HSNV122 - Thermoplasticity and metallurgy in large deformations in simple tension

Summarized:

One treats the determination of the mechanical evolution of a cylindrical bar subjected to known and uniform evolutions thermal and metallurgical (the metallurgical transformation is of bainitic type) and to a mechanical loading of tension.

The behavior model is a model of plasticity in large deformations (command STAT_NON_LINE, key word DEFORMATION: "SIMO_MIEHE") with linear isotropic hardening and plasticity of transformation.

The yield stress and the slope of curve of tension depend on the temperature and the metallurgical composition. The coefficient of thermal expansion depends on the metallurgical composition.

The bar is modelled by axisymmetric elements.

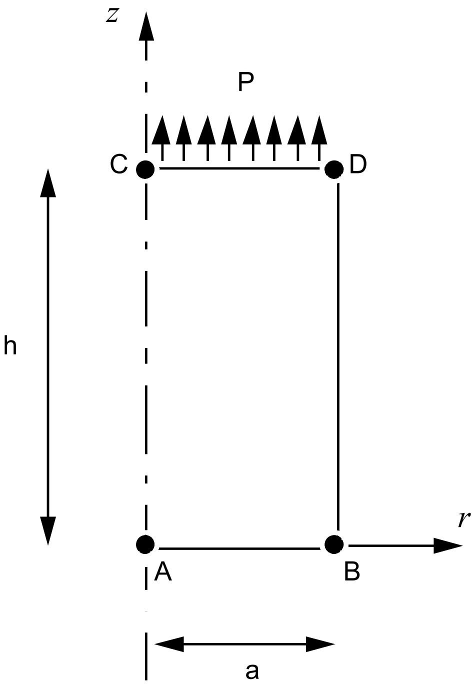
The mechanical loading applied is a following pressure.

This case test is identical to the case test HSNV101 (modelization B, [V7.22.101]) in the meaning where it acts of the same material, the same loading and the same thermal and metallurgical evolutions but in a version in large deformations.

1 Problem of reference

1.1 Geometry

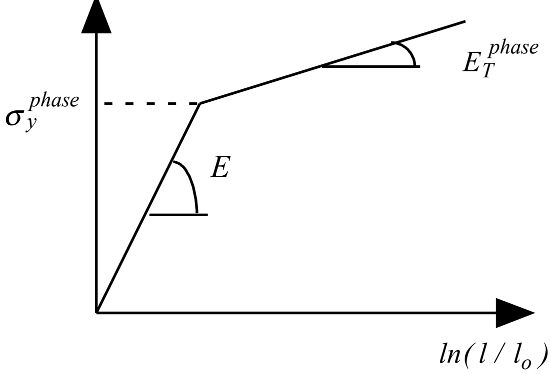
Rayon : a = 0.05 m
Hauteur : h = 0.2 m



1.2 Properties of the material

the material obeys a constitutive law in large deformations with linear isotropic hardening and plasticity of transformation. For each metallurgical phase, the hardening slope is given in the logarithmic strain plane - rational stress.

$$\sigma = \frac{F}{S} = \frac{F}{S_o} \cdot \frac{l}{l_o}$$



l_o and l are, respectively, the initial length and the current length of the useful part of the test-tube.

S_o and S are, respectively, surfaces initial and current.

$C_p = 2000000.\ \text{J m}^{-3}\,^\circ\text{C}^{-1}$	$\lambda = 9999.9\ \text{W m}^{-1}\,^\circ\text{C}^{-1}$
$E = 200000.\ 10^6\text{Pa}$	$\sigma_y^{aust} = 400.\ 10^6\text{Pa} + 0.5\,(T-T^o)\ 10^6\text{Pa}$
$\nu = 0.3$	$\sigma_y^{fbm} = 530.\ 10^6\text{Pa} + 0.5\,(T-T^o)\ 10^6\text{Pa}$
$\alpha_{fbm} = 15.\ 10^{-6}\ ^\circ\text{C}^{-1}$	$h_{aust} = 1250.\ 10^6\text{Pa} - 5.\ (T-T^o)\ 10^6\text{Pa}$
$\alpha_{aust} = 23.5\ 10^{-6}\ ^\circ\text{C}^{-1}$	$h_{fbm} = -50.\ 10^6\text{Pa}\ - 5.\ (T-T^o)\ 10^6\text{Pa}$
$\varepsilon_{fbm}^{ref} = 2.52\ 10^{-3}$	$K_f = 0.\ \text{Pa}^{-1}$
$T^{ref} = 900^\circ\text{C}$	$K_b = K_M = 10^{-10}\text{Pa}^{-1}$
	$F'_{fbm} = 2.\ (1 - Z_{fbm})$

with

C_p	=	thermal
λ	heat capacity	= conductivity
E	=	Young's modulus
ν	=	Poisson's ratio
$*^{aust}$	=	characteristic relating to the austenitic phase
$*^{fbm}$	=	characteristic relating to the phases ferritic, bainitic and martensitic
α	=	thermal coefficient of thermal expansion
ε_{fbm}^{ref}	=	strain of the phases ferritic, bainitic and martensitic the reference temperature, austenite being regarded as not deformed with this temperature
σ_y	=	yield stress
h	=	$\dfrac{EE_T}{E-E_T}$
K	=	coefficient relating to the plasticity of transformation
F'	=	function relating to the plasticity of transformation

the TRC used makes it possible to model a metallurgical evolution of bainitic type, on all structure, of the form:

$$Z_{fbm} = \begin{cases} 0. & \text{si } t \le \tau_1 \qquad \tau_1 = 60\text{s} \\ \dfrac{t-\tau_1}{\tau_2-\tau_1} & \text{si } \tau_1 \le t < \tau_2 \qquad \tau_2 = 112\,\text{s} \\ 1. & \text{si } t \ge \tau_2 \end{cases}$$

1.3 Boundary conditions and loadings

- $u_Z=0$ on the face AB (condition of symmetry).
- tension imposed (following pressure) on the face CD :

$$p(t) = \begin{cases} p_o\ t & \text{pour } t \le \tau_1 \qquad p_o = 6\ 10^6\ \text{Pa} \\ 360\ 10^6\ \text{Pa} & \text{pour } t \ge \tau_1 \qquad \tau_1 = 60\text{s} \end{cases}$$

Note:

In large deformations, it is essential to use the following pressure to take account of current surface and not of initial surface (before strain).

- $T=T^{o}+\mu t$, $\mu=-5\,°C.s^{-1}$ on all structure.

1.4 Initial conditions

$T^{o}=900°\text{C}=T^{ref}$

2 Reference solution

2.1 Computation of the reference solution (cf feeding-bottle [1] and [3])

For a traction test according to the direction x , the tensors of Kirchhoff τ and Cauchy σ are form:

$$\boldsymbol{\tau}=\begin{pmatrix}\tau & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \text{ and } \sigma=\begin{pmatrix}\sigma & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \text{ with } \tau=J\sigma$$

the variation of volume J is given by the resolution of

$$J^3-(3\varepsilon^{th}+\frac{2\sigma}{3K})J^2-J-3\varepsilon^{th}=0$$

where ε^{th} is the thermal strain. This one applies to an austenitic transformation – bainitic:

$$\varepsilon^{th}=Z_{aust}\ \alpha_{aust}(T-T^{ref})+Z_b\left[\alpha_{fbm}(T-T^{ref})+\varepsilon_{fbm}^{ref}\right]$$

Note:

The coefficient K is the bulk modulus (not to be confused with the coefficients K_{phase} relating to the model of plasticity of transformation)

In plastic load, for a linear isotropic R hardening, such as:

$$R=(Z_{aust}\ h_{aust}+Z_b\ h_{fbm})p$$

the cumulated plastic strain p is worth

$$p=\frac{J\sigma-\sigma_y}{Z_{aust}\ h_{aust}+Z_b\ h_{fbm}}$$

with

$$\sigma_y=Z_{aust}\ \sigma_y^{aust}+Z_b\ \sigma_y^{fbm}$$

the tensor gradients of the transformation $\boldsymbol{F}$ and $\overline{\boldsymbol{F}}$ the strain tensor plastics $\mathbf{G}^p$ are form:

$$\mathbf{F} = \begin{bmatrix} F & 0 & 0 \\ 0 & F_{yy} & 0 \\ 0 & 0 & F_{yy} \end{bmatrix} \quad \text{et} \quad J = \det \mathbf{F} = F F_{yy}^2 \quad \Rightarrow F_{yy} = \sqrt{J/F}$$

$$\overline{\mathbf{F}} = J^{-1/3}\mathbf{F} = \begin{bmatrix} \overline{F} & 0 & 0 \\ 0 & \overline{F}_{yy} & 0 \\ 0 & 0 & \overline{F}_{yy} \end{bmatrix} \quad \text{et} \quad \det \overline{\mathbf{F}} = 1 \quad \Rightarrow \begin{cases} \overline{F} = J^{-1/3} F \\ \overline{F}_{yy} = \overline{F}^{-1/2} \end{cases}$$

$$\mathbf{G}^p = \begin{bmatrix} G^p & 0 & 0 \\ 0 & G_{yy}^p & 0 \\ 0 & 0 & G_{yy}^p \end{bmatrix} \quad \text{et} \quad \det \mathbf{G}^p = 1 \quad \Rightarrow G_{yy}^p = (G^p)^{-1/2}$$

The law of evolution of the plastic strain G^P is written:

$$\dot{G}^p / G^p = -2\dot{p} - 4\tau K_b (1 - Z_b) \langle \dot{Z}_b \rangle$$

- For $0\text{s} \le t \le 60\text{s}$, there A. $\dot{Z}_b = 0$ It is not there plasticity of transformation. One obtains then:

$$G^p = e^{-2p}$$

- For $60\text{s} \le t \le 176\text{s}$, one A. $\sigma = \text{constante}$

to integrate the law of evolution of the plastic strain, it should be supposed that the stress of Kirchhoff τ varies very little, i.e. the variation of volume J is very small. Under this assumption, one obtains

$$G^p = e^{-2p} \; e^{-4\tau K_b \left(Z_b - Z_b^2/2 \right)}$$

the component $\overline{F}$ of the gradient of the transformation is given by the resolution of:

$$\overline{F}^3 - \frac{\tau}{\mu G^p} \overline{F} - \frac{1}{(G^p)^{3/2}} = 0$$

Lastly, the field of displacement u (in the initial configuration) is form $\mathbf{u} = u_x \mathbf{X} + u_y \mathbf{Y} + u_z \mathbf{Z}$. The components are given by:

$$u_x = (F - 1) X$$
$$u_y = \left(\sqrt{J/F} - 1 \right) Y$$
$$u_z = \left(\sqrt{J/F} - 1 \right) Z$$

2.2 Notice

In the case test HSNV101 (modelization B), the coefficients of the material were selected of such way not to have classical plasticity $\dot{p}=0$ during the metallurgical transformation which takes place between times 60 and 122s . Indeed if one writes the criterion of load-discharge in this time interval, one obtains

$$f=\sigma-2750\,p\ -\ 250 \text{ with } \sigma = 360 \text{ MPa}$$

whom cancels oneself only for only one value of the cumulated plastic strain p .
For the constitutive law written in large deformations, the criterion of load-discharge is written between these two times

$$f=J(t)\sigma-2750\,p\ -\ 250 \text{ with } \sigma = 360 \text{ MPa}$$

In this case, as long as the variable J remains lower than the value obtained at time $t=60\text{s}$, one will have $\dot{p}=0$. However the value of J is function only of the value of the thermal strain (the stress σ is constant and the coefficient K is independent of the metallurgical phases and the temperature). In this time interval, the thermal strain ε^{th} is given by the following equation:

$$\varepsilon^{th}=8.173\ 10^{-7}\ t^2-1.1807\ 10^{-4}\ t-2.90763\ 10^{-3}$$

One traces below the thermal strain as well as the variation of volume J , solution of the equation of the 3rd degree, according to time.

Thermal strain according to time

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Variation of volume J according to time

One notes that the variable J decreases and increases same way as the thermal strain. In this case, to know time from which the variable J is higher than the value obtained at time $60\mathrm{s}$, it is enough to know the time for which the thermal strain is identical to that obtained at time $t=60\mathrm{s}$. One finds by the resolution of the equation above $t=84.46\mathrm{s}$.

2.3 Uncertainty on the solution

the solution is analytical. Two mistakes are made on this solution. The first door on the computation of the bainitic proportion of phase created. The computation metallurgical precondition does not restore exactly the equation of [§1.2] giving Z_{fbm} according to time, this is why the results of reference presented below are calculated with the bainitic proportion of phase calculated by *Code_Aster*.

The second error is the assumption made on the stress of Kirchhoff τ which is not constant on the time interval understood enters 60 and $176\mathrm{s}$. This will impact the computation of displacement u_x and the plastic strain G^P.

2.4 Results of reference

One will adopt like results of reference displacement in the direction of the loading of tension, the stress of Cauchy σ, the Boolean indicator of plasticity χ and the cumulated plastic strain p. Various times of computations are $t=47, 48, 60, 83, 84, 85$ and $176\mathrm{s}$. For the computation of displacement, the initial length of the bar in the direction of loading is of $0.2\mathrm{m}$.

In all the cases, one has

- $3K = 500\,000\,MPa$ (bulk modulus) $\mu = 76923.077\,MPa$

At time $t = 47s$, one has $Z_b = 0$ $T = 665°C$, $\sigma = 282$ MPa

$\varepsilon^{th} = -5.5225\ 10^{-3}$ $J = 0.983855$ $\tau = 277.45$ MPa

$\sigma_y = 282.5$ MPa $p = 0$ $\chi = 0$

$G^p = 1$ $\overline{F} = 1.0012$ $u = -8.4347\ 10^{-4}$ m

At time $t = 48s$, one has $Z_b = 0$ $T = 660°C$, $\sigma = 288$ MPa

$\varepsilon^{th} = -5.64\ 10^{-3}$ $J = 0.983508$ $\tau = 283.25$ MPa

$\sigma_y = 280.$ MPa $p = 1.327\ 10^{-3}$ $\chi = 1$

$G^p = 0.997$ $\overline{F} = 1.00256$ $u = -5.9639\ 10^{-4}$ m

At time $t = 60s$, one has $Z_b = 0$ $T = 600°C$, $\sigma = 360$ MPa

$\varepsilon^{th} = -7.05\ 10^{-3}$ $J = 0.979337$ $\tau = 352.56$ MPa

$\sigma_y = 250.$ MPa $p = 3.7295\ 10^{-2}$ $\chi = 1$

$G^p = 0.9281$ $\overline{F} = 1.03959$ $u = 6.47595\ 10^{-3}$ m

At time $t = 83s$, one has $Z_b = 0.442138$ $T = 485°C$, $\sigma = 360$ MPa

$\varepsilon^{th} = -7.07867\ 10^{-3}$ $J = 0.979249$ $\tau = 352.53$ MPa

$\sigma_y = 249.978$ MPa $p = 3.7295\ 10^{-2}$ $\chi = 0$

$G^p = 0.8841277$ $\overline{F} = 1.06514$ $u = 1.15441\ 10^{-2}$ m

At time $t = 84$ **S**, one has $Z_b = 0.461361$ $T = 480°C$, $\sigma = 360$ MPa

$\varepsilon^{th} = -7.06031\ 10^{-3}$ $J = 0.979305$ $\tau = 352.55$ MPa

$\sigma_y = 249.977$ MPa $p = 3.7296\ 10^{-2}$ $\chi = 1$

$G^p = 0.8828104$ $\overline{F} = 1.06593$ $u = 1.17051\ 10^{-2}$

At time $t = 85s$, one has $Z_b = 0.480584$ $T = 475°C$, $\sigma = 360$ MPa

$\varepsilon^{th} = -7.04032\ 10^{-3}$ $J = 0.979367$ $\tau = 352.57$ MPa

$\sigma_y = 249.976$ MPa $p = 3.73044\ 10^{-2}$ $\chi = 1$

$G^p = 0.8815276$ $\overline{F} = 1.06671$ $u = 1.18644\ 10^{-2}$

At time $t = 176s$, one has $Z_b = 1$ $T = 20°C$, $\sigma = 360$ MPa

$\varepsilon^{th} = -1.068\ 10^{-2}$ $J = 0.968132$ $\tau = 348.527$ MPa

$\sigma_y = 90.$ MPa $p = 5.9432\ 10^{-2}$ $\chi = 1$

$G^p = 0.82814$ $\overline{F} = 1.10053$ $u = 1.7743\ 10^{-2}$ m

2.5 bibliographical References

One will be able to refer to:

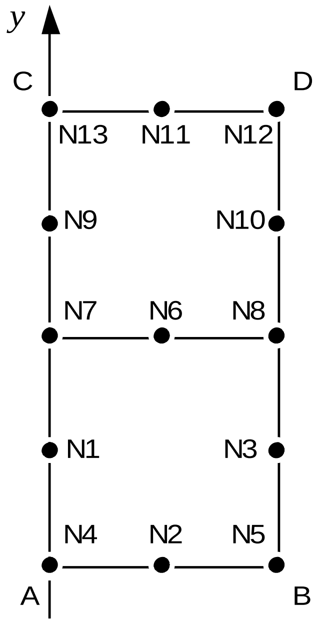
1) V. CANO, E. LORENTZ: Introduction into *the Code_Aster* of an elastoplastic model of behavior in large deformations with isotropic hardening – internal Note EDF DER HI - 74/98/006/0

2) A.M. DONORE, F. WAECKEL: Influence structure transformations in the elastoplastic constitutive laws Notes HI-74/93/024

3) . WAECKEL F, V. CANO: Constitutive law large deformations élasto (visco) plastic with metallurgical transformations [R4.04.03]

3 Modelization A

3.1 Characteristic of the modelization



$A=N4 \quad B=N5 \quad C=N13 \quad D=N12$.

Charge: the nombre total of increments is of 102 (4 increments of 0 with 46s, 2 increments of 46 with 48s , 6 increments of 48 with 60s , 26 of 60 with 112s , 4 of 112 with 116s and 60 increments until 176s). Convergence is carried out if residue (RESI_GLOB_RELA) is lower or equal to 10-6.

3.2 Characteristics of the mesh

Many nodes: 13
Number of meshes and types: 2 meshes QUAD8, 6 meshes SEG3

3.3 Values tested

Identification	Reference
$t=47$ Displacement $DY\ (N13)$	– 8.4347 10-4 m
$t=47$ Variable $p\ \ VARI\ (M1, PG1)$	0.
$t=47$ $\chi\ \ VARI\ (M1, PG1)$	0
$t=47$ Stress $SIGYY\ (M1, PG1)$	282. 10 $^{-6}$ Pa
$t=48$ Displacement $DY\ (N13)$	– 5.9639 10-4 m
$t=48$ Variable $p\ \ VARI\ (M1, PG1)$	1.3260 10-3
$t=48$ $\chi\ \ VARI\ (M1, PG1)$	1
$t=48$ Stress $SIGYY\ (M1, PG1)$	288. 106 Pa
$t=60$ Variable $DY\ (N13)$	Displacement 6.476 10-3
$t=60$ m $p\ \ VARI\ (M1, PG1)$	3.7295 10-2
$t=60$ $\chi\ \ VARI\ (M1, PG1)$	1
$t=60$ Stress $SIGYY\ (M1, PG1)$	360. 106 Pa

$t=83$ Variable DY $(N13)$	Displacement 10-2	1.1544
$t=83$ m p $VARI$ $(M1, PG1)$	3.7295 10-2	
$t=83$ χ $VARI$ $(M1, PG1)$	0	
$t=83$ Stress $SIGYY$ (M1, PG1)	360. 106 Pa	
$t=84$ Variable DY $(N13)$	Displacement 10-2	1.1705
$t=84$ m p $VARI$ $(M1, PG1)$	3.7296 10-2	
$t=84$ χ $VARI$ $(M1, PG1)$	1	
$t=84$ Stress $SIGYY$ $(M1, PG1)$	360. 106 Pa	
$t=85$ Variable DY $(N13)$	Displacement 10-2	1.1864
$t=85$ m p $VARI$ $(M1, PG1)$	3.7304 10-2	
$t=85$ χ $VARI$ $(M1, PG1)$	1	
$t=85$ Stress $SIGYY$ (M1, PG1)	360. 106 Pa	
$t=176$ Variable DY $(N13)$	Displacement 10-2	1.7743
$t=176$ m p $VARI$ $(M1, PG1)$	5.943 10-2	
$t=176$ χ $VARI$ $(M1, PG1)$	1	
$t=176$ Stress $SIGYY$ $(M1, PG1)$	360. 106 Pa	

4 Summary of the results

Them results found with *Code_Aster* are very satisfactory with percentages of error lower than 0.9%, knowing that the analytical solution of reference makes the dead end on certain aspects which into account precisely the solution of Code_Aster *takes*. This can explain the differences observed.