WTNP121 – Modelization of a bar saturated with linear compressible fluid (monophasic flow) subjected to a shock with pressure

Summarized:

This case test has a double purpose:

- to validate the diagrams finished volumes developed for the modelization of diphasic flows.
- to validate the modelization Finite elements saturated hydraulics (modelizations Puts N)

the diphasic problem here will be degenerated in a monophasic problem which one knows the analytical solution. It is the monodimensional modelization of a bar saturated with water subjected to a shock with pressure.

1 Problem of reference

the purpose of this case test is to compare the solution obtained with the various diagrams volumes finished with an analytical solution.

1.1 Analytical solution

the non stationary and monodimensional monophasic problem can be written in a general form of the type:

$$N\frac{\partial P}{\partial t}-K_{int.}\Delta P=0$$

$$P(t=0)=P_0$$

$$P(t,x=0)=0$$

$$\frac{\partial P}{\partial x}(t,x=L)=0$$

This problem admits an analytical solution obtained by development in Fourier series.

$$P=\sum_{k=0}^{\infty}\frac{4\mathrm{P}_0}{(2\mathrm{k}+1)\pi}\exp\left(\frac{-K_{int}}{N}\omega_k^2 t\right)\sin(\omega_k x) \quad \text{with} \quad \omega_k=\left(k+\frac{1}{2}\right)\frac{\pi}{L}$$

One can reduce this series to a finished number K of terms, according to calculated time.
This number of terms is in the following way given:

That is to say n_x the number of points x_i where the solution is evaluated at one time t

One poses: $a_k^i=\frac{4}{(2\mathrm{k}+1)\pi}\exp\left(\frac{-K_{int}}{N}\omega_k^2 t\right)\sin(\omega_k x_i)$

So that the solution can be written: $P(x_i)=\sum_{k=0}^{K} P_0 . a_k^i$

One chooses K such as: $\frac{1}{n_x}\sqrt{\sum_{i=1}^{nx}(a_k^i)^2}<\epsilon$

In practice, we took $\epsilon=10^{-10}$.
The paces of the analytical solution at times 1,10,100,1000 are shown on figure 1 :

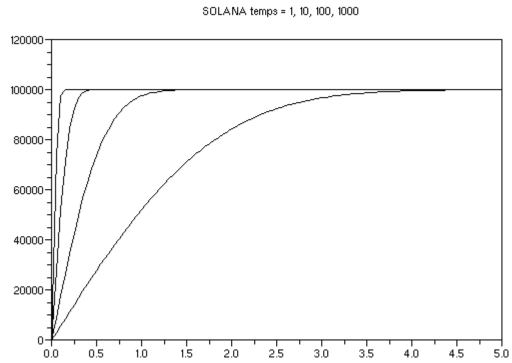


Figure 1: Analytical representation of the solutions

the following table gives the number of terms according to time:

Times	Number series terms
1.194	
10	64.100
	22
1000	8

Table 1.1-1 : Representation amongst term according to time

1.2 simplifying Assumptions

One considers that the medium is completely saturated with water and one imposes a gas pressure null on all the nodes. The biphasic system is then brought back to solve the following problem:

$$\frac{\partial(\varphi\,\rho_l)}{\partial t}-div\left(K_{int}\frac{\rho_l k_{rl}}{\mu_l}\nabla P_l\right)=0$$

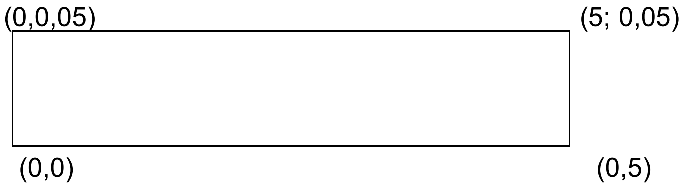
- The fluid is incompressible: $\rho_l=cst$
- The matrix is compressible and porosity evolves proportionally with the fluid pressure: $\frac{\partial\varphi}{\partial P_l}=E_m$

The conservation equation of the mass for the fluid is thus written:

$$\rho_l E_m\frac{\partial P_l}{\partial t}-div\left(K_{int}\frac{\rho_l k_{rl}}{\mu_l}\nabla P_l\right)=0$$

1.3 Geometries

a bar $1\mathrm{D}$ $5\mathrm{m}$ length is considered. Concretely the mesh area will make $[0\mathrm{m},5m]\times[0\mathrm{m};0{,}05m]$ (in the case of the modelization in triangle, it is important not to have too “flattened” triangles, the choice height of the field is thus not pain-killer).



1.4 Meshes

One tested this case on two meshes, one composed of 100 quadrangles (modelizations A with D and M) and the other of 200 triangles (modelizations E with H).

The modelizations I, J, K, L and N consituent an extension $3D$ (bar of section 1×1), the mesh consists of 100 hexahedrons.

1.5 Properties of the materials

One gives here only the properties whose solution depends, knowing that the command file contains other data of material which do not play any part in the solution of with the dealt problem.

Relative	Permeability Viscosity	1
	liquidates $(kg.m^{-1}.s^{-1})$	1
	Modulates compressibility	0
	Density of the fluid (kg/m^3)	1
homogenized Parameters	Permeability (m^2)	10^{-13}
	Porosity	$0,5$
	Storage	10^{-10}
	Saturation out of fluid	1

Table 1.5-1 : Properties of the materials

1.6 Boundary conditions and initial

the limiting conditions are the following ones:

- conditions of Neumann on the right of field: $\frac{\partial P_l}{\partial x}(t, x=5, y)=0$
- conditions of Dirichlet on the left part of the field: $P_l(t, x=0, y)=0$

The initial fluid pressure is of $P_l(t=0, x, y)=10^4 Pa$.

1.7 Lasted of simulation and time step

the period of simulation is of $100\,s$ and the number of time step is of 100.

Note: By preoccupations with an information, the following tests are presented until $1000\,s$ *whereas one limits without Code_Aster simulations to* $100\,s$ *.*

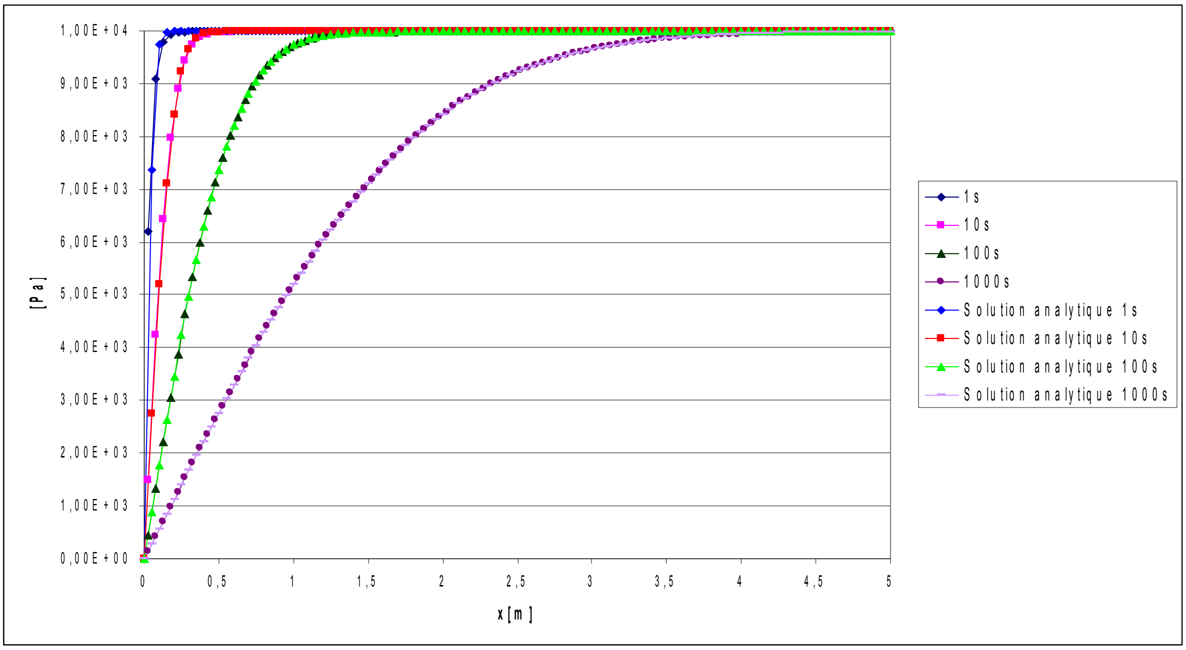
2 Modelization A

2.1 Characteristic of the modelization A

Modelization D_PLAN_HH2SUDM. This modelization corresponds to the modelization Volume Finished with decentring on the mesh close for the terms to mobilities (the fickiens terms are realized). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE. One uses a mesh made up of 100 elements QUAD9.

2.2 Results

One traces the profiles of pressure of fluid at various times as well as the analytical solution at these same times. The results are identical.



Drawing 2: Fluid Values pressure

2.3 tested

One carries out tests on 4 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node into non regression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	**Time (S)**	**PRE1** ***Aster***	**authorized relative Error (** $\%$ **)**
$(0{,}075\,;0)$ *N304*	100 S	-1,33E+003	7%.100
$(0{,}075\,;0{,}5)$ *NQ95*	S	-1.33E+003	9%.100
$(0{,}075\,;1)$ *N293*	S	-1.33E+003	7%.100
$(0{,}05\,;0{,}05)$ *N469*	S	-8,93E+002	7,5%

Table 2.3-1 : Values tested

3 Modelization B

3.1 Characteristic of the modelization B

Modelization D_PLAN_HH2SUDA. This modelization corresponds to the modelization Volume Finished eccentric on the edges for mobilities (the fickiens terms are centered). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE. One uses a mesh made up of 100 elements QUAD9.

3.2 Results

the results are identical to those obtained with the modelization volumes finished eccentric on the mesh.

3.3 Values tested

One carries out tests on 4 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node into non regression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**authorized relative Error (** $\%$ **)**
$(0{,}075\,;0)$ *N304*	100 S	-1,33E+003	7%.100
$(0{,}075\,;0{,}5)$ *NQ95*	S	-1.33E+003	9%.100
$(0{,}075\,;1)$ *N293*	S	-1.33E+003	7%.100
$(0{,}05\,;0{,}05)$ *N469*	S	-8,93E+002	7,5%

Table 3.3-1 : values tested

4 Modelization C

4.1 Characteristic of the modelization C

Modelization D_PLAN_HH2SUC. This modelization corresponds to the modelization centered Finished Volumes (for the darcéens terms and fickiens). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE. The mesh consists of 100 elements QUAD9.

4.2 Results

the results are identical that those obtained with the modelization Volumes finished eccentric on the mesh.

4.3 Values tested

One carries out tests on 4 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node into non regression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**authorized relative Error (** $\%$ **)**
$(0{,}075\,;0)$ $N304$	100 S	-1,33E+003	7%.100
$(0{,}075\,;0{,}5)$ $NQ95$	S	-1.33E+003	9%.100
$(0{,}075\,;1)$ $N293$	S	-1.33E+003	7%.100
$(0{,}05\,;0{,}05)$ $N469$	S	-8,93E+002	7,5%

Table 4.3-1 : Values tested

5 Modelization D

5.1 Characteristic of the modelization D

Modelization D_PLAN_HH2S. This modelization corresponds to the modelization Conventional finite elements. The coupling law hydraulic one is LIQU_AD_GAZ_VAPE. The mesh consists of 100 elements QUAD8.

5.2 Results

the results are identical to those obtained with the modelization volumes finished eccentric on the mesh.

5.3 Values tested

One carries out tests on 2 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node into non regression with a relative error authorized of $0{,}01\,\%$.

Points (x,y)	Time (s)	PRE1 *Aster*	authorized relative Error ($\%$)
$(0{,}05\,;0)$	100 S	-8.89E+002	1%.100
$(0{,}05\,;1)$	S	-8.89E+002	1%

Table 5.3-1 : Values tested

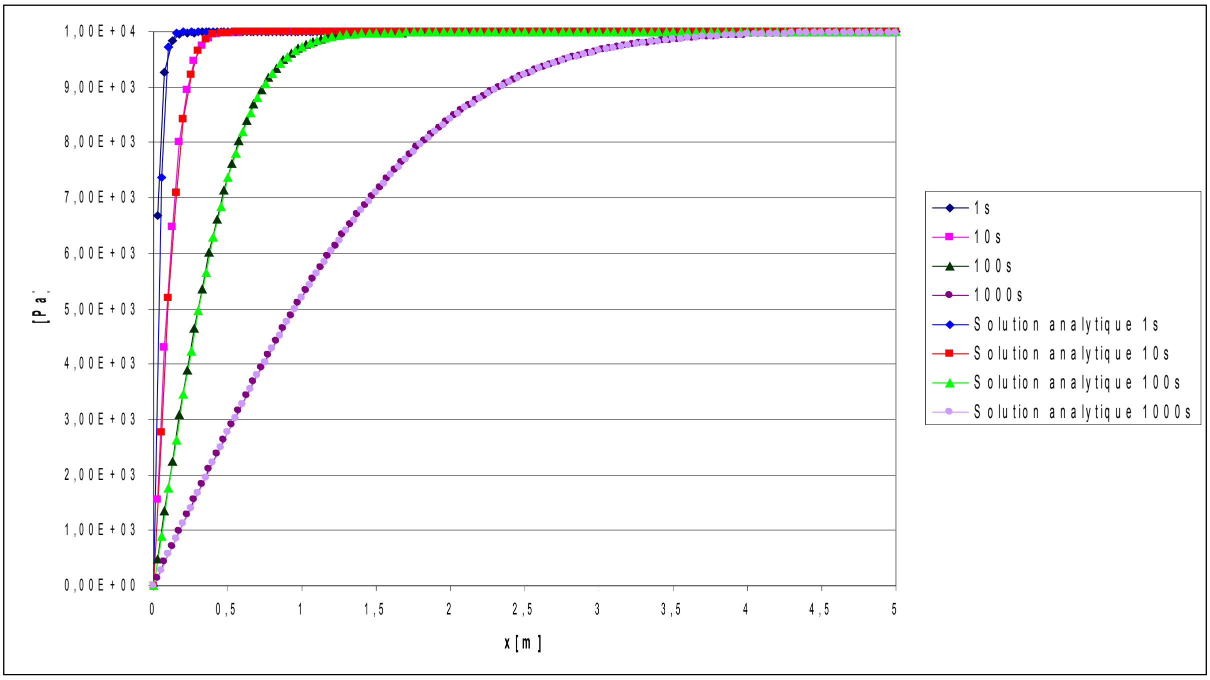
6 Modelization E

6.1 Characteristic of the modelization E

Modelization D_PLAN_HH2SUDM. This modelization corresponds to the modelization Volume Finished with decentring on the mesh close for the terms to mobilities (the fickiens terms are realized). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE. The mesh consists of 200 elements TRIA7. The goal is here to validate this diagram on meshes triangles.

6.2 Results

One traces the profiles of fluid pressure at various times. The results correspond well to the analytical solution.



Drawing 3: Fluid Values pressure

6.3 tested

On two nodes we compared the results with the analytical solution and on the third node we carried out a test of non regression.

Points (x, y)	**Time** (s)	**PRE1 *Aster***	**authorized relative Error ($\%$)**
$(0{,}075 ; 0)$ *N360*	100 S	-1,33E+003	3%.100
$(0{,}075 ; 0{,}025)$ *N505*	S	-1,33E+003	3%.100
$(1675 ; 0{,}0158)$ *NT70*	S	-3,13E+002	0.01%

Table 6.3-1 : Values tested

7 Modelization F

7.1 Characteristic of the modelization F

Modelization `D_PLAN_HH2SUDA`. This modelization corresponds to the modelization Volumes Finished eccentric on the edges for mobilities (the fickiens terms are centered). The coupling law hydraulic one is `LIQU_AD_GAZ_VAPE`. The mesh consists of 200 elements `TRIA7`.

7.2 Results

the results are identical to those obtained with the modelization volumes finished eccentric on the mesh.

7.3 Values tested

One carries out tests on 3 nodes with $t=100\,s$.
On two nodes we compared the results with the analytical solution and on the third node we carried out a test of non regression.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**authorized relative Error (** $\%$ **)**
$(0{,}075\,;0)$ *N360*	100 S	-1,33E+003	3%.100
$(0{,}075\,;0{,}025)$ *N505*	S	-1,33E+003	3%.100
$(1675\,;0{,}0158)$ *NT70*	S	-3,13E+002	0.01%

Table 7.3-1 : Values tested

8 Modelization G

8.1 Characteristic of the modelization G

Modelization D_PLAN_HH2SUC. This modelization corresponds to the modelization centered Volume Finished (for the darcéens terms and fickiens). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE. The mesh consists of 200 elements TRIA7.

8.2 Results

the results are identical that those obtained with the modelization Volumes finished eccentric on the mesh.

8.3 Values tested

On two nodes we compared the results with the analytical solution and on the third node we carried out a test of non regression.

Points (x, y)	Time (s)	PRE1 *Aster*	authorized relative Error ($\%$)
$(0,075;0)$ *N360*	100 S	-1,33E+003	3%.100
$(0,075;0,025)$ *N505*	S	-1,33E+003	3%.100
$(1675;0,0158)$ *NT70*	S	-3,13E+002	0.01%

Table 8.3-1 : Values tested

9 Modelization H

9.1 Characteristic of the modelization H

Modelization D_PLAN_HH2S. This modelization corresponds to the modelization Finite elements. The coupling law hydraulic one is LIQU_AD_GAZ_VAPE. The mesh consists of 200 elements TRIA6.

9.2 Results

the results are identical to those obtained with the modelization volumes finished eccentric on the mesh.

9.3 Values tested

One carries out tests on 2 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node into non regression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**authorized relative Error (** $\%$ **)**
$(0{,}05\,;0{,})$ *N203*	100 S	-8,93E+002	3%.100
$(0{,}05\,;0{,}005)$ *N103*	S	-8,93E+002	3%

Table 9.3-1 : Values tested

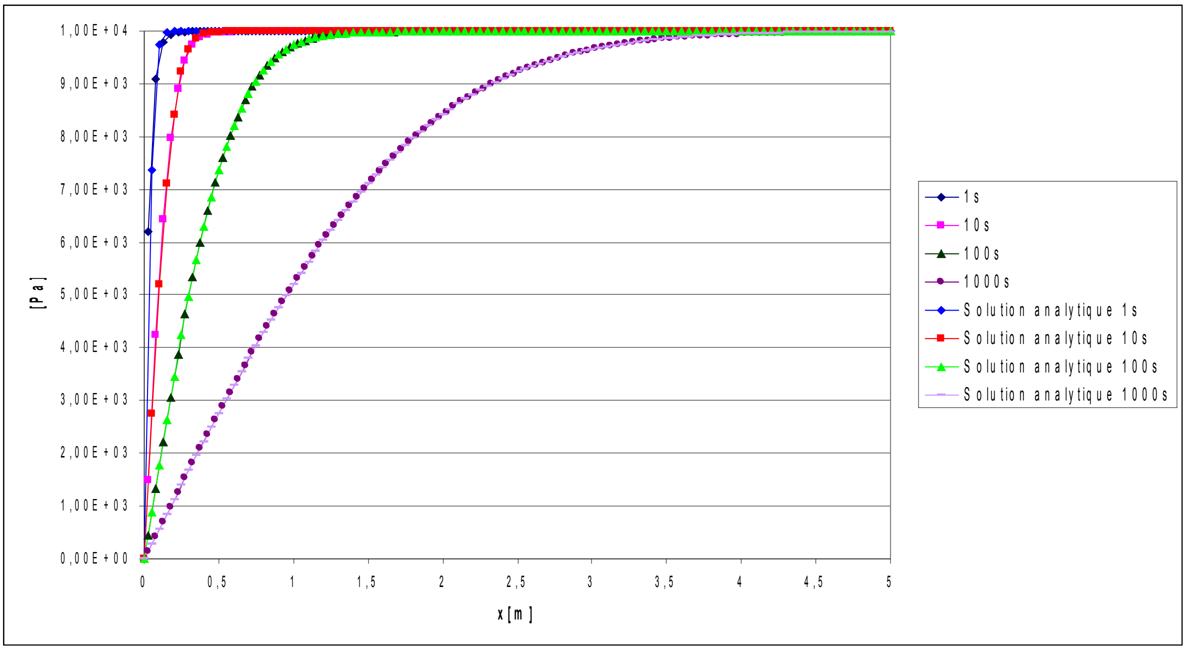
10 Modelization I

10.1 Characteristic of the modelization I

Modelization `3D_HH2SUDM`. This modelization corresponds to the modelization Volume Finished with decentring on the mesh close for the terms to mobilities (the fickiens terms are realized). The coupling law hydraulic one is `LIQU_AD_GAZ_VAPE`. The mesh consists of 100 elements `HEXA27`. It is a question here of validating the diagrams on a test $3D$.

10.2 Results

One traces the profiles of fluid pressure at various times. The results are identical to the analytical solution.



Drawing 4: Fluid Values pressure

10.3 tested

One carries out tests on 3 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node into non regression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	Time (s)	PRE1 *Aster*	authorized relative Error
$(-49{,}5\,;0{,}5\,;0)$ *NH4*	100 S	-8.263E+03	1%.100
$(-1{,}5\,;0\,;-0{,}5)$ *NH195*	S	-9.990E+03	1%.100
$(-49{,}5\,;0\,;0{,}5)$ *NH1*	S	-8.263E+03	1%

Table 10.3-1 : Values tested

11 Modelization J

11.1 Characteristic of the modelization J

Modelization `3D_HH2SUDA`. This modelization corresponds to the modelization Volume Finished eccentric on the edges for mobilities (the fickiens terms are centered). The coupling law hydraulic one is `LIQU_AD_GAZ_VAPE`. The mesh consists of 100 elements `HEXA27`.

11.2 Results

the results are identical to those obtained with the modelization volumes finished eccentric on the mesh.

11.3 Values tested

One carries out tests on 3 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node into non regression with a relative error authorized of $0{,}01\%$.

Points (x,y)	Time (s)	PRE1 *Aster*	authorized relative Error ($\%$)
$(-49{,}5\,;0{,}5\,;0)$ *NH4*	100 S	-8.263E+03	1%.100
$(-1{,}5\,;0\,;-0{,}5)$ *NH195*	S	-9.990E+03	1%.100
$(-49{,}5\,;0\,;0{,}5)$ *NH1*	S	-8.263E+03	1%

Table 11.3-1 : Values tested

12 Modelization K

12.1 Characteristic of the modelization K

Modelization `3D_HH2SUC`. This modelization corresponds to the modelization Volume Finished centered (for the darcéens terms and fickiens). The coupling law hydraulic one is `LIQU_AD_GAZ_VAPE`. The mesh consists of 100 elements `HEXA27`.

12.2 Results

the results are identical that those obtained with the modelization Volumes finished eccentric on the mesh.

12.3 Values tested

One carries out tests on 3 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node into non regression with a relative error authorized of $0{,}01\,\%$.

Points (x,y)	**Time** (s)	**PRE1** ***Aster***	**authorized relative Error (** $\%$ **)**
$(-49{,}5\,;0{,}5\,;0)$ $NH4$	100 S	-8.263E+03	1%.100
$(-1{,}5\,;0\,;-0{,}5)$ $NH195$	S	-9.990E+03	1%.100
$(-49{,}5\,;0\,;0{,}5)$ $NH1$	S	-8.263E+03	1%

Table 12.3-1 : Values tested

13 Modelization L

13.1 Characteristic of the modelization L

Modelization 3D_HH2S. This modelization corresponds to the modelization Finite elements. The coupling law hydraulic one is LIQU_AD_GAZ_VAPE. The mesh consists of 100 elements HEXA20.

13.2 Results

the results are very close to those obtained with the modelization volumes finished eccentric on the mesh.

13.3 Values tested

One carries out tests on 3 nodes with 1 time by comparing the results with the analytical solution. One also tests the first node into non regression with a relative error authorized of $0{,}01\%$.

Points (x, y)	Time (s)	PRE1 *Aster*	authorized relative Error ($\%$)
$(-49; -0{,}5; 0{,}5)$ *N6*	100	-9.090E+03	1%.100
$(-47: -0{,}5; 0{,}5)$ *N16*	100	-9.989E+03	1%.100
$(-48; -0{,}5; -0{,}5)$ *N716*	100	-9.947E+03	1%

Table 13.3-1 : Values tested

14 Modelization M

14.1 Characteristic of the modelization M

Modelization D_PLAN_HS. This modelization corresponds to the modelization Finite elements. The coupling law hydraulic one is LIQU_SATU. The mesh consists of 100 elements QUAD8.

14.2 Results

the results are very close to those obtained with the modelization finite elements D_PLAN_HH2S (modelization D).

14.3 Values tested

One carries out tests on 2 nodes with 1 time by comparing the results with the analytical solution.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**authorized relative Error (** $\%$ **)**
$(0{,}5 ; 0)$ *N104*	100	-8.89E+02	1%.100
$(0{,}5 ; 1)$ *N103*		-8.89E+02	1%

Table 14.3-1 : Values tested

15 Modelization N

15.1 Characteristic of the modelization N

Modelization 3D_HS. This modelization corresponds to the modelization Finite elements. The coupling law hydraulic one is LIQU_SATU. The mesh consists of 100 elements HEXA20.

15.2 Results

the results are very close to those obtained with the modelization finite elements D_PLAN_HH2S (modelization D).

15.3 Values tested

One carries out tests on 2 nodes with 1 time by comparing the results with the analytical solution.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**authorized relative Error (** $\%$ **)**
$(0{,}05;-0{,}5;-0{,}5)$ *N108*	100	-8.89E+02	1%.100
$(0{,}05;0{,}5;0{,}5)$ *N306*		-8.89E+02	1%

Table 15.3-1 : Values tested

16 Summary of the results

This case test makes it possible to test the diagrams volumes finished in various configurations:

- the 3 diagrams finished volumes: centered, decentred edge, decentred mesh
- in $2D$ and $3D$
- on various types of meshes (triangles and rectangles for $2D$, hexahedrons for $3D$)

These same cases are also realized with the diagrams conventional finite elements. All the results are identical to the analytical solution.