HSNV124 - Volume element in tension and temperature variables

Summarized:

This test, suggested by the IPSI for the Phi2As day of March 30th, 2000 on the nonlinear behaviors makes it possible to validate the good taking into account of the variation of the coefficients with the temperature for four models of behavior (nonlinear isotropic hardening, linear kinematic hardening, and two types of nonlinear kinematic hardening) in $3D$.

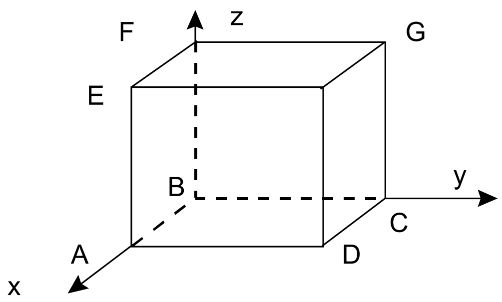
Four modelizations make it possible to validate each one of these behaviors.

The reference solution is analytical for the first three behaviors, and the results will be compared with those of the other participants in the Phi2as day for the fourth.

1 Problem of reference

1.1 Geometry

Volume element materialized by a unit cube on side:



1.2 Properties of the materials

$E=2.10^5\,MPa \quad \nu=0.3\,, \quad \alpha=10^{-5}\,{}^\circ C^{-1}$

the material is elastoplastic with various types of behaviors:

$C1$: Isotropic hardening: curve of tension is form:

$\sigma(\varepsilon^P, T)=\sigma_y(T)+Q(T)(1-e^{-b(T)\varepsilon^P})$

$SIGY=200.-1.7.\mathrm{T}$ (in MPa)

$Q(T)=100.+1.7.\mathrm{T}$ (in MPa)

$b(T)=50.+2.\mathrm{T}$

$C2$: Linear kinematic hardening:

$\sigma(\varepsilon^P, T)=\pm\sigma_y(T)+C(T)\varepsilon^P$

$SIGY=200.-1.7.\mathrm{T}$ (in MPa)

$C(T)=1000+2990.\mathrm{T}$ (in MPa)

$C3$: Nonlinear kinematic hardening (I):

$$\sigma(\varepsilon^P, \dot{\varepsilon}^P, T) = \pm\sigma_y(T) + Q(T)\alpha$$

$$\dot{\alpha} = \dot{\varepsilon}^P - D(T)\alpha\left|\dot{\varepsilon}^P\right|$$

$SIGY=200.-1.7.\mathrm{T}$ (in MPa)

$C(T)=(100+1.7.\mathrm{T})(50+$ (in MPa)

$D(T)=50$

$C4$: Nonlinear kinematic hardening (II):

$$\sigma(\varepsilon^P, \dot{\varepsilon}^P, T) = \pm\sigma_y(T) + Q(T)\alpha$$

$$\dot{\alpha} = \dot{\varepsilon}^P - D(T)\alpha\left|\dot{\varepsilon}^P\right|$$

same characteristics as for the behavior $C3$, except $D(T)=50+2\mathrm{T}$

1.3 Boundary conditions and loadings

Such as the stress state and of strain are uniform in the volume element:
B Not blocked in x , y and z . A Not blocked in z , $DY=0$ on the face $ABFE$
Distributed force on the face $CDHG$: Fy
Uniform temperature $T(t)$ on the cube. The reference temperature is worth $0°C$.
Fy and T vary according to time in the following way:

time t	0	1	2
$Fy(t)$	0	$210\,MPa$	$210\,MPa$
$T(t)$	0	0	$100°C$

2 Reference solution

2.1 Method of calculating used for the reference solution [bib1]

2.1.1 isotropic Hardening: analytical

$t=1\text{s}$ ($T=0°C$): $\varepsilon^P=\frac{1}{b}\ln\left(\frac{Q+\sigma_y-\sigma}{Q}\right)$ with $\sigma=210MPa=cste$ is $\varepsilon^P=0.21072\%$

Heating: Maximum plastic strain with $t=1.45222\text{s}$ ($T=45.222°C$):

$$\varepsilon^P=\frac{1}{b(T)}\ln\left(\frac{Q(T)+\sigma_y(T)-\sigma}{Q(T)}\right)$$ that is to say

$$\varepsilon^P=0.48108\%$$

Then: the plastic strain does not evolve any more.

2.1.2 Linear kinematic hardening: analytical

$t=1\text{s}$ ($T=0°C$): $\varepsilon^P=1\%$

Heating: Constant plastic strain until $t=356/316=1.12658\text{s}$ ($T=12.658°C$):

Then, the plastic strain decreases to reach with $t=2\text{s}$: $\varepsilon^P=0.08\%$

2.1.3 Nonlinear kinematic hardening I: analytical

$t=1\text{s}$ ($T=0°C$)$:\varepsilon^P=\frac{1}{D}\ln\left(\frac{A+\sigma_y-\sigma}{A}\right)$ with $A=\frac{C}{D}=100$ is $\varepsilon^P=0.21072\%$

Heating: Maximum plastic strain with $t=1.26011\text{s}$ ($T=26.011°C$):

$$\varepsilon^P=\frac{1}{D}\ln\left(\frac{A(T)+\sigma_y(T)-\sigma}{A(T)}\right)$$ that is to say

$$\varepsilon^P=0.40729\%=\varepsilon_0^p$$

the plastic strain does not evolve any more until $t_1=1.98332\text{s}$ ($T_1=98.332°C$) where one meets the other end of the field of elasticity.

Then: the plastic strain decreases to reach with $t=2\text{s}$:

$$\varepsilon^P=\varepsilon_0^p+\frac{1}{D}\ln\left(\frac{A(T)+\sigma_y(T)-\sigma}{X_0(T_1)+A(T)}\right)$$ with $X_0(T_1)=A(T_1)\left(1-e^{-D\varepsilon_0^p}\right)$ is $\varepsilon^P=0.4037229\%$

2.1.4 nonlinear Kinematic hardening II

Comparison with the reference solution suggested to the day $\Phi^2 As$. (result numerical obtained with 10 time step), and comparison with the results got with *Code_Aster* with a discretization in time very fine

Plastic strain YY	Computation fine Aster: 100 steps until $1.26\mathrm{s}$, 100 steps between 1.98 and 2s	Référence $\Phi^2 As$: Result for 10 steps
$t=1\mathrm{s}$	2.1072 10-03	2.1072 10-03
$1.26\mathrm{s}<t<1.98\mathrm{s}$	4.18947 10-03	4.38 10-03
$t=2\mathrm{s}$	4.12131 10-03	4.32 10-03

2.2 Accuracy on the results of reference

One has an analytical solution for the first three behaviors, uncertainty is thus null. It is estimated at 4% for the fourth (difference between result for 10 steps and that for 200 steps, the solution strongly depending on the temporal discretization).

2.3 References bibliographical

1) IPSI: day of Phi2AS study of March 30th, 2000 on the nonlinear behaviors of the materials.

3 Modelization A

3.1 Characteristic of the modelization

Behavior $C1$: isotropic hardening, in $3D$. It is modelled in two ways:

- either using behavior VMIS_ISOT_TRAC, with the curves of tension given all, $10°C$ and interpolated them for each temperature, which can be vague,
- or using behavior VMIS_CIN1_CHAB, by cancelling nonlinear kinematic hardening, and by keeping only the isotropic hardening which is precisely expressed in the form:

$$\sigma\left(\varepsilon^{P}, T\right) = \sigma_{y}(T) + Q(T)\left(1 - e^{-b(T)\varepsilon^{P}}\right)$$

It is enough to take then:

$R_0 = SIGY = 200.$ -1.7.T1
$R_I = SIGY + Q(T) = 200. - 1.7.\mathrm{T} + 100. + 1.7.\mathrm{T} = 300$
$C_I = G_0 = 0$

time step enters $t=0\mathrm{s}$ and $t=1\mathrm{s}$ 20 time step enters $t=1\mathrm{s}$ and $t=2\mathrm{s}$.

3.2 Characteristics of the mesh

The mesh comprises a mesh HEXA8.

3.3 Quantities tested and results

Behavior VMIS_ISOT_TRAC :

Time (s)	Plastic strain according to Y	Reference	*Aster*	% difference
$t=1.$	$EPYY$	2.1072 10-03	2.1096 10-03	0.1
$t1=1.45$	$EPYY$	4.8108 10-03	4.8135E-03	0.056
2	$EPYY$	4.8108 10-03	4.8135E-03	0.056

Behavior VMIS_CIN1_CHAB :

Time (s)	Plastic strain according to Y	Reference	*Aster*	% Difference
$t1=1.$	$EPYY$	2.1072 10-03	2.1096 10-03	0.1
$t1=1.45$	$EPYY$	4.8108 10-03	4.81079 10-03	0.0001
2	$EPYY$	4.8108 10-03	4.81079 10-03	0.0001

4 Modelization B

4.1 Characteristic of the modelization

Behavior $C2$: linear kinematic hardening, in $3D$. It is modelled in three ways:

- maybe using behavior VMIS_CINE_LINE, while taking: $\mathrm{D\_SIGM\_EPSI} = E.C(T)/(E+C(T))$ with $C(T)=(1000+2990.\mathrm{T})$
- is using behavior VMIS_ECMI_LINE, while taking: $\mathrm{D\_SIGM\_EPSI}=E.C(T)/(E+C(T))$ and the constant of Prager $PRAG=2/3\,C(T)$
- is using behavior VMIS_CIN1_CHAB, by keeping only linear kinematic hardening: It is enough to take then: $R_0=R_I=SIGY$ $b=0$ $C_I=C(T)$, $G_0=0$

temporal Discretization: 1 time step enters $t=0\mathrm{s}$ and $t=1\mathrm{s}$ 20 time step enters $t=1\mathrm{s}$ and $t=2\mathrm{s}$.

4.2 Characteristics of the mesh

The mesh comprises a mesh HEXA8.

4.3 Quantities tested and results

Behavior VMIS_CINE_LINE :

Time (s)	Plastic strain according to Y	Reference	*Aster*	% difference
$t1=1.1$	$EPYY$	0.01	0.01	0.
2	$EPYY$	8.E-4	8.E-4	0

Behavior VMIS_ECMI_LINE :

Time (s)	Plastic strain according to Y	Reference	*Aster*	% difference
$t1=1.1$	$EPYY$	0.01	0.01	0.
2	$EPYY$	8.E-4	8.E-4	0

Behavior VMIS_CIN1_CHAB :

Time (s)	Plastic strain according to Y	Reference	*Aster*	% difference
$t1=1.1$	$EPYY$	0.01	0.01	0.
2	$EPYY$	8.E-4	8.E-4	0

5 Modelization C

5.1 Characteristic of the modelization

Behavior $C3$: nonlinear kinematic hardening (I) in 3D . It is modelled in two ways:

- maybe using behavior VMIS_CIN1_CHAB. It is enough to take then:
  $R_0=R_I=SIGY \quad b=0 \quad C_I=C(T)=(100+1.7.\text{T})(50+2.\text{T})$, $G_0=50$
- that is to say using behavior VMIS_CIN2_CHAB, by choosing the parameters in such way that the two kinematical variables are identical: It is enough to take then:
  $R_0=R_I=SIGY \quad b=0 \quad C1_I=C2_I=C(T)/2$, $G1_0=G2_0=50$

temporal Discretization: 20 time step enters $t=0\text{s}$ and $t=1\text{s}$ 60 time step enters $t=1\text{s}$ and $t=2\text{s}$.

5.2 Characteristics of the mesh

The mesh comprises a mesh HEXA8.

5.3 Quantities tested and results

Behavior VMIS_CIN1_CHAB :

Time (S)	Plastic strain according to Y	Reference	*Aster*	% difference
1	$EPYY$	2.1072 10-03	2.113 10-03	0.27
1.26	$EPYY$	4.0729 10-03	4.0875 10-03	0.36
1.98	$EPYY$	4.0729 10-03	4.0875 10-03	0.36
2	$EPYY$	4.0372 10-03	3.978 10-03	1.46

Behavior VMIS_CIN2_CHAB :

Time (S)	Plastic strain according to Y	Reference	*Aster*	% difference
1	$EPYY$	2.1072 10-03	2.113 10-03	0.27
1.26	$EPYY$	4.0729 10-03	4.0875 10-03	0.36
1.98	$EPYY$	4.0729 10-03	4.0875 10-03	0.36
2	$EPYY$	4.0372 10-03	3.978 10-03	1.46

5.4 Notices

the variation with the reference solution comes from the temporal discretization. While refining more, the solution approaches the analytical solution. One chose a compromise between a reasonable temporal discretization in time computation and nevertheless rather precise.

6 Modelization D

6.1 Characteristic of the modelization

Behavior $C4$: nonlinear kinematic hardening (II) in $3\mathrm{D}$. It is modelled in two ways:

- maybe using behavior VMIS_CIN1_CHAB. It is enough to take then:
  $R_0=R_I=SIGY \quad b=0 \quad C_I=C(T)=(100+1.7.\mathrm{T})(50+2.\mathrm{T})$, $G_0=D(T)$
- that is to say using behavior VMIS_CIN2_CHAB, by choosing the parameters in such way that the two kinematical variables are identical: It is enough to take then:
  $R_0=R_I=SIGY \quad b=0 \quad C1_I=C2_I=C(T)/2$, $G1_0=G2_0=D(T)$

temporal Discretization: 40 time step enters $t=0\mathrm{s}$ and $t=1\mathrm{s}$ 30 time step enters $t=1\mathrm{s}$ and $t=2\mathrm{s}$.

6.2 Characteristics of the mesh

The mesh comprises a mesh HEXA8.

6.3 Quantities tested and results

Behavior VMIS_CIN1_CHAB :

Time (S)	Plastic strain according to Y	Reference	*Aster*	% difference
1	$EPYY$	2.1072 10-03	2.11 10-03	0.14
1.26	$EPYY$	4.18947 10-03	4.231 10-03	0.99
1.98	$EPYY$	4.18947 10-03	4.231 10-03	0.99
2	$EPYY$	4.12131 10-03	4.163 10-03	1.00

Behavior VMIS_CIN2_CHAB :

Time (S)	Plastic strain according to Y	Reference	*Aster*	% difference
1	$EPYY$	2.1072 10-03	2.11 10-03	0.14
1.26	$EPYY$	4.18947 10-03	4.231 10-03	0.99
1.98	$EPYY$	4.18947 10-03	4.231 10-03	0.99
2	$EPYY$	4.12131 10-03	4.163 10-03	1.00

6.4 Notices

the variation with the reference solution (obtained for a very fine temporal discretization) comes from the temporal discretization. One chose here a compromise between a reasonable temporal discretization in time computation and nevertheless rather precise.

7 Summary of the results

This test makes it possible to highlight the effects of variation of the coefficients of the elastoplastic behaviors with the temperature.

The results are identical to the analytical solution for the linear kinematic hardening (where the solution does not depend on the temporal discretization). For the other behaviors, the accuracy is less good (lower deviation than 1.5%) because the solution strongly depends on the selected temporal discretization.

This test thus makes it possible to validate the integration of behaviors VMIS_ISOT_TRAC, VMIS_CINE_LINE, VMIS_ECMI_LINE, VMIS_CIN1_CHAB, and VMIS_CIN2_CHAB compared to the variation of the coefficients with the temperature.