HSNS102 – Plate out of reinforced concrete with thermal loading

Abstract:

The purpose of this test is principal to check the modelization of the elements reinforced concrete under thermal loading according to three techniques:

Modelization A: Model shell: DKT + GRILLE_EXCENTREE

Modelization B: Model solid: Voluminal + GRILLE_MEMBRANE

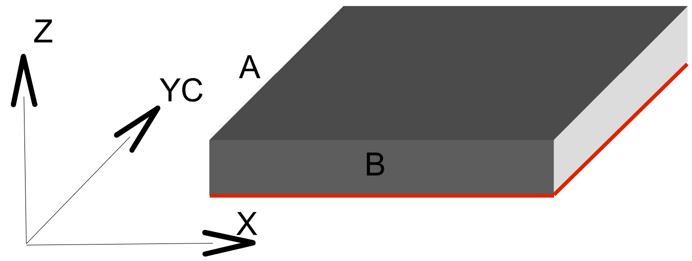
Modelization C: Model solid: Voluminal + MEMBRANE

the purpose is to check the mechanical response by comparison with a reference solution obtained analytically.

Moreover, it tests, in the modelization B, functionality MATRICE=ELASTIQUE of STAT_NON_LINE for elements GRILLE_MEMBRANE.

1 Problem of reference

1.1 Geometry



square Plate:
Length: $l=1.0\,m$
Thickness: $ep=0.2\,m$

Reinforcements:
Section: $S_a=0.01\,m^2$
Eccentring: $e=-0.01\,m$

1.2 Properties of the materials

Concrete:
Young's modulus, $E=3\,10^{10}\,Pa$
Poisson's ratio, $\nu=0.0$
thermal Coefficient of thermal expansion, $\alpha=10^{-5}\,K^{-1}$
Steel:
Young's modulus, $E=2\,10^{11}\,Pa$
Poisson's ratio, $\nu=0.0$
thermal Coefficient of thermal expansion, $\alpha=10^{-5}\,K^{-1}$

1.3 Boundary conditions and loadings

On with dimensions one A one blocks displacements according to X and Z rotation around Y :

$$U_x=0.0\,;U_z=0.0\,;R_y=0.0$$

On edge B one blocks displacement according to Y :

$$U_Y=0.0$$

The initial temperature is $20\,°C$ for steel and the concrete.

One increases the temperature of steel to reach $120\,°C$.

2 Reference solution

2.1 Method of calculating

the total deflections in steel and the concrete are:

$\epsilon_a^t=\epsilon_a^m+\epsilon_a^{th}$ etavec $\epsilon_b^t=\epsilon_b^m$ $\epsilon_a^{th}=\alpha \Delta T$

Is ϵ the strain of the average plan of the plate and χ the curvature of the plate, the two unknowns to be found. By respecting the kinematics (the sections remain plane), steel being perfectly related to the concrete, one a:

$\epsilon_a^t=\epsilon-e\chi$ and $\epsilon_b^t=\epsilon-y\chi$

the normal force imposed on the plate are null:

$$N=N_a+N_b=E_a S_a \epsilon_a^m+E_b\int \epsilon_b^m=E_a S_a(\epsilon-e\chi-\epsilon_a^{th})+E_b S_b \epsilon=0$$

In the same way, the bending moment imposed on the plate is null:

$$M=M_a+M_b=e\,E_a S_a \epsilon_a^m+E_b\int y\,\epsilon_b^m=E_a S_a(e\,\epsilon-e^2\chi-e\,\epsilon_a^{th})+E_b I_b \chi=0$$

One thus obtains two equations to determine the two unknowns:

$$(E_a S_a+E_b S_b)\epsilon-E_a S_a e\chi=E_a S_a \epsilon_a^{th}$$
$$E_a S_a e\epsilon+(E_b I_b-E_a S_a e^2)\chi=E_a S_a e\,\epsilon_a^{th}$$

One obtains:

$$\epsilon=\frac{\alpha \Delta T}{A} \quad \text{and} \quad \chi=\frac{-\alpha \Delta T}{B}$$

with:

$$A=1+\frac{E_b S_b}{E_a S_a}+\frac{S_b e^2}{I_b} \quad \text{and} \quad B=e+(\frac{1}{E_a S_a}+\frac{1}{E_b S_b})+\frac{E_b I_b}{e}$$

From these values one can calculate:

- the lengthening of the plate: $\Delta L=\epsilon L$
- the rotation of the plate: $R_y=\chi L$
- the deflection at the end of the plate: $f=-\chi\frac{L^2}{2}$ or in the middle of the plate $f=-\chi\frac{L^2}{8}$
- the normal stress in steel: $\sigma_a=E_a(\epsilon-e\chi-\epsilon_a^{th})$
- normal force in the concrete: $N_b=E_b S_b \epsilon$

2.2 Quantities and results of reference

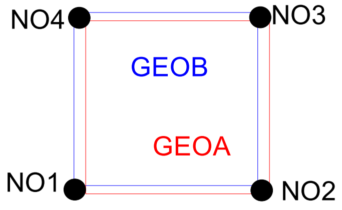
One calculates the lengthening and the deflection of the plate (displacement U_x and U_z of a node of edge C of the plate), rotation (R_y constant over the length), the normal force in reinforcements, the normal force in the concrete N_b.

2.3 Uncertainties on the solution

exact Solution.

3 Modelization A

3.1 Characteristic of modelization



Modelization:
1 element DKT (GEOB) superimposed to 1 element GRILLE_EXCENTRE (GEOA) leaned on the same nodes.

Boundary conditions:
Nodes $NO1$ and $NO4$: $DX=0., DZ=0., DRY=0.$

Nodes $NO1$ and $NO2$: $DY=0$

Thermal loading:

The concrete remains with $20°C$.
Steel passes from $20°C$ to $120°C$ between times 0 and 1,
the reference temperature is in both cases of $20°C$.

3.2 Characteristics of the mesh

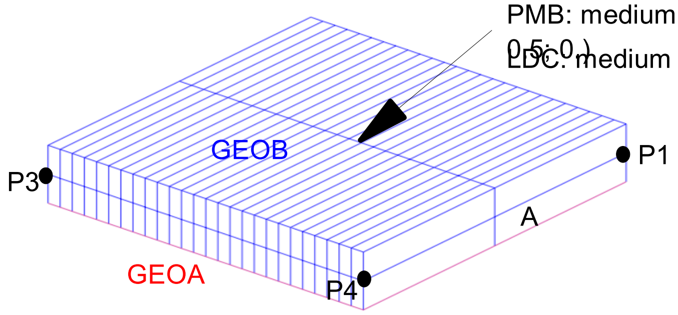
Nodes: 4
Meshes: 1 QUAD4 for the concrete and 1 QUAD4 for steel

3.3 Values tested and Standard

Identification	results of Reference	Value of reference	Tolerance
DX Displacement in NO2	"ANALYTIQUE"	1.4285714E-04	0.0001%
Displacement DZ in NO2	"ANALYTIQUE"	2.1428571E-03	0.0001%
Rotation RY in NO2	"ANALYTIQUE"	-4.2857143E-03	0.0001%
Stress SIXX in the mesh steel AMA1	"ANALYTIQUE"	-8.571429E+07	0.0001%
Force NXX in the mesh concrete MA1	"ANALYTIQUE"	8.571429E+05	0.0001%

4 Modelization B

4.1 Characteristic of modelization



Modelization :
Elements **3D** linear (GEOB) and elements **GRILLE_MEMBRANE** (GEOA) leaned on the nodes of the lower face.

Boundary conditions :

Face A : blocked by $DX=0$
Line $P1-P4$: blocked by $DZ=0$
Point: $P1$ blocked by $DY=0$

thermal Loading:

The concrete remains with $20°C$.
Steel passes from $20°C$ to $120°C$ between times 0 and 1,
the reference temperature is in both cases of $20°C$.

4.2 Characteristics of the mesh

Nodes: 243
Meshes: 104 CUB8 for the concrete and 52 QUAD4 for steel

4.3 Values tested and results

Note: the values of displacements and stresses are evaluated in the center of the plate because in modelization $3D$ there are edge effects free with the end, not considered by the analytical solution.

Standard	identification of reference	Value of reference	Tolerance
DX Displacement to point PMB	"ANALYTIQUE"	7.19892100E-05	1.00%
Displacement DZ to point PMB	"ANALYTIQUE"	5.35714274E-04	1.05%
Stress SIXX in steel to the point LDCS	"ANALYTIQUE"	-8.571429E+07	1.00%
Stress SIXX in the concrete to point PMB	"ANALYTIQUE"	4.52857145E+06	1.00%

5 Modelization C

5.1 Characteristic of the modelization

Modelization :
Elements **3D** linear (GEOB) and elements **MEMBRANE** (GEOA) leaned on the nodes of the lower face.

Boundary conditions and thermal loading :
Identical to the modelization B.

5.2 Characteristic of the mesh

Mesh identical to the modelization B.

5.3 Values tested and Values

results identical to the modelization B.

6 Summary of the results

This test compares the solutions obtained with three types of modelization with an analytical solution in the case of a reinforced concrete plate subjected to a thermal loading.

–Model shell: DKT + GRILLE_EXCENTREE

–Models solid: Voluminal + GRILLE_MEMBRANE

–Models solid: Voluminal + MEMBRANE

the assessment of the comparisons indicates a negligible difference between the results. Only one finite element is sufficient to find the analytical solution in the case DKT + GRILLE_EXCENTREE.
In the voluminal cases with GRILLE_MEMBRANE or MEMBRANE , the error is lower than 1% for a sufficiently fine mesh (26 elements in the length).