{E} + \alpha_{aust}(T - T^o)$$

The yield stress is reached for  $\tau_1 = \frac{\sigma_o^{aust}}{p_o - s^{aust} \times \mu} = 47.06 \text{ s}$ .

Before transformation, thermoelastoplastic solution  $\tau_1 \leq t \leq \tau_1$   $\tau_1 = 60 \text{ s}$ .

$$\begin{aligned} \sigma(t) &= p_o t & \varepsilon_{zz}(t) &= \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) + \varepsilon_{zz}^p(t) \\ \varepsilon_{zz}^e(t) &= \frac{\sigma(t)}{E} & \varepsilon_{zz}^{th}(t) &= Z_{aust} \times \alpha_{aust}(T - T^o) \\ \varepsilon_{zz}^p(t) &= \frac{\sigma(t) - (\sigma_y^{aust} + S^{aust} \mu t)}{H_o^{aust} + \lambda^{aust} \mu t} \end{aligned}$$

During the transformation, thermo-élasto-metallurgical solution  $\tau_1 < t < \tau_2$   $\tau_2 = 112 \text{ s}$ .

$$\begin{aligned} \sigma(t) &= 360 \cdot 10^6 \text{ Pa} & \varepsilon_{zz}(t) &= \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) + \varepsilon_{zz}^{pt}(t) + \varepsilon_{zz}^p(60) \\ \varepsilon_{zz}^{th}(t) &= Z_{aust} \times \alpha_{aust}(T - T^o) + Z_{fbm} \times \alpha_{fbm}(T - T^o) + Z_{fbm} \times \varepsilon_{ref_{fbm}} \\ \varepsilon_{zz}^p(t) &= k^{fbm} F(Z_{fbm}) P_o \tau_1 \end{aligned}$$

After the transformation, thermoelastoplastic solution  $\tau_2 < t < \tau_3$   $\tau_3 = 176 \text{ s}$ .

$$\begin{aligned} \sigma(t) &= 360 \cdot 10^6 \text{ Pa} & \varepsilon_{zz}(t) &= \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) + \varepsilon_{zz}^p(t) + \varepsilon_{zz}^{pt}(112) \\ \varepsilon_{zz}^p(t) &= \frac{\sigma(t) - (\sigma_o^{fbm} + s^{fbm} \mu t)}{H_o^{fbm} + \lambda^{fbm} \mu t} \end{aligned}$$

### 2.2 Results of reference

$\varepsilon_{zz}^p$ ,  $\chi$ ,  $\sigma$  and  $\varepsilon_{zz}$  for  $t=47, 48, 64$  and  $114$  seconds.

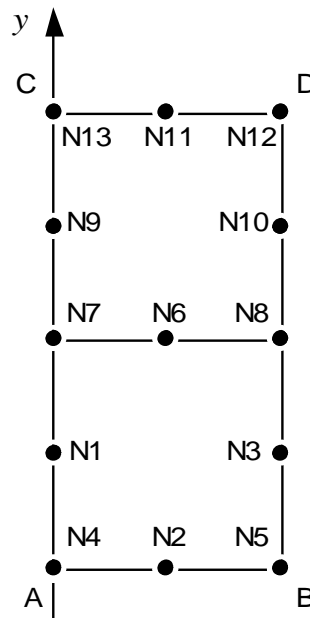
$\varepsilon_{zz}^p$  for  $t=60$  and  $176$  seconds.

### 2.3 Bibliographical references

- 1) DONORE A.M. - WAECKEL F.: Influence structure transformations in the elastoplastic constitutive laws Notes HI-74/93/024.

## 3 Modelization A

### 3.1 Characteristic of the modelization



$$A=N4 \quad B=N5 \quad C=N13 \quad D=N12 .$$

### 3.2 Characteristics of the mesh

Many nodes: 13

Number of meshes and types: 2 meshes QUAD8, 6 meshes SEG3

## 3.3 Quantities tested and results

One tests the parameters of the data structure results:

Identification	Reference	Test	Tolerance
INST for NUME_ORDRE= 7 0.176	176	ANALYTIQUE	0,10%
ITER_GLOB for NUME_ORDRE=70	5	NON_REGRESSION	0.00%

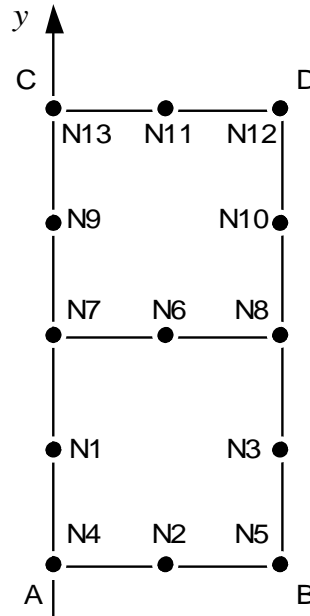
Identification	Reference	Test	Tolerance
$\varepsilon_{zz}^P$ $t=47 s$	0	NON_DEFINI	1,0E-12 (absolute)
$\chi$ $t=47 s$	0	DEFINITE NON_	1,0E-12 (absolute)
$\sigma$ $t=47 s$	282. 106	DEFINITE NON_	0,1%
$\varepsilon_{zz}$ $t=47 s$	- 4.1125 10-3	DEFINITE NON_	0,1%
$\varepsilon_{zz}^P$ $t=48 s$	3.2653 10-3	DEFINITE NON_	0,15%
$\chi$ $t=48 s$	1	DEFINITE NON_	0,1%
$\sigma$ $t=48 s$	288. 106	DEFINITE NON_	0,1%
$\varepsilon_{zz}$ $t=48 s$	- 9.3469 10-4	DEFINITE NON_	0,007%
$\varepsilon_{zz}^P$ $t=60 s$	0.04	DEFINITE NON_	0,1%
$\varepsilon_{zz}^P$ $t=64 s$	0.040	DEFINITE NON_	0,022%
$\chi$ $t=64 s$	0	DEFINITE NON_	1,0E-12 (absolute)
$\sigma$ $t=64 s$	360. 106	DEFINITE NON_	0,01%
$\varepsilon_{zz}$ $t=64 s$	3.4683 10-2	DEFINITE NON_	0,025%
$\varepsilon_{zz}^P$ $t=114 s$	0.04107	DEFINITE NON_	0,01%
$\chi$ $t=114 s$	1	DEFINITE NON_	0,1%
$\sigma$ $t=114 s$	360. 106	DEFINITE NON_	0,020%
$\varepsilon_{zz}$ $t=114 s$	0.03684	DEFINITE NON_	0,026%
$\varepsilon_{zz}^P$ $t=176 s$	0.06206	DEFINITE NON_	0,20%

## 3.4 Remarks

In this modelization:  $\varepsilon_{zz}^{Pt}(T, Z) = 0$

## 4 Modelization B

### 4.1 Characteristic of the modelization



$$A=N4 \quad B=N5 \quad C=N13 \quad D=N12 .$$

### 4.2 Characteristics of the mesh

Many nodes: 13

Number of meshes and types: 2 meshes QUAD8, 6 meshes SEG3

## 4.3 Quantities tested and results

Identification	Reference	Aster	% difference
$\varepsilon_{zz}^P$ $t=47 s$	0	0	0
$\chi$ $t=47 s$	0	0	0.282
$\sigma$ $t=47 s$	. 106.282	. 106	0
$\varepsilon_{zz}$ $t=47 s$	- 4.1125 10-3	- 4.1125 10-3	0
$\varepsilon_{zz}^P$ $t=48 s$	3.2653 10-3	3.26535 10-3	0.011
$\chi$ $t=48 s$	1	1	0.288
$\sigma$ $t=48 s$	. 106.288	. 106	0
$\varepsilon_{zz}$ $t=48 s$	- 9.3469 10-4	- 9.34644 10-4	- 0.005
$\varepsilon_{zz}^P$ $t=60 s$	0.04	0.04	0
$\varepsilon_{zz}^P$ $t=64 s$	0.04	4.0 10-2	0
$\chi$ $t=64 s$	0	0	0.360
$\sigma$ $t=64 s$	. 106	359.99 106	- 0.004
$\varepsilon_{zz}$ $t=64 s$	4.00085 10-2	4.000268 10-2	- 0.015
$\varepsilon_{zz}^P$ $t=114 s$	0.041071	4.10751 10-2	+0.004
$\chi$ $t=114 s$	1	1	0.360
$\sigma$ $t=114 s$	. 106	360.01 106	0.000
$\varepsilon_{zz}$ $t=114 s$	0.072841	7.144112 10-2	- 1.915
$\varepsilon_{zz}^P$ $t=176 s$	0.06206	6.2066 10-2	0.000

## 4.4 Remarks

In this modelization, one takes into account the term due to the plasticity of transformation:

$$\varepsilon^{pl}(T, Z) \neq 0 \text{ when } \dot{Z} \neq 0$$



## 5 Summary of the results

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the results found with *Code\_Aster* are very satisfactory, with percentages of error lower than 0.025% except for the strain than time 114s when the error reaches 2% for the modelization B.