HSNV121 - Tension in plastic large deformations of a bar under thermal loading

Abstract:

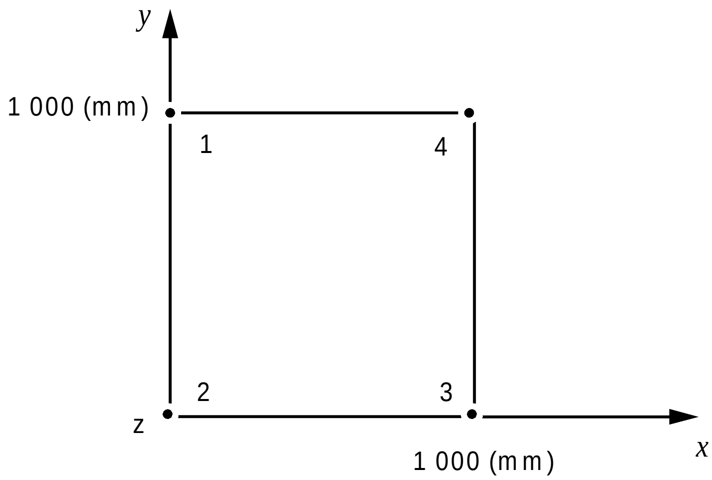
This quasi-static thermomechanical test consists in heating uniformly a bar of section rectangular ($3D$) or cylindrical ($2D$ axisymmetric) then to subject it to a tension. One thus validates the kinematics of the large deformations in plasticity (command STAT_NON_LINE, key word DEFORMATION: "SIMO_MIEHE" or "PETIT_REAC") for a behavior model in large deformations with linear isotropic hardening (command STAT_NON_LINE, key word RELATION: "VMIS_ISOT_LINE" and "VMIS_ISOT_TRAC") with thermomechanical loading. With modelizations shell or plate, the large deformations in plasticity are accessible thanks to key word DEFORMATION: "PETIT_REAC" provided that rotations remain weak.

The bar is modelled by an element voluminal (HEXA20, modelization A) or quadrangular (QUAD4, for an axisymmetric modelization, modelization B) or by shell elements or of shell (DKT for the modelization C and COQUE_3D for the modelization D).

The solution is analytical.

1 Problem of reference

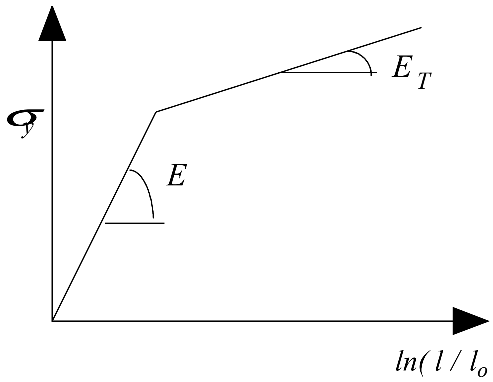
1.1 Geometry



1.2 Properties of the material

the material obeys a plastic constitutive law in large deformations with linear isotropic hardening, whose characteristics depend on the temperature.
Curve of tension is given in the logarithmic strain plane - rational stress.

$$\sigma = \frac{F}{S} = \frac{F}{S_o} \cdot \frac{l}{l_o}$$



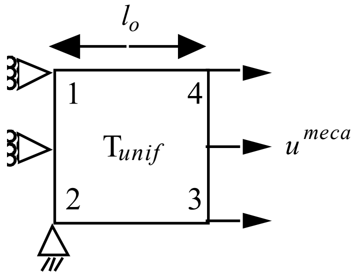
$\nu = 0.3$
$\alpha = 10^{-4} K^{-1}$
$\sigma_y = 1000$ MPa
à $T = 20°$C
$E = 250000$MPa
$E_T = 2500$ MPa
à $T = 120°$C
$E = 200000$MPa
$E_T = 2000$ MPa

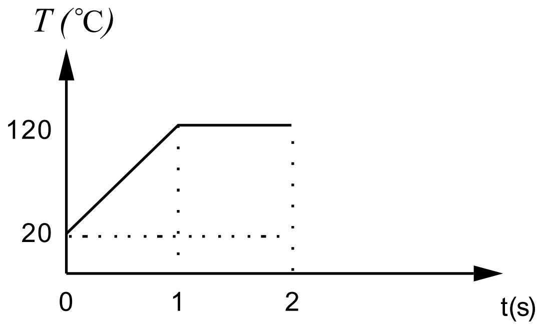
l_o and l are, respectively, the initial length and the current length of the useful part of the test-tube.

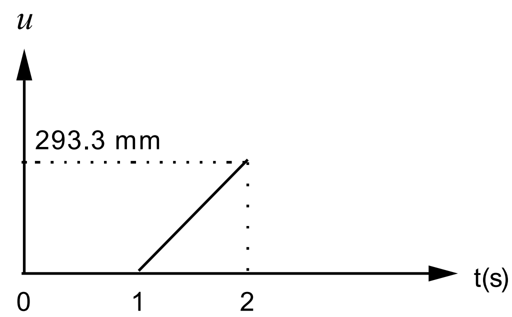
S_o and S are, respectively, initial and current surface. Between the temperatures $20 °C$ and $120 °C$, the characteristics are interpolated linearly.

1.3 Boundary conditions and loadings

the bar, initial length l_o, blocked in the direction $O\mathbf{x}$ on the face [1,2] is subjected to a uniform temperature T and a mechanical displacement of tension u^{meca} on the face $[3,4]$. The sequences of loading are the following ones:







Reference temperature: $T_{réf}$ = 20°C.

Note:

Mechanical displacement is measured from the configuration deformed by the thermal loading ($t=1\text{s}$). To have total displacement, it is thus necessary to add the thermal displacement obtained at time $t=1\text{s}$.

2 Reference solution

2.1 Result of the reference solution

For a traction test according to the direction x , the tensor of Kirchhoff τ is form:

$$\boldsymbol{\tau}=\begin{pmatrix}\tau & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

The tensor gradients of the transformation $\mathbf{F}$ and $\overline{\mathbf{F}}$ the isochoric tensor of plastic strains $\mathbf{G}^p$ are form:

$$\mathbf{F}=\begin{bmatrix}F & 0 & 0\\ 0 & F_{yy} & 0\\ 0 & 0 & F_{yy}\end{bmatrix} \quad \text{et} \quad J=\det\mathbf{F}=FF_{yy}^2 \quad \Rightarrow F_{yy}=\sqrt{J/F}$$

$$\overline{\mathbf{F}}=J^{-1/3}\mathbf{F}=\begin{bmatrix}\overline{F} & 0 & 0\\ 0 & \overline{F}_{yy} & 0\\ 0 & 0 & \overline{F}_{yy}\end{bmatrix} \quad \text{et} \quad \det\overline{\mathbf{F}}=1 \quad \Rightarrow \begin{cases}\overline{F}=J^{-1/3}F\\ \overline{F}_{yy}=\overline{F}^{-1/2}\end{cases}$$

$$\mathbf{G}^p=\begin{bmatrix}G^p & 0 & 0\\ 0 & G_{yy}^p & 0\\ 0 & 0 & G_{yy}^p\end{bmatrix} \quad \text{et} \quad \det\mathbf{G}^p=1 \quad \Rightarrow G_{yy}^p=(G^p)^{-1/2}$$

By the constitutive law, the following relation is obtained:

$$\tau=\frac{3K}{2}(J^2-1)-\frac{9K\alpha(T-T_{ref})}{2}(J+\frac{1}{J})$$

that is to say

$$J^3-3\alpha(T-T_{ref})J^2-J(1+\frac{2\tau}{3K})-3\alpha(T-T_{ref})=0$$

the stress of Cauchy is written:

$$J\sigma=\tau$$

In plastic load for a linear isotropic R hardening, such as:

$$R(p)=\frac{EE_T}{E-E_T}p$$

one a:

$$p=\frac{E-E_T}{E\,E_T}(\tau-\sigma_y)$$

the integration of the flow model of the plastic strain G^P gives (knowing that $G^P(p=0)=1$):

$$G^P=e^{-2p}$$

The component $\bar{F}$ of the gradient of the transformation is given by the resolution of:

$$\bar{F}^3-\frac{\tau}{\mu G^p}\bar{F}-\frac{1}{(G^p)^{3/2}}=0$$

The field of displacement $\boldsymbol{u}$ (in the initial configuration) is form $\boldsymbol{u}=u_x\boldsymbol{X}+u_y\boldsymbol{Y}+u_z\boldsymbol{Z}$. The components are given by:

$$u_x=\frac{\tilde{u}}{l_o}X \quad \text{avec } \tilde{u}=(F-1).l_o$$

$$u_y=\frac{\tilde{v}}{l_o}Y \quad \text{avec } \tilde{v}=\left(\sqrt{\frac{J}{F}}-1\right)l_o$$

$$u_z=\frac{\tilde{v}}{l_o}Z$$

2.2 Results of reference

One will adopt like results of reference displacements, the stress of Cauchy σ and the cumulated plastic strain p .

At time $t=2\ s$ ($\Delta T=100°C$, tension u)

One seeks total displacement (thermal + mechanical) such as the stress τ is equal to:
$\tau=1500\,Mpa$ (with $T=120°C$)

$3K=500\,000\,MPa$ $\mu=76923\,MPa$
$J=1.03$
$\sigma=1453\,MPa$
$p=0,2475$
$G^p=0,609$
$\bar{F}=1,289$
$F=1,303$
$\tilde{u}=303\,mm$
$\tilde{v}=-110\,mm$

With these quantities, it is possible to determine the elastic strain energy of the bar. Attention, the presence of thermal generates a raised jacobian, requiring a specific correction as described in R5.03.21. With final, one obtains at the material point: $\Psi_{elas}=5,6\,MPa$

2.3 Uncertainty on the solution

the solution is analytical. With the rounding errors near, one can consider it exact.

2.4 Bibliographical references

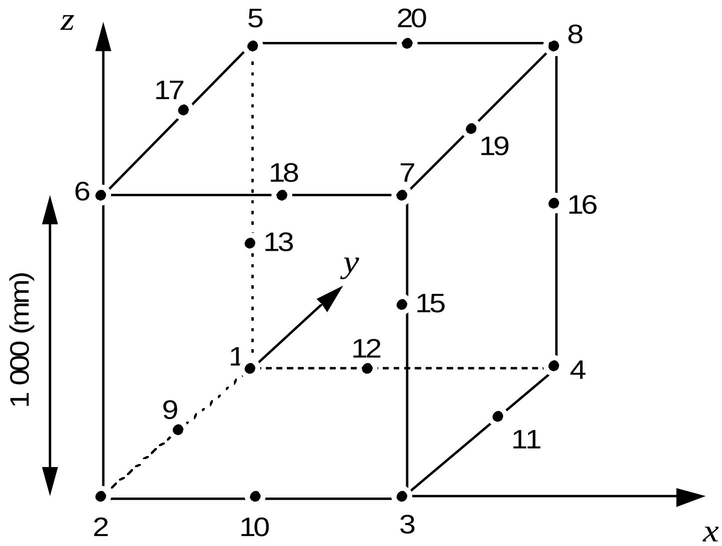
One will be able to refer to:

1) V. CANO, E. LORENTZ: Introduction into *the Code_Aster* of an elastoplastic model of behavior in large deformations with isotropic hardening - internal Note EDF DER HI - 74/98/006/0

3 Modelization A

3.1 Characteristic of the voluminal

modelization Modelization: 1 mesh HEXA20
1 nets QUAD8



Boundary conditions:

$N2$: $U_x = U_y = U_z = 0$ $N9 \quad N13 \quad N14 \quad N5 \quad N17 : U_x = 0$

$N1$: $U_x = U_z = 0$

$N6$: $U_x = U_y = 0$

Charge: Tension on the face $[3\,4\,8\,7\,11\,16\,19\,15]$ + assignment of the same temperature on all the nodes.
The nombre total of increments is of 21 (1 increment enters $t=0\text{s}$ and 1s, 20 increments enters $t=1\text{s}$ and 2s)
convergence is carried out if residue RESI_GLOB_RELA is lower or equal to 10-6.

3.2 Characteristics of the mesh

Many nodes: 20
Number of meshes: 2

1 HEXA20
1 QUAD8

3.3 Quantities tested and results

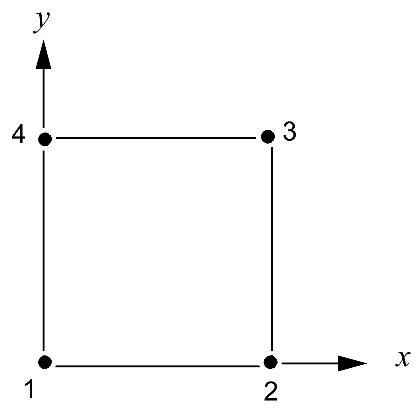
Identification	Reference	Tolerance
$t=2$ Displacement DX ($N8$)	303	1.00%
$t=2$ Displacement DY ($N8$)	– 110	1.00%
$t=2$ Displacement DZ ($N8$)	– 110	1.00%
$t=2$ Stresses $SIGXX$ ($PG1$)	1453	1.00%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	1,50%
$t=2$ $INDIPLAS$ ($PG1$)	1.	0,10%
$t=2$ ENER_ELAS, TOTAL	5.60E+009	5.00%

With $INDIPLAS$ the indicator of plasticity.

4 Modelization B

4.1 Characteristic of the axisymmetric

modelization $2D$ Modelization: 1 mesh QUAD4
1 nets SEG2



Boundary conditions:

$N1$: $U_y = 0$

$N2$: $U_y = 0$

Loading:

Tension on the face $[3\ 4]$ (mesh SEG2) + assignment of the same temperature on all the nodes the nombre total of increments is of 21 (1 increment enters $t=0\text{s}$ and 1s , 20 increments enters $t=1\text{s}$ and 2s)

convergence is carried out if residue RESI_GLOB_RELA is lower or equal to 10-6.

4.2 Characteristics of the mesh

Many nodes: 4
Number of meshes: 2

1 QUAD4
1 SEG2

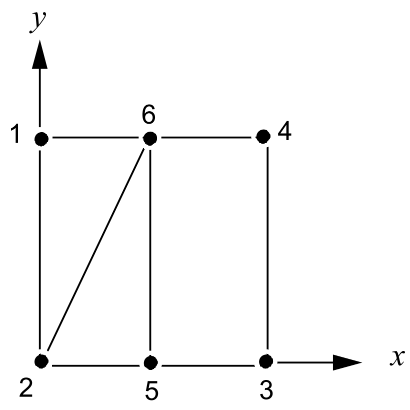
4.3 Quantities tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX ($N3$)	– 110	1.00%
$t=2$ Displacement DY ($N3$)	303	1.00%
$t=2$ Stresses $SIGYY$ ($PG1$)	1453	1.00%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	1.00%
$t=2$ ENER_ELAS, TOTAL	5.60E+009	5.00%

5 Modelization C

5.1 Characteristic of the modelization

Modelization plates DKT of thickness $1000\,mm$: 1 mesh QUAD4, 2 meshes TRIA3
1 nets SEG2



Boundary conditions:

$N2$: $U_x=0 \quad U_y=0 \quad U_z=0 \quad \theta_x=0 \quad \theta_y=0 \quad \theta_z=0$

$N1$: $U_x=0 \quad U_z=0$

Loading:

Tension on the face $[3\ 4]$ (mesh SEG2) + assignment of the same temperature on all the nodes the nombre total of increments is of 21 (1 increment enters $t=0\mathrm{s}$ and $1\mathrm{s}$, 20 increments enters $t=1\mathrm{s}$ and $2\mathrm{s}$)

convergence is carried out if residue RESI_GLOB_RELA is lower or equal to 10-6.

5.2 Characteristics of the mesh

Many nodes: 8
Number of meshes: 4

1 QUAD4
2 TRIA3
1 SEG2

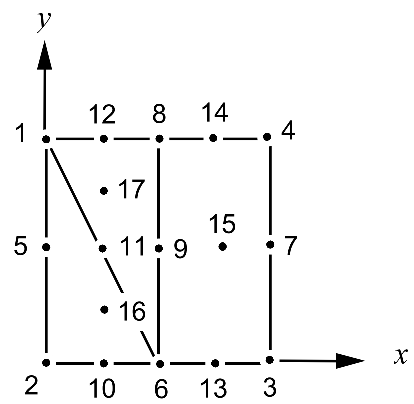
5.3 Quantities tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX ($N3$)	– 110	1.50%
$t=2$ Displacement DY ($N3$)	303	1.00%
$t=2$ Force NXX ($PG1$)	1497	1.00%
$t=2$ Force NYY ($PG1$)	0.79	1.00%
$t=2$ Force NXY ($PG1$)	0.61	1.00%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	1.50%

6 Modelization D

6.1 Characteristic of the modelization

Modelization `COQUE_3D` of thickness $1000\,mm$: 1 mesh `QUAD9`, 2 meshes `TRIA7`
1 nets `SEG3`



Boundary conditions:

$N2$: $U_x=0 \quad U_y=0 \quad U_z=0 \quad \theta_x=0 \quad \theta_y=0 \quad \theta_z=0$

$N5$: $U_x=0 \quad U_z=0$

$N1$: $U_x=0 \quad U_z=0$

Loading:

Tension on the face $[3\;4]$ (mesh `SEG3`) + assignment of the same temperature on all the nodes the nombre total of increments is of 21 (1 increment enters $t=0\mathrm{s}$ and $1\mathrm{s}$, 20 increments enters $t=1\mathrm{s}$ and $2\mathrm{s}$)

convergence is carried out if residue `RESI_GLOB_RELA` is lower or equal to 10-6.

6.2 Characteristics of the mesh

Many nodes: 17
Number of meshes: 4

1 `QUAD9`
2 `TRIA7`
1 `SEG3`

6.3 Quantities tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX ($N3$)	– 110	1.50%
$t=2$ Displacement DY ($N3$)	303	1.00%
$t=2$ Stress $SIXX$ ($PG1$)	1453	3.10%
$t=2$ Variable p $VARI$ ($PG1$)	0.2475	1.50%

7 Summary of the results

results found with *Code_Aster* and `DEFORMATION: "SIMO_MIEHE"` are very satisfactory with percentages of error lower than 0.4% on the stress and than 1.2% the variable of hardening. For shell elements and of shell the use of `DEFORMATION: "PETIT_REAC"` gives results satisfactory with percentages of error of 3% on the force or the stress and lower than 0.7% on the variable of hardening.