HSNV103 - Thermoplasticity and metallurgy in plane strains

Summarized:

One treats the determination of the mechanical evolution of a right-angled parallépipède in plane strains subjected to evolutions known $T(t)$ thermal and $Z(t)$ formula and uniforms (the metallurgical transformation is of bainitic type).

The elements used are two-dimensional elements in plane strains and the behavior model is the plasticity of von Mises with linear isotropic hardening (for the modelization B, one also takes account of the plasticity of transformation).

The yield stress and the slope of curve of tension depend on the temperature and the metallurgical composition.

The coefficient of thermal expansion α depends on the metallurgical composition.

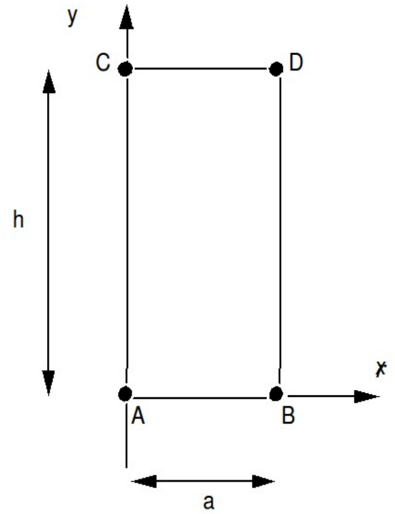
For the modelization A (without plasticity of transformation), the reference solution is obtained by the analytical resolution of the problem. For the modelization B (with plasticity of transformation), the reference solution is obtained by the numerical resolution of the problem by means of the axisymmetric elements for which one imposes the condition of plane strains.

The results provided by *Code_Aster* are very satisfactory with errors lower than 0,5%.

1 Problem of reference

1.1 Geometry

Width: $a=0.05\,m$
Height: $h=0.2\,m$



1.2 Properties of the materials

$E=200000.10^6$ Pa	$\sigma_y^{aust}=\sigma_o^{aust}+s^{aust}\left(T-T^o\right)$	notons $H(t)=\dfrac{\alpha(t)\cdot E(t)}{E(t)-\alpha(t)}$
$\nu=0.3$	$\sigma_o^{aust}=400.\ 10^6$ Pa	$H^{aust}=H_o^{aust}+\lambda^{aust}\left(T-T^o\right)$
$\alpha_{fbm}=15.\ 10^{-6}\ {}^\circ C^{-1}$	$s^{aust}=0.5\ 10^6$ Pa.$^\circ C^{-1}$	$H_o^{aust}=1250\ 10^6$ Pa
$\alpha_{aust}=23.5\ 10^{-6}\ {}^\circ C^{-1}$	$\sigma_y^{fbm}=\sigma_o^{fbm}+s^{fbm}\left(T-T^o\right)$	$\lambda^{aust}=-5\ 10^6$ Pa.$^\circ C^{-1}$
$\varepsilon_{ref_{fbm}}=2.52\ 10^{-3}$	$\sigma_o^{fbm}=530.\ 10^6$ Pa	$H^{fbm}=H_o^{fbm}+\lambda^{fbm}\left(T-T^o\right)$
$T^{ref}=900^\circ C$	$s^{fbm}=0.5\ 10^6$ Pa.$^\circ C^{-1}$	$H_o^{fbm}=-50\ 10^6$ Pa
$\rho.cp=2\,000\,000$ J.m$^{-3}\cdot{}^\circ C^{-1}$	$\lambda=9999.9$ W.m$^{-1}\cdot{}^\circ C^{-1}$	$\lambda^{fbm}=-5\ 10^6$ Pa$\cdot{}^\circ C^{-1}$
		$k^{fbm}=1.\ 10^{-10}$ Pa^{-1}

$*^{aus}$ = characteristic relating to the austenitic phase
$*^{fbm}$ = characteristic relating to the phases ferritic, bainitic and martensitic
α_{fbm} = thermal coefficient of thermal expansion of the phases ferritic, bainitic and martensitic
α_{aus} = coefficient of thermal expansion of the austenitic phase
$\varepsilon_{réf\ fbm}$ = strain of the phases ferritic, bainitic and martensitic with the reference temperature, austenite being regarded as not deformed with this temperature: translated the difference in compactness between cubic crystallographic structures with centered sides (austenite) and cubic centered (ferrite).

TRC to model a metallurgical evolution of bainitic type, on all structure, of the form:

$$Z_{fbm}=\begin{cases}0. & si \;\; t\leq\tau_1 & \tau_1=60\,s\\ \dfrac{t-\tau_1}{\tau_2-\tau_1} & si \;\; \tau_1\leq t<\tau_2 & \tau_2=112\,s\\ 1. & si \;\; t\geq\tau_2 & \end{cases}$$

Model of plasticity of transformation: $\dot{\varepsilon}^{pt}=K^{fbm}F(Z_{fbm})\dot{Z}_{fbm}$
with $F(Z_{fbm})=Z_{fbm}(Z-Z_{fbm})$

Notations: $T(\tau_1)=T_1$
$T(\tau_2)=T_2$

1.3 Boundary conditions and loadings

$u_Y=0$ on the side AB ; $u_X=0$ in A .
$T=T^0+\mu t$, $\mu=-5°C.s^{-1}$ on all structure.
The loading on the structure is due to the phenomena of thermal and metallurgical thermal expansion constrained in the direction z by the condition of plane strains.

1.4 Initial conditions

$T^0=900°C=T^{ref}$

2 Reference solution (for the modelization A)

2.1 Method of calculating used for the reference solution

Before transformation, thermoelastic solution until in t_1 such as:

$$\sigma_{zz}=-E\,\varepsilon^{th}=\sigma_y \quad\Leftrightarrow\quad T-T^0=\frac{-\sigma_y^{o\,y}}{E\,\alpha+s}=76.92\,°C$$
$$\Leftrightarrow\quad t_1=15.38\,s$$

thus for $t\le t_1$: $\sigma_{zz}=-E\,\alpha_\gamma(T-T^0)$

Before transformation, and for $t\ge t_1$, thermoelastoplastic solution such as:

$$\varepsilon_{zz}=0 \text{ and } \sigma_{zz}=R\,\varepsilon_{zz}^p+\sigma_y$$

from where $\varepsilon_{xx}^p=\dfrac{-\sigma_y(T)-E\,\alpha_\gamma(T-T^0)}{E+R(T)}$ and $\sigma_{zz}=-E(\varepsilon_{zz}^p+\alpha_\gamma(T-T^0))$

During the transformation, one remains in load as long as $\dot{\varepsilon}^{th}<0$

$$\dot{\varepsilon}^{th}=0 \quad\Leftrightarrow\quad T=\frac{(\alpha_\alpha-\alpha_\gamma)T^0-\varepsilon_{réf\ fbm}+\alpha_\alpha T_1-\alpha_\gamma T_2}{2(\alpha_\alpha-\alpha_\gamma)}=538.82\,°C$$
$$\Leftrightarrow\quad t=t_2=72.23\,s$$

as $T>538.82\,°C$

there thus has a thermoelastoplastic soution with phase change:

$$\varepsilon_{zz}^p=\frac{-\sigma_y(T,Z)-E\left[\alpha(Z)(T-T^0)+Z\,\varepsilon_{réf\ fbm}\right]}{E+R(T,Z)} \text{ and } \sigma_{zz}=-E(\varepsilon_{zz}^p+\alpha(Z)(T-T^0)+Z\,\varepsilon_{réf\ fbm})$$

and as $T<538.82\,°C$

there thus has a thermoelastic soution with phase change:

$$\sigma_{zz}=-E\left[\alpha(Z)(T-T^0)+Z\,\varepsilon_{réf\ fbm}+\varepsilon_{zz}^p(t_2)\right]$$

After the transformation, one replastifie when: $\sigma_{zz}=R(T,Z)\,\varepsilon_{zz}^p+\sigma_y(T,Z)$

with $\varepsilon_{zz}=\dfrac{\sigma_{zz}}{E}+\alpha_\alpha(T-T^0)+\varepsilon_{réf\ fbm}=0$

either $T=267.06\,°C$

and one thus has for $T<267.06\,°C$:

$$\varepsilon_{zz}^p=\frac{E\left[-\varepsilon_{réf\ fbm}-\alpha_\alpha(T-T^0)\right]-\sigma_y(T,Z)}{R(Z,T)+E} \text{ and } \sigma_{zz}=R(T,Z)\,\varepsilon_{zz}^p+\sigma_y(T,Z)$$

2.2 Results of reference

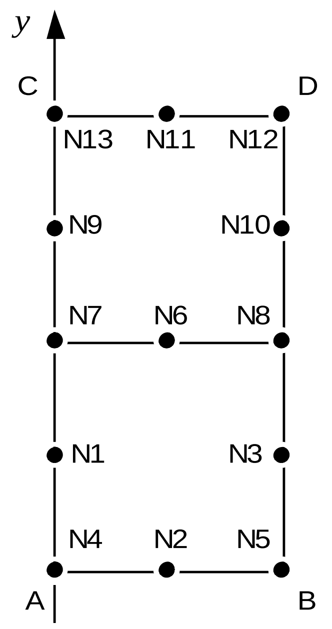
To $t=16\,s$:	χ	p	ε_{xx}	σ_{zz}
With $t=60\,s$:	χ	p	ε_{xx}	σ_{zz}
With $t=72\,s$:	χ	p		
With $t=112\,s$:	χ	p		σ_{zz}
With $t=176\,s$:	χ		ε_{xx}	σ_{zz}

2.3 Bibliography

1) DONORE A.M. - WAECKEL F. - Influence of structure transformations in the elastoplastic constitutive laws Notes HI-74/93/024.

3 Modelization A

3.1 Characteristic of the modelization



$A=N4 \quad B=N5 \quad C=N13 \quad D=N12$.

3.2 Characteristics of the mesh

Many nodes: 13.

Number of meshes and types: 2 meshes QUAD8, 6 meshes SEG3.

4 Results of the modelization A

4.1 Values tested

One tests the parameters of the data structure results:

Identification	Reference
INST for NUME_ORDRE= 176	176,0
ITER_GLOB for NUME_ORDRE=176	2

Identification	Reference
ε_{xx} $t=16\,s$	– 2.4599 10-3
χ $t=16\,s$	1
σ $t=16\,s$	360.13 106
p $t=16\,s$	7.9345 10-5
ε_{xx} $t=60\,s$	– 1.0309 10-2
p $t=60\,s$	5.7213 10-3
σ $t=60\,s$	265.73 106
p $t=72\,s$	5.8420 10-3
χ $t=112\,s$	0
σ $t=112\,s$	12.82 106
p $t=112\,s$	5.8421 10-3
ε_{xx} $t=176\,s$	– 1.5886 10-2
χ $t=176\,s$	1
σ $t=176\,s$	133.55 106

4.2 Remarks

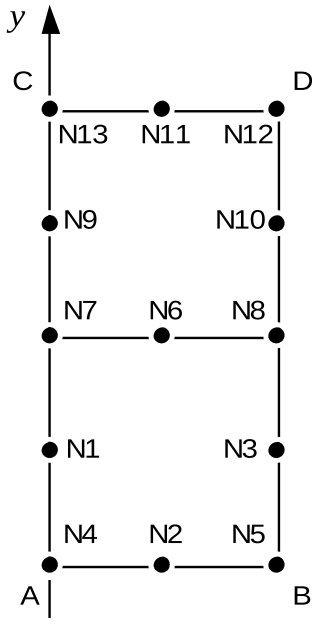
In this modelization:

$$\varepsilon_{zz}^{pt}(T, Z)=0$$

The error on the plastic strain cumulated at 72 seconds comes by way of the mistake made on the numerical description of the metallurgical transformation which is, at this time, from approximately 0,5%.

5 Modelization B

5.1 Characteristic of the modelization



$A = N4 \quad B = N5 \quad C = N13 \quad D = N12$.

5.2 Characteristics of the mesh

Many nodes: 13.

Number of meshes and types: 2 meshes QUAD8, 6 meshes SEG3.

6 Results of the modelization B

6.1 Values tested

Identification	Reference
σ $t=60\,s$	265.73 106
p $t=60\,s$	5.7213 10-3
ε_{yy} $t=89\,s$	– 1.0325 10-2
p $t=89\,s$	5.7213 10-3
σ $t=89\,s$	– 13.545 106
ε_{yy} $t=112\,s$	– 8.9197 10-3
σ $t=112\,s$	101.39 106
p $t=112\,s$	5.7213 10-3
ε_{yy} $t=176\,s$	– 1.5884 10-2
p $t=176\,s$	9.3610 10-2
σ $t=176\,s$	130.72 106

6.2 Remarks

In this modelization, one takes into account the term due to the plasticity of transformation:

$$\dot{\varepsilon}^{pt}(Z,T)\neq 0 \text{ when } \dot{Z}\neq 0$$

the reference solution is obtained by the numerical resolution of the problem with axisymmetric elements for which the condition of plane strains is imposed.

7 Summary of the results

the results found with *Code_Aster* are very satisfactory, with percentages of error lower than 0.5%.