HSNV132 - Fissure X-FEM in thermoelasticity

Summarized

This test is validating the taking into account of a thermal loading for a computation of cracking by the method X-FEM [bib1] on an academic case $2D/3D$. The elements X-FEM play a part only on the level it mechanical computation to represent the discontinuity of displacement through crack. For the thermal part, the temperature is considered continuous through the interface and the elements X-FEM do not intervene.

This test brings into play a square plate with a right crack leading, embedded to lower edge, and subjected to a horizontal variation in temperature. This loading causes to open crack. One compares displacement for ending node in top with right, for FEM and X-FEM. This test comprises a call to THER_LINEAIRE, then MECA_STATIQUE.

Four modelizations are considered:

- modelization A : $\text{FEM}\,2\text{D}$ (taken as reference),
- modelization B : $\text{X-FEM}\,2\text{D}$, crack in the middle of the elements,
- modelization C : $\text{X-FEM}\,3\text{D}$ (one blocks displacements according to z), crack in the middle of the elements,
- modelization D : $\text{X-FEM}\,3\text{D}$, modelization in truth 3d, without putting $DZ=0$, to validate the computation of G in 3D.

One finds a lower deviation than 1% for displacement, and a variation of $3{,}5\%$ on K_I and $1{,}4\ \%$ on K_{II}.

1 Problem of reference

1.1 Geometry

the structure $2\mathrm{d}$ is a unit square ($LX=1\,m$, $LY=1\,m$), comprising an on the right emerging right crack, located at middle height. [Figure 1.1-1]. One calls line left line in $x=0$, line the line line in $x=LX$ and line lower line in $y=0$.

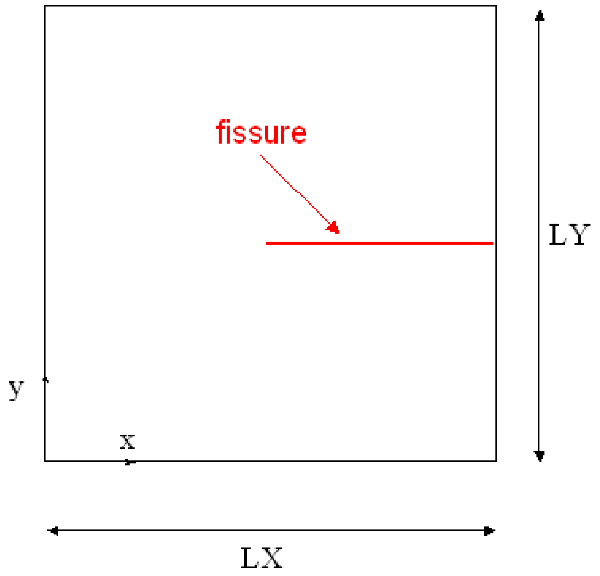

Figure 1.1-1 : geometry of the fissured square plate

1.2 Properties of the material

Modulus Young: $E=205000\,MPa$
Poisson's ratio: $\nu=0.3$
Thermal coefficient of thermal expansion: $\alpha=1{,}282\,10^{-5}$
Thermal conductivity: $\lambda=1\,W.m^{-1}.K^{-1}$
Voluminal heat capacity: $\rho\,C_p=0\,J.m-3.\mathrm{K}^{-1}$

1.3 Boundary conditions and loadings

1.3.1 For the thermal part

the thermal loading consists in applying a temperature imposed $T=20\,°C$ to the nodes of line of left and a temperature imposed $T=220\,°C$ on the nodes of line of right. The variation in temperature is uniform and opens crack in mixed mode.

1.3.2 For the mechanical part

the nodes of line lower are clamped.

1.4 Bibliography

1. GENIAUT S., MASSIN P.: eXtended Finite Method Element, Handbook of reference of *Code_Code_Aster*, [R7.02.12]

2. CRYSTAL E., PROIX J.M.: Computation of the stress intensity factors, [R7.02.08].

2 Modelization a: fissures with a grid in dimension 2

In this modelization, one considers structure in $2D$. The classical finite element method is used. This modelization is used as reference for the continuation.

2.1 Characteristics of the mesh

the structure is modelled by a regular mesh composed of 100×100 QUAD8, respectively along the axes x y. The crack is with a grid.

2.2 Quantities tested and results

One tests the values of following displacement X and Y the point end "PTEXTR" of coordinates $(1;1)$.

One tests also the value of the stress intensity factor K_I given by CALC_G, option CALC_K_G like that given by K_I of POST_K1_K2_K3 [bib2].

Finally, one tests G given by CALC_G, option CALC_G.

All these tests are tests of NON-regression and will be used as reference for the following modelizations.

Identification	*Code_Aster*
DX (PTEXTR)	- 8,7404263 10-4
DY (PTEXTR)	3,826096 10-3
K1 (CALC_G/CALC_K_G)	9,0328413 106
K1 (POST_K1_K2_K3)	8,4543655 106
G (CALC_G/CALC_G)	492.82

3 Modelization b: fissure NON-with a grid in dimension 2

In this modelization, one considers structure in $2D$. The wide finite element method (X-FEM) is used.

3.1 Characteristics of the mesh

the structure is modelled by a regular mesh composed of 101×101 QUAD4, respectively along the axes x y . The crack is not with a grid.

3.2 Quantities tested and results

One tests the values of following displacement X and Y the point end "PTEXTR" of coordinates $(1,1)$.

One tests also the value of the stress intensity factor K_I given by CALC_G, option CALC_K_G like that given by K_I of POST_K1_K2_K3.

Finally, one tests G given by CALC_G, option CALC_G.

The values of reference are those obtained by Standard modelization

A.	*Identification of reference*	Reference	% Tolerance
DX (PTEXTR)	AUTRE_ASTER	- 8,740426 10 -4.2,0	
DY (PTEXTR)	AUTRE_ASTER	3,826096 10 -3.1,0	
K1 (CALC_G/CALC_K_G)	AUTRE_ASTER	9,0328413 106.2,0	
K1 (POST_K1_K2_K3)	AUTRE_ASTER	8,4543655 106.4,0	
G (CALC_G/CALC_G)	AUTRE_ASTER	492.82	2,0

4 Modelization C: crack NON-with a grid out of forgery 3D

In this modelization, one considers structure in $3\mathrm{d}$, but all the degrees of freedom according to z (not only displacements) are put at zero to be reduced to the case $2\mathrm{d}$. The wide finite element method ($\mathrm{X\text{-}FEM}$) is used.

4.1 Characteristics of the mesh

the structure is modelled by a regular mesh composed of $11\times11\times1$ HEXA8, respectively along the axes x, y and z. The crack is not with a grid.

4.2 Boundary conditions and loadings

to be reduced to the case $2\mathrm{D}$, it is necessary to block all the following degrees of freedom z.
To block displacements according to z is not enough, the degrees of freedom nouveau riches have a strong importance. Nodes thus should $DZ=0$ be imposed on all the, and to also impose $H1Z=0$ on the nodes nouveau riches by Heaviside and $E1Z=E2Z=E3Z=E4Z=0$ on all the nodes nouveau riches by the asymptotic functions.

4.3 Quantities tested and results

One tests the values of following displacement X and Y the point end "PTEXTR" of coordinates $(1,1)$.

One does not test the value of rate of energy restitution G given by CALC_G nor that of the stress intensity factor K_I given by K_I of POST_K1_K2_K3 because the fact of forcing displacements according to Z is not in accordance with $2\mathrm{D}$ from the energy point of view.

The values of reference are those obtained by Standard modelization

A.	*Identification of reference*	Reference	% tolerance
DX (PTEXTR)	AUTRE_ASTER	$-8{,}740426\ 10^{-4}$	2
DY (PTEXTR)	AUTRE_ASTER	$3{,}826095\ 10^{-3}$	1

5 Modelization D: crack NON-with a grid in truth 3D

In this modelization, one considers structure in $3D$. The wide finite element method ($X\text{-FEM}$) is used.

5.1 Characteristics of the mesh

the structure is modelled by a regular mesh composed of $51\times51\times2$ HEXA8, respectively along the axes x , y and z . The crack is not with a grid.

5.2 Quantities tested and results

One tests the value of the stress intensity factors K_I and K_{II} given by CALC_G. These values cannot be compared with the values resulting from the other modelizations. They are compared with values resulting from a classical computation $3d$ with the same boundary conditions and a mesh $2\times100\times100$ HEXA8. This case is not part of the case test.

Standard	*identification of reference*	FEM - Reference	% tolerance
K_I	AUTRE_ASTER	6,8941 10 ^{6}	4
K_{II}	AUTRE_ASTER	3,0767 10 ^{6}	2

One can be astonished to find 4% error on K_I whereas the mesh $X\text{-FEM}$ is fine ($51x51$). In fact, if one looks at the contributions of the thermal for K_I, and the contribution NON-thermal for K_I, one has very weak variations on each contribution:

	thermal	NON-thermal	total
FEM	464,0 10 ^{6}	- 457,1 106.6,9	106
X-FEMs	462,9 10 ^{6}	- 456,2 106.6,7.106	
variation	-0,24%	-0,19%	-3,43%

6 Summaries of the results

the purposes of this test are reached:

- to validate on a simple case the taking into account of the temperature on mechanical computation with X-FEM ,
- to validate the computation of the stress intensity factors for the elements X-FEM , in particular the terms related to the thermal.