FDLV107 - Stiffness added under Summarized

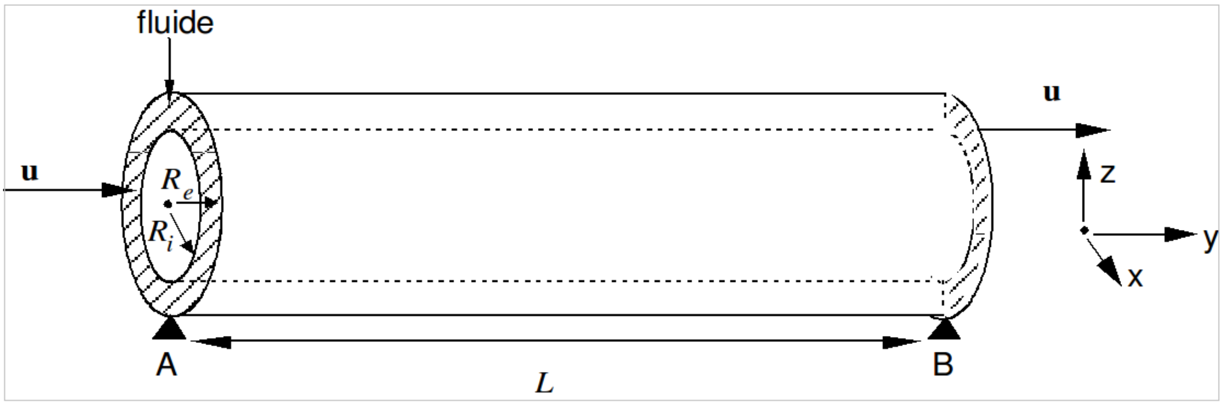
annular flow:

This test of the field of the interaction fluid-structure, validates the computation of stiffness added on a circular cylinder excited on its first mode of bending kneecap-kneecap and subjected to annular flows various velocities.

One calculates the stiffness added (function velocity) on the first mode of bending of the cylinder. One checks the decrease of the eigenfrequency of the mode, up to zero value for a critical velocity of flow of the fluid.

1 Problem of reference

1.1 Geometry



the system represented on the diagram above is composed of two coaxial cylinders and a fluid flow at the speed U in annular space between the two cylinders. Dimensions are:

interior radius: $R_i=1\,m$;
external radius: $R_e=1.05\,m$;
length: $L=100\,m$.

1.2 Properties of the materials

Structure:

Young modulus: $E=2.10^{11}\,Pa$;
Poisson's ratio: $\nu=0.3$;
density: $\rho_s=7800\,kg/m^3$.

Fluid:

density: $\rho=1000\,kg/m^3$.

1.3 Boundary conditions and loadings

Structure:

blocking of the nodes of the interior cylinder;
hinge at the points A and B of the external cylinder.

Fluid:

one imposes various velocities as starter of the fluid field with normal heat fluxes equal to $4\,m/s$ $0.5\,m/s$ $1.5\,m/s$ $2\,m/s$, $2.2\,m/s$ and $2.688\,m/s$ (critical velocity).

1.4 Initial conditions

Without object.

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is an approximate analytical solution. The analytical fluctuating potentials approximate to compute: the added stiffness are written [bib1]:

$$\begin{cases} \varphi_1(r,\theta,y)= \dfrac{R_e^2}{R_e^2-R_i^2}\left(r+\dfrac{R_i^2}{r}\right) \sin\theta \sin\dfrac{\pi(y+l/2)}{l} \\ \varphi_2(r,\theta,y)= \dfrac{R_e^2}{R_e^2-R_i^2}\left(r+\dfrac{R_i^2}{r}\right) \sin\theta \cos\dfrac{\pi(y+l/2)}{l} \end{cases}$$

The stiffness added on the first mode of bending of the external cylinder considered as a beam kneecap-kneecap is written [bib1]:

$$K_A=-\frac{\rho}{2}\frac{V_0^2\pi^3 R_e^3}{(R_e^2-R_i^2)\,l}\left(R_e+\frac{R_i^2}{R_e}\right)$$

This stiffness, calculated on a cylindrical geometry, is then assigned to a model with an equivalent degree of freedom.

The system with an equivalent degree of freedom is an equivalent spring-mass system to which one affects a mass equal to the mass of the system increased by the added mass by the fluid and a stiffness equal to the stiffness of the system increased by the stiffness added by flow for various velocities.

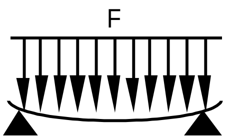
The mass of the system in air is of:

$$M=10292\,kg$$

for an external cylindrical shell of thickness:

$$C=2.10^{-3}\,m$$

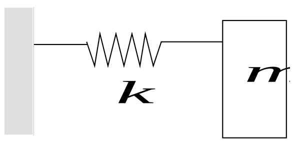
For equivalent stiffness in air of the system "outer shell", one takes the stiffness of a beam subjected to a distributed force over all its length:



$$K=\frac{384EI}{5L^3} \quad \text{avec} \quad I=\frac{\pi d^3 e}{8}=1.649\ 10^{-3}\ m$$

thus $K=2.533\,10^4\,N/m$

the equivalent system coupled with flow is represented by the following diagram:



with $m=M+M_A \quad k=K+K_A$

the own pulsation of the coupled system evolves according to the square rate of flow. If one calls V_{0_c} the critical velocity of flow for which the stiffness k is cancelled:

$$\exists V_{0_c}, K+K_A(V_{0_c})=0 \text{ with } V_{0_c}^2=\frac{2(R_e^2-R_i^2)\,l\,K}{\left(R_e+\frac{R_i^2}{R_e}\right)\rho\pi^3 R_e^3}$$

then one shows that:

$$\omega(V_0)=\omega_e(0)\sqrt{1-x^2}$$

where one posed:

$$\omega_e(0)=\sqrt{\frac{K}{M+M_A}}$$ (own pulsation of the fluid system at rest)

$$x=\frac{V_0}{V_{0_c}}$$ (fallback speed of flow)

the pulsation of the fluid at rest is worth: $\omega=0.085\,rad/s$.

2.2 Results of reference

One calculates for various rates of flow the eigenfrequency of the system.

$V_0(m/s)$	0.5.1.5		2.	2.2	2.688
$M_A(kg)$	3.486E6	3.486E6	3.486E6	3.486E6	3.486E6
$K_A(N/m)$	– 876.5	– 7888.50	– 14023.95	– 16968.98	– 25330
$M_{total}(kg)$	3.491E+6	=	=	=	=
$K_{total}(N/m)$	24453.5	17441.5	11306.05	8361.00	0.
$f(V_0)\times10^{-2}(Hz)$	1.318	1.112	0.896	0.772	0.

2.3 Uncertainty on the semi-analytical

solution Solution.

2.4 References bibliographical

1) ROUSSEAU G., LUU H.T. - Mass, damping and stiffness added for a vibrating structure placed in a potential flow - internal Note EDF/DER, HP-61/95/064/A (1995).

3 Modelization A

3.1 Characteristic of the modelization

For the geometry on which one evaluates the added coefficients:

Fluid: 1800 thermal `elements` THER_HEXA8 1560 thermal elements `THER_FACE4` of interface;

Structure: 1200 shell elements `QUAD4` modelization `"DKT"`.

For the system with 1 degree of freedom are equivalent: 2 discrete finite elements modelization `"DIS_T"`.

3.2 Characteristics of the mesh

Mesh 1 (shells cylinders): 1800 meshes HEXA8 1560 meshes QUAD4

Mesh 2 (discrete system): 1 mesh SEG2 1 nets POI1

3.3 Values tested

Identification	Reference $\times 10^{-2}$
frequency to $0.5\,m/s$	1.318
frequency with $1.5\,m/s$	1.112
frequency with $2\,m/s$	0.896
frequency to $2.2\,m/s$	0.772
frequency with $2.688\,m/s$	0.

4 Summary of the results

the variation on the eigenfrequencies increases owing to the fact that when one is close the critical velocity of buckling, the stiffness of the equivalent system must tend towards zero. However, with the errors rounding (since one assigns "to the hand" the values of stiffness added calculated by the operator to a discrete model) do not allow null to obtain an own pulsation of the system at the critical velocity.
Variations on the added values of stiffness also remain because the reference solution is built on an semi-analytical solution which leaves the approximation according to which there the separation of the variables between the dimension and the orthoradial coordinates is possible. It will be noticed that the potentials chosen to describe the disturbance generated by the vibration of structure in the fluid do not check the equation of Laplace compète but only in one cross-sectional area of the fluid in orthoradial coordinates. This approximation carried out on the reference solution can explain certain variations with numerical computation.