FDLV101 - Two cylinders separated by a Summarized incompressible

fluid:

This test of the field of the fluids (fluid coupling/structure) validates the computation of added mass matrix if there are several structures immersed in the same fluid.

By a modal analysis, one thus determines the coupled modes of two structures because of the mass of fluid which separates them. A plane modelization is adopted (thermal for the fluid, and plane strain for the cylinders).

One finds the modes coupled of the system with less 0.1% of result analytical.

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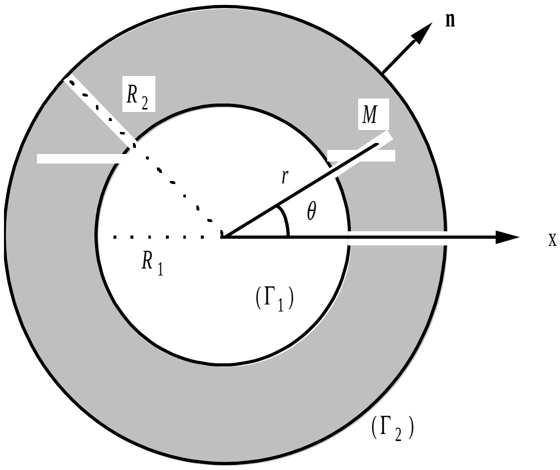
1 Problem of reference

1.1 Geometry



Two cylinders separated by incompressible fluid:

interior radius $R_1 = 1.0\,m$ external radius $R_2 = 1.1\,m$

1.2 Material properties

Fluid:

Water: $\rho_0 = 1000.0\,Kg.m^{-3}$

Solid:

Steel: $\rho_S = 7800.0\,Kg.m^{-3}$; $E = 2.E11\,Pa$; $\nu = 0.3$

Arises connecting the piston to the solid mass:

One places a discrete element on mesh POI1 in the center of the cylinder Γ_1 of stiffness $K1$ and two discrete elements on mesh POI1 on the cylinder Γ_2 on the level of the axis Ox whose stiffness is worth $K2$.

Discrete elements of the type K_T_D_L : $K1 = (1.E7, 1.E7, 1.E7)\,N/m$
$K2 = (5.E6, 5.E6, 5.E6)\,N/m$

1.3 Boundary conditions and loading

One imposes a pressure (i.e. by analogy thermal a temperature null [R4.07.03]) in an unspecified node of the fluid.

One imposes a null displacement of the cylinders according to Oy.

1.4 Initial conditions

Without object for the computation of added mass and the modal analysis.

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2 Reference solution

2.1 Method of calculating used for the analytical reference solution

Computation:

One will suppose that motions of the cylinders and the fluid are primarily plane. The longitudinal effects will be neglected in front of the transverse effects. The problem is two-dimensional. Taking into account symmetry, the reference used is a cylindrical coordinate system (r, θ) related to the central cylinder (see figure above). In this coordinate system and with this particular geometry, the normal derivative $\frac{\partial .}{\partial n}$ is equal to derivative $\frac{\partial .}{\partial r}$ compared to r .

In all this part, the variable p indicates the hydrodynamic field of pressure in the fluid created by natural vibrations of structures, $X_{1/2}$ indicates the eigen modes of the cylinder 1 or 2 respectively.

The eigen modes of the shells of border (Γ_1) and (Γ_2) in the absence of fluid are of the form (n the order of the mode indicates):

$$X_{1\mathrm{n}}(r)=\begin{cases}\cos n\theta \quad \text{ou} \quad \sin n\theta \\ 0\end{cases} \text{ and } X_{2\mathrm{n}}(r)=\begin{cases}0 \\ \cos n\theta \quad \text{ou} \quad \sin n\theta\end{cases}$$

θ is the azimuth angle. These modes are decoupled of course. The first component corresponds to the normal displacement of the interior shell, the second with that of the external shell. In fluid volume, there are thus two problems to solve:

$$\Delta p_{1\mathrm{n}}=0 \quad \left(\frac{\partial p_{1\mathrm{n}}}{\partial n}\right)_{\Gamma_1}=-\rho_f\begin{cases}\cos n\theta \\ \text{ou} \\ \sin n\theta\end{cases} \quad \left(\frac{\partial p_{1\mathrm{n}}}{\partial n}\right)_{\Gamma_2}=0 \qquad \text{éq 2.1-1}$$

and:

$$\Delta p_{2\mathrm{n}}=0 \quad \left(\frac{\partial p_{2\mathrm{n}}}{\partial n}\right)_{\Gamma_1}=0 \quad \left(\frac{\partial p_{2\mathrm{n}}}{\partial n}\right)_{\Gamma_2}=-\rho_f\begin{cases}\cos n\theta \\ \text{ou} \\ \sin n\theta\end{cases} \qquad \text{éq 2.1-2}$$

the field $p_{1\mathrm{n}}$ corresponds to the field of pressure generated in the fluid if the central shell Γ_1 only vibrates, the field $p_{2\mathrm{n}}$ is that created by the outer shell Γ_2 if it only vibrates. The linearity of the equation of Laplace makes it possible to solve each problem independently and then to superimpose them to find the field of pressure total.

The solution of the problem [éq 2.1-1] is, in polar coordinates, of the type [bib1]:

$$p_{1\mathrm{n}}(r, \theta)=\left\{A r^n+B\left(\frac{1}{r}\right)^n\right\}\begin{Bmatrix}\cos n\theta \\ \text{ou} \\ \sin n\theta\end{Bmatrix}$$

One must have $n \neq 0$, because if not one has the NON-conservation of the volume of the fluid.

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The constants A and B are determined by the boundary conditions:

$$\left(\frac{\partial p_{1n}}{\partial n}\right)_{R_1}=-\rho_f\begin{Bmatrix}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{Bmatrix} \text{ and } \left(\frac{\partial p_{1n}}{\partial n}\right)_{R_2}=0$$

It is found whereas the field of pressure for each of the two problems is written:

$$p_{1n}(r,\theta)=\frac{\rho_f R_1}{n}\frac{(r/R_1)^n+(R_2/R_1)^n(R_2/r)^n}{(R_2/R_1)^{2n}-1}\begin{Bmatrix}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{Bmatrix}$$

and:

$$p_{2n}(r,\theta)=\frac{\rho_f R_2}{n}\frac{(R_2/R_1)^n(r/R_1)^n+(R_2/r)^n}{(R_2/R_1)^{2n}-1}\begin{Bmatrix}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{Bmatrix}$$

The modal coefficients of added mass m^A_{ijnm} are calculated from the following formula [R4.07.03] if $i=1$ or 2, $j=1$ or 2, (n,m) belongs to i^2.

$$m^A_{ijnm}=\int_{\Gamma_j} p_{jn} X_{\text{im}}(r).n(\Gamma_j)\,d\Gamma_j$$

The indexing is a little more complex here than in the formula presented in [R4.07.03]: indices I and J refer to the shells Γ_1 and Γ_2, and the indices m and n are associated with the modes of shell. It is noticed that there is coupling of the modes of the various shells, external and intern.

It is noticed, on the one hand, that the fluid does not couple the modes of different n indices because the integrals $\int_\Gamma \cos n\theta\cos m\theta\, d\Gamma$ are cancelled; in addition, the fluid does not couple either the modes $\cos n\theta$ and $\sin n\theta$ because $\int_{(\Gamma)}\cos n\theta\sin n\theta\, d\Gamma=0$. The only existing coupling is a coupling between the two shells for the modes of comparable nature.

A each mode n, one associates a matrix of order 4 symmetric. A submatrix corresponding to projection on the mode n is written:

$$M^A_1=\begin{pmatrix} m^A_{11\text{nn}} & m^A_{12\text{nn}}\\ m^A_{21\text{nn}} & m^A_{22\text{nn}}\end{pmatrix}$$

The total matrix is written: $\begin{bmatrix} M^A_1 & 0\\ 0 & M^A_2\end{bmatrix}$ with $M^A_1=M^A_2$

$$m^A_{11\text{nn}}=L R_1\int_0^{2\pi} p_1(R_1,\theta)\begin{Bmatrix}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{Bmatrix}d\theta$$

Is:

$$m^A_{11\text{nn}}=\frac{\pi}{n}\rho_f R_1^2 L\frac{(R_2/R_1)^{2n}+1}{(R_2/R_1)^{2n}-1} \qquad \text{éq 2.1-3}$$

one will obtain:

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$$m^A_{22nn}=\frac{\pi}{n}\rho_f R_2^2 L\frac{(R_2/R_1)^{2n}+1}{(R_2/R_1)^{2n}-1}$$ éq 2.1-4

and:

$$m^A_{21nn}=m^A_{12nn}=-\frac{\pi}{n}\rho_f R_1 R_2 L\frac{2(R_2/R_1)}{(R_2/R_1)^{2n}-1}$$ éq 2.1-5

L indicates here the height of the shells cylinders in the longitudinal direction.

In our case, only the modes of order of $n=1$ the shells are considered: they correspond respectively to the modes of translation of each shell along an axis passing by the center of the central tube: one takes those arbitrarily corresponding to Ox the axis: the coefficients of linear added mass are written:

$$m^A_{11}=\pi\rho_f R_1^2\frac{(R_2/R_1)^2+1}{(R_2/R_1)^2-1}$$

$$m^A_{22}=\pi\rho_f R_2^2\frac{(R_2/R_1)^2+1}{(R_2/R_1)^2-1}$$

$$m^A_{21}=m^A_{12}=-\pi\rho_f R_1 R_2\frac{2(R_2/R_1)}{(R_2/R_1)^2-1}$$

The equation of the generalized motion of the two coupled shells is written:

$$\begin{pmatrix}m_1 & 0\\ 0 & m_2\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_2\end{pmatrix}+\begin{pmatrix}k_1 & 0\\ 0 & k_2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=-\begin{pmatrix}m_{11} & m_{12}\\ m_{12} & m_{22}\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_2\end{pmatrix}$$

The own pulsations of the coupled system are given by the equation of degree 4:

$$det\left[\begin{pmatrix}m_1+m_{11} & m_{12}\\ m_{12} & m_2+m_{22}\end{pmatrix}\Omega^2-\begin{pmatrix}k_1 & 0\\ 0 & k_2\end{pmatrix}\right]=0$$

Numerical application:

$K_1=10^7 N/m \quad K_2=10^7 N/m$
$m_{11}=33\,060\,kg/m$
$m_{22}=40\,004\,kg/m$
$m_{12}=-36\,200\,kg/m$

Two eigenfrequencies are obtained:

$f_1=1.696\,Hz \quad f_2=4.128\,Hz$

2.2 Results of Analytical

reference

2.3 bibliographical References

R.J GIBERT. Vibrations of Structures. Interactions with fluids. Eyrolles (1988).

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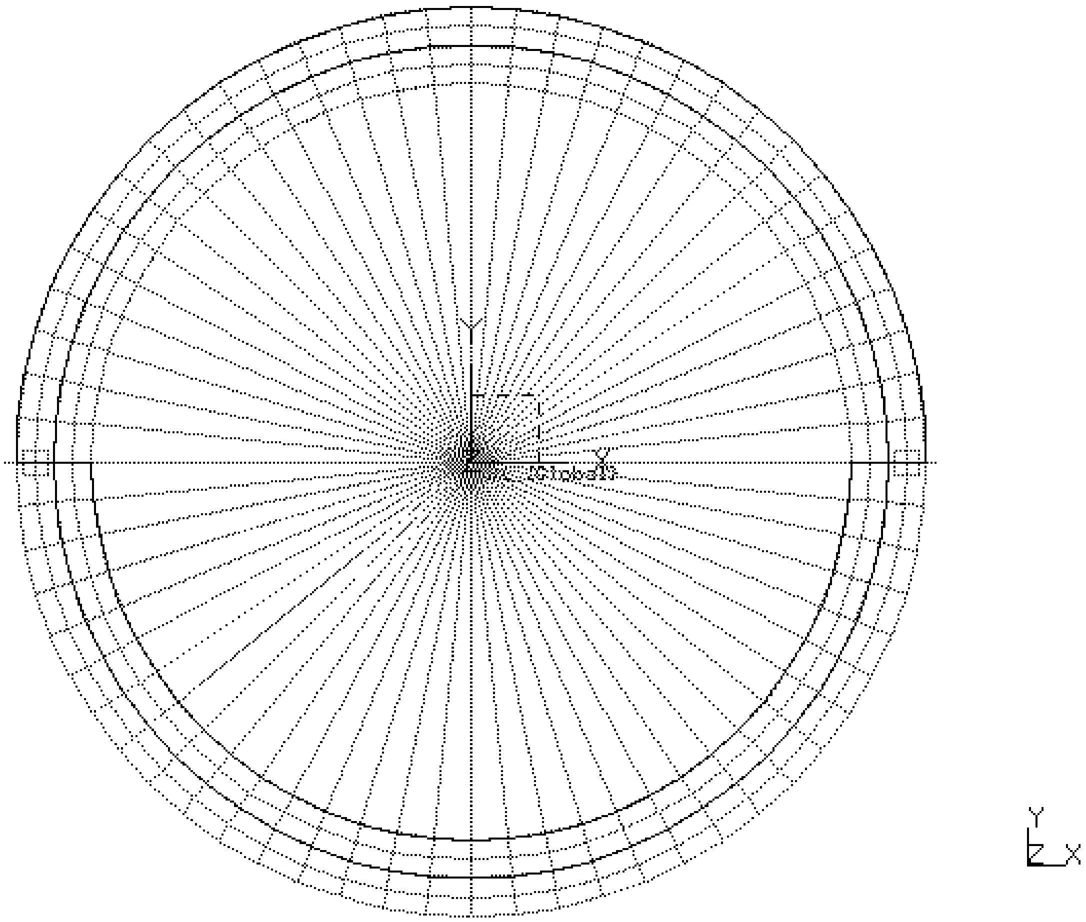
3 Modelization A

3.1 Characteristic of the modelization

plane thermal Formulation for fluid (QUAD4 and SEG2)
Formulation plane strain and discrete for solid (TRIA3, QUAD4 and POI1)



This modelization is designed to determine the modes of order $n=1$ of the cylinders. The modes of shells of a higher nature cannot be simulated by this kind of model, but by a modelization of the type `COQUE_CYL` [U4.22.01].

Cutting = 64 meshes QUAD4 on the circumference of the cylinders
64 meshes TRIA3 on the interior of the interior cylinder
64 meshes SEG2 on the fluid interface/cylinders
2 meshes QUAD4 according to the thickness of the fluid
2 meshes QUAD4 according to the thickness of the external cylinder

Boundary conditions:

```
DDL_IMPO=_F  (GROUP_NO= HANGS,  DY= 0.  )
DDL_IMPO=_F  (GROUP_NO= ACCREXT,   DY=0.   )
TEMP_IMPO=_F (GROUP_NO= TEMPIMPO,  TEMP= 0.)
```

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3.2 Characteristics of the mesh

Many nodes: 356 QUAD4

Number of meshes and types: 64 TRIA3, 128 SEG2, 3 POI1

3.3 Values tested

Identification	Reference (Hz)	% tolerance
Order of the eigen mode i : 1	1.696	0.1%
Order of the eigen mode i : 2	4.128	0.1%

3.4 Remarks

Computations of modes carried out by:

```
MODE_ITER_SIMULT     OPTION = "PLUS_PETITE"     NMAX_FREQ= 2.
```

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