FDLV110 - Computation of added mass on modes obtained by substructuring

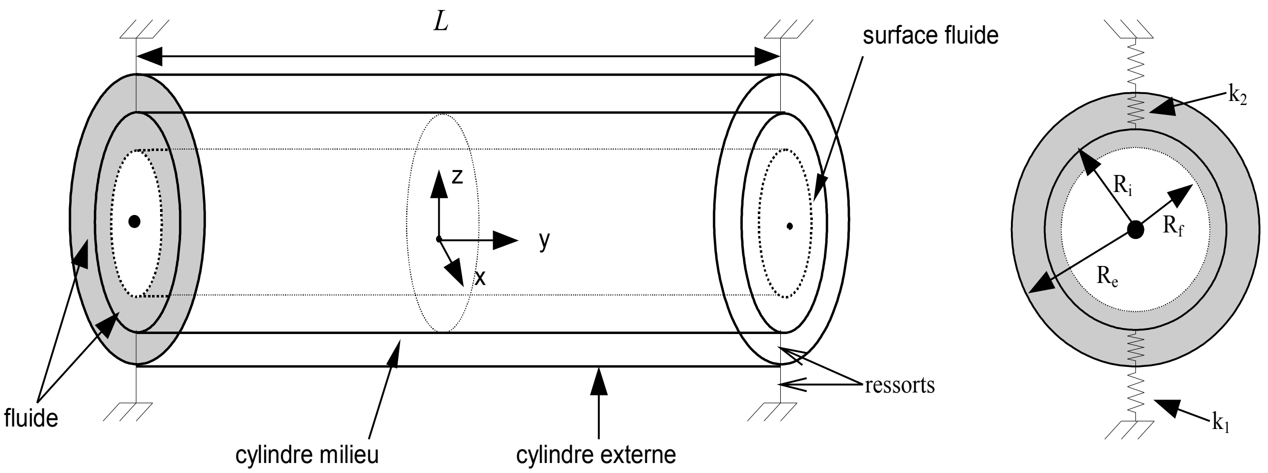
Summarized:

This test of the field of the modal analysis and the interaction fluid-structure implements the computation of added mass on a structure made up of three concentric cylinders separated by two fluid rings (water) which one supposes the behavior governed by the potential theory (fluid true, incompressible at rest). The model is three-dimensional for water. The structure is represented by elements of type thin shell in the modelization A (the structure is rigid in the reference solution). This one is characterized by two eigen modes evaluated by dynamic substructuring, with interface of the type CRAIG-BAMPTON.

The interest of the test lies in the use of functionality “NOEUD_DOUBLE `of L” operator “CALC_MATR_AJOU `. This functionality makes it possible to calculate the effects of added mass D” a structure represented by a surface mesh (without thickness) which is bathed in a fluid. The fluids chosen in this benchmark are different densities on both sides of the intermediate cylinder (water with different temperatures).

1 Problem of reference

1.1 Geometry



$L=50\text{m}$; $R_f=1\text{m}$; $R_i=\frac{5}{3}m$; $R_e=3\text{m}$; $k_1=10^9 N.m^{-1}$; $k_2=0.5\,10^7 N.m^{-1}$; $\rho_{fluide}=1000\,kg.m^{-3}$; $\rho_s=7800\ kg.m^{-3}$; thickness of the shell: $50\,cm$.

1.2 Properties of the materials

Fluid: density $\rho_1=1000\,kg.m^{-3}$; $\rho_2=750\,kg.m^{-3}$.

Structure: $\rho_s=7800\,kg/m^3$; $E=2.1\,10^{11}\,Pa$; $\nu=0.3$ (steel).

1.3 Boundary conditions and loadings

the external cylinder on the one hand is connected to a fixed frame via four springs of unit stiffness k_1 , connected on the other hand to the cylinder medium by four springs of unit stiffness k_2 . The two structures are rigid in this reference solution.

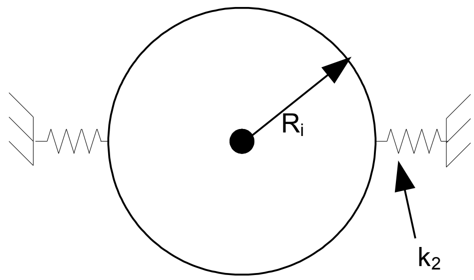
2 Reference solution

One calculates the eigen modes of the system after having checked those of each substructure. One evaluates then the added mass on the modes in air.

2.1 Decomposition of substructures

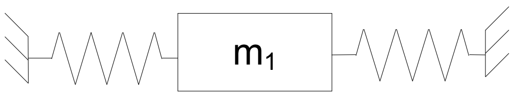
First substructure: roll intermediate

the first substructure is consisted by the intermediate cylinder and of four springs of stiffness $k_2 = 10^7 N.m^{-1}$. These springs are embedded with the interface with the external cylinder which constitutes the second substructure (interface of the type CRAIG-BAMPTON).



Mass cylinder 1: $m_1 = 2.041\,10^6 kg$

The cylinder being rigid, its motion can be modelled by a spring-mass system with a degree of freedom:

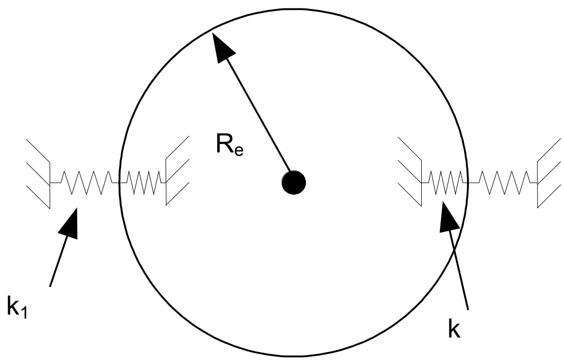


Achacune of its ends, the cylinder is connected to two springs in parallel: the equivalent stiffness of each one is $k' = 2k_2$

the eigenfrequency is worth then: $f' = \frac{1}{2\pi}\sqrt{\frac{2k'}{m_1}} = \frac{1}{2\pi}\sqrt{\frac{4k_2}{m_1}}$, that is to say: $f' = 0.705\,Hz$

Second substructure: roll external

the second substructure is the external cylinder connected on the one hand to the interface by same springs, on the other hand with a fixed frame:



Mass cylinder 2: $m_2 = 3{,}674\,10^6\,kg$

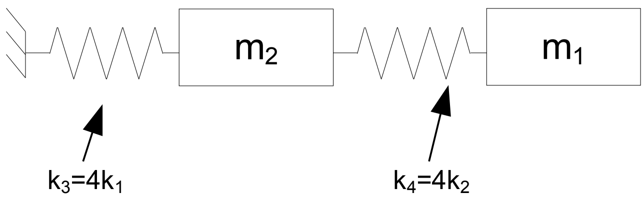
The equivalent stiffness of a fastener of this cylinder by the system of springs in series k_1 and k_2 being worth $9{,}9\,10^6\,N.m^{-1}$ (four fasteners of the same type connect in parallel the cylinder to a fixed support), the eigenfrequency is given by:

$$f'' = \frac{1}{2\pi}\sqrt{\frac{4k_2k_1/(k_2+k_1)}{m_1}}$$, that is to say: $f'' = 0.522\,Hz$

N.B.: the third cylinder (interior cylinder) was not modelled in our case because it agint of a fixed cylinder. It thus constitutes a fixed wall of the fluid field.

Modes in air of structure supplements (intermediate cylinder and external cylinder)

It is a system with two degrees of freedom:



The eigenfrequencies of this system are given by the exact formula [bib2]:

$$f_i = \frac{1}{2^{3/2}\pi}\sqrt{\frac{k_3}{m_2}+\frac{k_4}{m_2}+\frac{k_4}{m_1}\pm\sqrt{\left(\frac{k_3}{m_2}+\frac{k_4}{m_2}+\frac{k_4}{m_1}\right)^2-4\frac{k_3k_4}{m_1m_2}}}\ ,$$

that is to say

$$f_1 = 0.497\,Hz \text{ and } f_2 = 5.263\,Hz\,.$$

The two eigen modes admit, for numerical value:

$$\begin{pmatrix}X_1\\X_2\end{pmatrix}_1 = \begin{pmatrix}1\\5.\,10^{-3}\end{pmatrix} \text{ et } \begin{pmatrix}X_1\\X_2\end{pmatrix}_2 = \begin{pmatrix}-9.\,10^{-3}\\1\end{pmatrix}.$$

2.2 Beginning again computation of the added

mass matrix Potential

fluids [bib1], one establishes that:

$$\varphi_1^{(1)} = \left(R_i^2 \left(\frac{R_i^2 + R_f^2}{R_i^2 - R_f^2} + \frac{R_e^2 + R_i^2}{R_e^2 - R_i^2} \right); \left(5.e^{-3}\right) \times R_e^2 \frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right)$$

et

$$\varphi_1^{(2)} = \left(\left(-9.e^{-3}\right) \times R_i^2 \left(\frac{R_i^2 + R_f^2}{R_i^2 - R_f^2} + \frac{R_e^2 + R_i^2}{R_e^2 - R_i^2} \right); R_e^2 \frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right)$$

The shape of the added mass matrix, in this configuration, is:

$$M_a = \begin{bmatrix} M_a^{11} & M_a^{12} \\ M_a^{21} & M_a^{22} \end{bmatrix}$$

With:

$$M_a^{11} = \rho\ \pi L \left\{ R_i^2 \left(\frac{R_i^2 + R_f^2}{R_i^2 - R_f^2} + \frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right) \right\} = 1{,}753.10^6 \text{ kg},$$

$$M_a^{22} = \rho\ \pi L \left\{ R_e^2 \left(\frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right) \right\} = 2{,}676.10^6 \text{ kg},$$

$$M_a^{12} = \rho\ \pi L \left\{ (5.10^{-3}) \times R_i^2 \left(\frac{R_i^2 + R_f^2}{R_i^2 - R_f^2} + \frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right) - (9.10^{-3}) \times R_e^2 \left(\frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right) \right\} = -15318 \text{ kg}.$$

The coefficient of inertial coupling M_a^{12} is regarded as negligible in front of the coefficients of added auto--mass M_a^{11} and M_a^{22}. The eigenfrequencies of the system depend, at first approximation, only of these two last coefficients.

2.3 Results of reference

Result analytical.

2.4 References bibliographical

•ROUSSEAU G., LUU H.T. : Mass, damping and stiffness added for a vibrating structure placed in a potential flow - Bibliography and establishment in *the Code_Aster* - HP-61/95/064
•BLEVINS R.D.: Formulated for Natural frequency and shape mode, ED. Krieger

3 Modelization A

3.1 Characteristic of the modelization

For the system 3D on which one calculates the added coefficients:

Roll:	2400 meshes `QUAD4` shell elements `MEDKQU4` 12 meshes `SEG2` elements Fluid `springs`
MECA_DIS_T_L:	3600 meshes `thermal` QUAD4 elements `THER_FACE4` on cylindrical surfaces
	7200 meshes `thermal` HEXA8 elements `THER_HEXA8` in fluid annular volume

3.2 Values tested

	Frequencies analytical in air (Hz)
First mode in air	0.497
Second mode in air	5.263

	theoretical Added mass (kg)
M^{11}	1.753 106
M^{22}	2.675 106

	Frequencies analytical of the modes out of water (Hz)
First mode out of water	0.365
Second mode out of water	4.004

4 Summary of the results

The computation of added mass on modes estimated by substructuring are satisfactory. This made it possible to validate" of the command "option "NOEUD_DOUBLE CALC_MATR_AJOU". The variation observed on the second coefficient of added mass is explained by the discretization of the second cylinder. The number of elements is a little insufficient to compute: in an exact way the integral of the field of pressure on the structure.