Model of Rousselier for the ductile rupture

Summary

The model of Rousselier describes the damage due to the plastic growth of cavities in a metal. It makes it possible to model cracking and the ductile rupture. The relation of behavior is elastoplastic or viscoplastic with isotropic work hardening. It allows the changes of plastic volume and is written in small deformations. The writing in great deformations with a formulation of Simo and Miehe modified, in the elastoplastic case only, is described in [R5.03.06].
This model is available in the order STAT_NON_LINE via the keyword RELATION = 'ROUSS_PR' or 'ROUSS_VISC' under the keyword factor BEHAVIOR and with the keyword DEFORMATION = 'PETIT_REAC'.
This model is established for modelings three-dimensional (3D), axisymmetric (AXIS), in plane constraints and plane deformations (C_PLAN, D_PLAN).

One presents the writing and the digital processing of this model.

Contents

1 Introduction

The mechanisms at the origin of the ductile rupture of metals are associated with the development of cavities within material. Three phases are generally distinguished:

- germination: it is starting or nucleation the cavities, into cubes sites which correspond preferentially to the particles of second phase present in material,
- growth: it is the phase which corresponds to the development itself of the cavities, controlled primarily by the plastic flow of the metal matrix which surrounds these cavities,
- coalescence: it is the phase which corresponds to the localization of the deformation between the cavities to create macroscopic cracks.

The model of Rousselier [bib1], [bib2], [bib3] presented here is based on microstructural assumptions which introduces a microstructure made up of cavities and of a matrix whose elastic strain negligible are compared with the plastic deformations. In this case, and in the absence of nucleation of new cavities, porosity f, definite like the relationship between the volume of the cavity V^c and total volume V representative ground volume, is directly connected to the macroscopic plastic deformation by:

$$\frac{\rho_0}{\rho}=\frac{1-f_0}{1-f} \text{ with } f=\frac{V^c}{V}\Leftrightarrow \dot{f}=(1-f)\operatorname{tr}\dot{\varepsilon}^p \qquad \text{éq 1-1}$$

where f_0 indicate initial porosity, ρ_o and ρ are respectively the density in the configurations initial and current (one takes in the continuation $\rho_o=1$) and $\dot{\varepsilon}^p$ the rate of plastic deformation of total volume V.

The construction of the model rests on a thermodynamic and phenomenologic analysis which brings to write the potential *plastic* F in the following form:

$$F(\boldsymbol{\tau},\ p,\ f)=\tau_{eq}+\sigma_1 D_1 f\exp\left(\frac{\tau_m}{\sigma_1}\right)-R(p) \qquad \text{éq 1-2}$$

where $\boldsymbol{\tau}=\boldsymbol{\sigma}/\rho$ is the constraint of Kirchhoff, $\boldsymbol{\sigma}$ is the constraint of Cauchy, R isotropic work hardening function of the cumulated plastic deformation p, σ_1 and D_1 parameters of material. The presence in the plastic potential of the hydrostatic constraint τ_m authorize the changes of plastic volume.

In the event of nucleation of new cavities, one considers that the voluminal fraction created is proportional to the cumulated plastic deformation. It is thus enough to replace f by $f+A_n p$ in the equations of the model. A_n is a parameter of material. The equation [éq 1-1] is not modified.

In the viscoplastic case, one writes the potential *viscoplastic* F^{vp} like a function of the plastic potential F :

$$F^{vp}=\Lambda(F,p,f) \qquad \text{éq 1-3}$$

One will consider only the typical case such as:

$$\dot{p}=\frac{\partial\Lambda}{\partial F}=\dot{\varepsilon}_0\left[sh\left(\frac{F}{\sigma_0}\right)\right]^m \qquad \text{éq 1-4}$$

who is reduced to a function power (law of the type Norton) when two parameters of material $\dot{\varepsilon}_0$ and σ_0 are very large.

Thereafter, one presents the relations of behavior of the model of Rousselier and his digital integration.

2 Notations

One will note by:

$\mathbf{Id}$	tensor second-order identity
$\mathbf{II}$	tensor identity of the fourth order
$\mathrm{tr}\,\mathbf{A}$	trace of the second-order tensor $\mathbf{A}$
$\tilde{\mathbf{A}}$	deviatoric part of the tensor $\mathbf{A}$ defined by $\tilde{\mathbf{A}}=\mathbf{A}-(\frac{1}{3}\mathrm{tr}\,\mathbf{A})\mathbf{Id}$
A_m	hydrostatic part of the tensor $\mathbf{A}$ defined by $A_m=\frac{\mathrm{tr}\,\mathbf{A}}{3}$
A_{eq}	equivalent value of von Mises defined by $A_{eq}=\sqrt{\frac{3}{2}\tilde{\mathbf{A}}:\tilde{\mathbf{A}}}$
$:$	doubly contracted product: $\mathbf{A}:\mathbf{B}=\sum_{i,j}A_{ij}B_{ij}=\mathrm{tr}(\mathbf{A}\mathbf{B}^{\mathbf{T}})$
$\otimes$	tensorial product: $(\mathbf{A}\otimes\mathbf{B})_{ijkl}=A_{ij}B_{kl}$
λ,μ,E,ν,K	moduli of the isotropic elasticity
$\dot{p}$	speed of equivalent plastic deformation $\dot{p}=\sqrt{\frac{2}{3}\tilde{\dot{\varepsilon}}^{p}:\tilde{\dot{\varepsilon}}^{p}}$

In addition, within the framework of a discretization in time, all the quantities Q evaluated at the previous moment are subscripted by $^-$, quantities evaluated at the moment $t=t^-+\Delta t$ are not subscripted and the increments are indicated by Δ. One has as follows:

$$Q=Q^-+\Delta Q$$

The digital resolution is carried out by one θ - method, with $0\leq\theta\leq 1$. For all the quantities, one defines:

$$Q^{\theta}=Q^-+\theta\Delta Q$$

3 Model of Rousselier

We now describe the derivation of the equations of the model of Rousselier presented in introduction.

3.1 Derivation of the equations of the model

It is supposed that the free energy *specific* breaks up into three parts: a hyperelastic part which depends only on the elastic strain, a part related to the mechanism of work hardening and a part related to the damage:

$$\Phi\left(\boldsymbol{\varepsilon}^{\mathbf{e}},\ p,f\right)=\Phi^{e}\left(\boldsymbol{\varepsilon}^{\mathbf{e}}\right)+\Phi^{p}\left(p\right)+\Phi^{f}\left(f\right) \qquad \text{éq 3.1-1}$$

The inequality of Clausius-Duhem is written (one does not consider the thermal part):

$$\boldsymbol{\tau}:\dot{\boldsymbol{\varepsilon}}-\dot{\Phi}\geq 0 \qquad \text{éq 3.1-2}$$

expression in which $\dot{\boldsymbol{\varepsilon}}=\dot{\boldsymbol{\varepsilon}}^{e}+\dot{\boldsymbol{\varepsilon}}^{p}$ represent the rate of deformation.

Dissipation is still written:

$$\left(\boldsymbol{\tau}-\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}^{\mathbf{e}}}\right):\dot{\boldsymbol{\varepsilon}}^{e}+\boldsymbol{\tau}:\dot{\boldsymbol{\varepsilon}}^{p}-\frac{\partial\Phi}{\partial p}\dot{p}-\frac{\partial\Phi}{\partial f}\dot{f}\geq 0 \qquad \text{éq 3.1-3}$$

The second principle of thermodynamics then requires the following expression for the elastic relation stress-strain:

$$\boldsymbol{\tau}=\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}^{e}} \qquad \text{éq 3.1-4}$$

One defines the thermodynamic forces associated with the elastic strain, the cumulated plastic deformation and porosity in accordance with the framework of generalized standard materials:

$$\boldsymbol{\tau}\left(\boldsymbol{\varepsilon}^{e}\right)=\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}^{e}} \qquad \text{éq 3.1-5}$$

$$A\left(p\right)=\frac{\partial\Phi}{\partial p} \qquad \text{éq 3.1-6}$$

$$B\left(f\right)=\frac{\partial\Phi}{\partial f} \qquad \text{éq 3.1-7}$$

It remains then for dissipation:

$$\boldsymbol{\tau}:\dot{\boldsymbol{\varepsilon}}^{p}-A\,\dot{p}\ -B\,\dot{f}\geq 0 \qquad \text{éq 3.1-8}$$

The principle of maximum dissipation applied starting from the potential *viscoplastic* $F^{\mathrm{vp}}(\boldsymbol{\tau},\mathrm{A},\mathrm{B})$ allows to deduce the laws of evolution from them from the plastic deformation, cumulated plastic deformation and porosity, is:

$$\dot{\boldsymbol{\varepsilon}}^{p}=\frac{\partial F^{vp}}{\partial\boldsymbol{\tau}} \qquad \text{éq 3.1-9}$$

$$\dot{p}=-\frac{\partial F^{vp}}{\partial A} \quad \text{éq 3.1-10}$$

$$\dot{f}=-\frac{\partial F^{vp}}{\partial B} \quad \text{éq 3.1-11}$$

It is supposed that $F^{\mathrm{vp}}(\boldsymbol{\tau}, A, B)$ is a function of the potential *plastic* $F(\boldsymbol{\tau}, A, B)$ and that this last breaks up into two terms depending respectively on the second invariant on $\boldsymbol{\tau}$ coupled to A and of the first invariant of $\boldsymbol{\tau}$ coupled to B :

$$F^{vp}=\Lambda(F)=\Lambda\left(F_{vM}(\tau_{eq}, A)+F_m(\tau_m, B)\right) \quad \text{éq 3.1-12}$$

By assumption, the first term breaks up in an additive way like the potential of von Mises:

$$F_{vM}(\tau_{eq}, A)=\tau_{eq}-A(p)-R_0=\tau_{eq}-R(p) \quad \text{éq 3.1-13}$$

Not to get a commonplace result, the decomposition of the second term must be multiplicative:

$$F_m(\tau_m, B)=g(\tau_m)h(B) \quad \text{éq 3.1-14}$$

Taking into account the equation [éq 1-1], the laws of evolution for $\mathrm{tr}\,\dot{\boldsymbol{\varepsilon}}^p$ and $\dot{f}$ lead to the equality:

$$\frac{g'(\tau_m)}{g(\tau_m)}=\left(\frac{-1}{1-f}\right)\frac{h'(B(f))}{h(B(f))} \quad \text{éq 3.1-15}$$

The two members of this equation are functions of the two independent variables τ_m and f, therefore it is equal to a constant of dimension the reverse of a constraint, it is the parameter of material $1/\sigma_1$. The parameter without dimension D_1 appears in the integration of g'/g :

$$g(\tau_m)=D_1\sigma_1\exp\left(\frac{\tau_m}{\sigma_1}\right) \quad \text{éq 3.1-16}$$

The function $B(f)$ and the opposite function $f=h_1(B)$ are unknown. The simplest choice and most natural is to take $h_1\equiv h$, which gives:

$$h(B)\equiv h_1(B)=f \quad \text{éq 3.1-17}$$

$$h'(B)=\frac{df}{\mathrm{dB}}=-\frac{1}{\sigma_1}f(1-f) \quad \text{éq 3.1-18}$$

The plastic potential is written finally:

$$F=\tau_{eq}+\sigma_1 D_1 f\exp\left(\frac{\tau_m}{\sigma_1}\right)-R(p) \quad \text{éq 3.1-19}$$

The law of evolution for $\dot{p}$ give:

$$\dot{p}=\frac{d\Lambda(F)}{\mathrm{dF}}=V(F) \quad \text{éq 3.1-20}$$

The function $V(F)$ the viscosity of material defines. One will consider only the typical case such as:

$$V(F)=\dot{\varepsilon}_0\left[sh\left(\frac{F}{\sigma_0}\right)\right]^m \quad \text{éq 3.1-21}$$

who is reduced to a function power (law of the type Norton) when two parameters of material $\dot{\varepsilon}_0$ and σ_0 are very large. Conversely one a:

$$F-S(\dot{p})=0 \quad \text{éq 3.1-22}$$

$$S(\dot{p})=\sigma_0 sh^{-1}\left[\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{\frac{1}{m}}\right] \quad \text{éq 3.1-23}$$

In the case of plasticity independent of time, the preceding equation becomes $F=0$ (criterion or threshold of plasticity) and $\dot{p}$ is given by the equation of consistency $\dot{F}=0$ if $F=0$ and $\dot{p}=0$ if $F<0$.

The equations of the model are now completely defined, in the case without nucleation of new cavities. In the event of nucleation of new cavities, one considers that the voluminal fraction created is proportional to the cumulated plastic deformation. It is thus enough to replace f by $f+A_n p$ in the equations of the model. A_n is a parameter of material. The equation [éq 1-1] is not modified.

3.2 Equations of the model

One summarizes the equations of the model deduced from the thermodynamic and phenomenologic analysis which precedes:

$$\Phi_{vp}=\tau_{eq}+\sigma_1 D_1\left(f+A_n p\right)\exp\left(\frac{\tau_m}{\sigma_1}\right)-R(p)-\sigma_0 sh^{-1}\left[\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{\frac{1}{m}}\right]=0 \qquad \textbf{éq 3.2-1}$$

$$\boldsymbol{\tau}=\frac{\boldsymbol{\sigma}}{\rho}=\left[\lambda\left(\mathbf{Id}\otimes\mathbf{Id}\right)+2\mu\,\mathbf{II}\right]:\boldsymbol{\varepsilon}^e \qquad \text{éq 3.2-2}$$

$$\rho=\frac{1-f-A_n p}{1-f_0} \qquad \text{éq 3.2-3}$$

$$\tilde{\boldsymbol{\varepsilon}}^p=\dot{p}\frac{3\tilde{\boldsymbol{\sigma}}}{2\sigma_{eq}}=\dot{p}\frac{3\tilde{\boldsymbol{\tau}}}{2\tau_{eq}} \qquad \text{éq 3.2-4}$$

$$tr\,\dot{\boldsymbol{\varepsilon}}^p=\dot{p}\,D_1\left(f+A_n p\right)\exp\left(\frac{\tau_m}{\sigma_1}\right) \qquad \text{éq 3.2-5}$$

$$\dot{f}=A_1\left(1-f\right)tr\,\dot{\boldsymbol{\varepsilon}}^p \qquad \text{éq 3.2-6}$$

with $A_1=1$, this parameter being introduced only for digital reasons.

4 Digital formulation

4.1 Keywords, given internal material and variables

For the foreseeable applications, the model was established under two distinct keywords: 'ROUSS_PR'for the plastic model with nucleation of cavities or 'ROUSS_VISC'for the viscoplastic model without nucleation. That allows D' to avoid useless digital calculations. The corresponding simplified equations are obtained starting from the general equations while posing respectively $\sigma_0=0$ or $A_n=0$.

The whole of the parameters of the model is provided under the keywords factors 'ROUSSELIER'or 'ROUSSELIER_FO'and 'TRACTION'(to define the traction diagram) order DEFI_MATERIAU ([U4.43.01]). Parameters of the viscoplastic model (σ_0 , $\dot{\varepsilon}_0$ and m) are provided by the keyword 'VISC_SINH'.

Internal variables produced in *Code_Aster* are:

- V1, cumulated plastic deformation p ,
- V2, porosity f ,
- V3 with V8, the tensor of elastic strain ε^e ,
- V9, the indicator of plasticity (0 if the last calculated increment is elastic, 1 if regular plastic solution, 2 if singular plastic solution).

We now present the digital integration of the law of behavior and give the form of the tangent matrix (options FULL_MECA and RIGI_MECA_TANG).

4.2 Expression of the discretized model

The digital resolution is carried out by one θ - method, with $0\leq\theta\leq1$, and in an incremental way. For all the quantities Q , one defines:

$$Q=Q^-+\Delta Q$$
$$Q^\theta=Q^-+\theta\Delta Q$$

The incremental writing requires the taking into account of the possible variation of the properties material (of the fact, for example, of a change of temperature during the step of time).
The system of equations discretized is:

$$\tilde{\boldsymbol{\tau}}^\theta=2\mu\theta\Delta\tilde{\boldsymbol{\varepsilon}}^e+\frac{2\mu\theta+(1-\theta)2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^-=2\mu\theta\left(\Delta\tilde{\boldsymbol{\varepsilon}}-\Delta\tilde{\boldsymbol{\varepsilon}}^p\right)+\frac{2\mu\theta+(1-\theta)2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^- \quad \text{éq 4.2-1}$$

$$\tau_m^\theta=K\theta\,\mathrm{tr}\Delta\boldsymbol{\varepsilon}^e+\frac{3K*\theta+(1-\theta)3K^-}{3K^-}\tau_m^-=K\theta\left(\mathrm{tr}\Delta\boldsymbol{\varepsilon}-\mathrm{tr}\Delta\boldsymbol{\varepsilon}^p\right)\frac{3K*\theta+(1-\theta)3K^-}{3K^-}\tau_m^- \quad \text{éq 4.2-2}$$

$$\Delta\tilde{\boldsymbol{\varepsilon}}^p=\Delta p\frac{3\tilde{\boldsymbol{\tau}}^\theta}{2\tau_{eq}^\theta} \quad \text{éq 4.2-3}$$

$$\mathrm{tr}\,\Delta\boldsymbol{\varepsilon}^p=\Delta pD_1\left(f^\theta+A_n p^\theta\right)\exp\left(\frac{\tau_m^\theta}{\sigma_1}\right) \quad \text{éq 4.2-4}$$

$$\Delta f=A_1\left(1-f^\theta\right)\mathrm{tr}\Delta\boldsymbol{\varepsilon}^p \quad \text{éq 4.2-5}$$

$$\Phi_{vp}^\theta=\tau_{eq}^\theta+\sigma_1 D_1\left(f^\theta+A_n p^\theta\right)\exp\left(\frac{\tau_m^\theta}{\sigma_1}\right)-R(p^\theta)-\sigma_0 sh^{-1}\left[\left(\frac{\Delta p}{\dot{\varepsilon}_0\Delta t}\right)^{\frac{1}{m}}\right]=0 \quad \textbf{éq 4.2-6}$$

This system is reduced to the solution of only one equation scalar for the unknown factor Δf, knowing $\Delta \varepsilon$, Δt and quantities Q^-. It is noted that ρ does not intervene in the algorithm, on the other hand it will intervene in the calculation of the coherent tangent matrix. One calculates successively:

$$\tau_m^\theta = \frac{3K*\theta+(1-\theta)3K^-}{3K^-}\tau_{m^-} + K\theta\left(\operatorname{tr}\Delta\varepsilon - \frac{\Delta f}{A_1(1-f^\theta)}\right) \quad \text{éq 4.2-7}$$

Δp is the positive root of the quadratic equation:

$$A_n\theta(\Delta p)^2 + \left(f^\theta + A_n p^-\right)\Delta p - \frac{\Delta f}{A_1(1-f^\theta)}\frac{1}{D_1\exp\left(\tau_m^\theta/\sigma_1\right)} = 0 \quad \text{éq 4.2-8}$$

$$\tilde{\boldsymbol{\tau}}^\theta = \left(1 - \frac{3\mu\,\theta\Delta p}{\left[\frac{2\mu\theta+(1-\theta)2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^- + 2\mu\,\theta\Delta\tilde{\varepsilon}\right]_{eq}}\right)\left(\frac{2\mu\theta+(1-\theta)2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^- + 2\mu\,\theta\Delta\tilde{\varepsilon}\right) \quad \text{éq 4.2-9}$$

$$\tau_{eq}^\theta = \left[\frac{2\mu\theta+(1-\theta)2\mu^-}{2\mu^-}\tilde{\boldsymbol{\tau}}^- + 2\mu\,\theta\Delta\tilde{\varepsilon}\right]_{eq} - 3\mu\,\theta\Delta p \quad \text{éq 4.2-10}$$

The scalar equation for Δf is the equation [éq 4.2-6] $\Phi_{vp}^\theta = 0$.

Notice 1 :

Like Δf is very weak in most of the structure, it would be preferable to use Δp like principal unknown factor. But in this case it is not possible to be brought back to a scalar equation, which makes more difficult the use of a method of the Newton type. It is also one of the reasons why equations [éq 1-1], [éq 3.2-6] and [éq 4.2-5] *were not modified by the introduction of the nucleation of the cavities.*

Notice 2 :

The equation [éq 3.2-6] *can be integrated exactly:*

$$\operatorname{tr}\varepsilon^p = \frac{1}{A_1}\ln\left(\frac{1-f_0}{1-f}\right)$$

from where:

$$\operatorname{tr}\Delta\varepsilon^p = \frac{1}{A_1\theta}\ln\left(\frac{1-f^-}{1-f^\theta}\right)$$

Like the digital parameter A_1 can be modified in a discontinuous way, the derived form [éq 4.2-5] *was preserved, including in the calculation of the coherent tangent matrix. If the use of the parameter A_1 was to be abandoned in a later version, it would be necessary to consider the use of the integrated form.*

Notice 3 :

The integrated form $\Phi_{vp}^\theta = 0$ is used, including in plasticity instead of the relation of consistency $\dot{F} = 0$ who gives $\dot{p}$. The coherent tangent matrix is calculated with this integrated form.

4.3 Resolution of the nonlinear scalar equation

The resolution of the equation $\Phi_{vp}^{\theta}(\Delta f)=0$ be carried out by an algorithm of Newton on terminals controlled in the routine LCROUS. $\Phi_{vp}^{\theta}(\Delta f)$ and its derivative compared to Δf are calculated in the routine RSLPHI called by LCROUS. The initial values of the terminals are:

- limit lower: $\Delta f_1=0$ since $\Phi_{vp}^{\theta}(0)<0$ (it was checked as a preliminary that the elastic branch (negative threshold) is not solution),
- limit higher: Δf_2 such as $\Phi_{vp}^{\theta}(0)>0$ sought by dichotomy between 0 and $1-f^-$ (first value for this research: $\frac{1-f^-}{2}$).

The algorithm of Newton begins with the value $\Delta f=0$. Whatever the value found for Δf one thus notes for the continuation that the function $\Phi_{vp}^{\theta}(\Delta f)$ and its derivative compared to Δf are at least calculated for $\Delta f=0$ and $\frac{1-f^-}{2}$.

The developments carried out to improve convergence and the robustness of the algorithm are described in [bib5].

4.4 Form of the tangent matrix of the behavior

One gives the form of the tangent matrix here (option FULL_MECA during iterations of Newton, option RIGI_MECA_TANG for the first iteration).

For the option RIGI_MECA_TANG, the tangent operator is the same one as that which connects $\boldsymbol{\varepsilon}^e$ with $\boldsymbol{\sigma}$ in [éq 3.2-2].

For the option FULL_MECA, the tangent matrix is obtained by linearizing the system of equations which governs the law of behavior: [éq 4.2-1] with [éq 4.2-6]. It is thus about a tangent matrix *coherent*.

To simplify the expressions, one notes in this paragraph [§4.5]: Q for Q^{θ} , quantities all being expressed at the moment $t^{\theta}=t^-+\theta\Delta t$. The coherent tangent matrix is:

$$\frac{\delta\boldsymbol{\sigma}}{\delta\boldsymbol{\varepsilon}}=\rho\left[a_3\mathbf{II}+\mathbf{Id}\otimes\left(\frac{a_1-a_3}{3}\mathbf{Id}+a_2\tilde{\boldsymbol{\tau}}\right)+\tilde{\boldsymbol{\tau}}\otimes\left(a_4\tilde{\boldsymbol{\tau}}+\frac{a_5}{3}\mathbf{Id}\right)+\boldsymbol{\tau}\otimes\left(y_4\left(\frac{a_1}{3\mathrm{K}}-1\right)\mathbf{Id}+\frac{y_5}{\mathrm{K}}\tilde{\boldsymbol{\tau}}\right)\right] \quad \text{éq 4.4-1}$$

This operator is calculated in the routine RSLJPL. The coefficients are calculated as follows:

$$a_1=3\mathrm{K}+y_1K\tau_{eq}(z_7+z_2\theta\Delta p) \quad \text{éq 4.4-2}$$

$$a_2=\mu(y_1+y_3)\sigma_1 \quad \text{éq 4.4-3}$$

$$a_3=\frac{2\mu\tau_{eq}}{z_5} \quad \text{éq 4.4-4}$$

$$a_4=3\mu y_2x_2 \quad \text{éq 4.4-5}$$

$$a_5=3\mu y_1\sigma_1 \quad \text{éq 4.4-6}$$

$$a_6=3\mu K\theta\Delta p-a_2\tau_{eq}\sigma_1 \quad \text{éq 4.4-7}$$

$$y_1=-\frac{3Kz_6z_1(f+A_np)}{x_1\tau_{eq}} \quad \text{éq 4.4-8}$$

$$y_2=-\frac{3\mu}{x_1z_5\tau_{eq}^2} \quad \text{éq 4.4-9}$$

$$y_3=-\frac{3Kz_6z_1A_n\theta\Delta p}{x_1\tau_{eq}} \quad \text{éq 4.4-10}$$

$$y_4=\frac{A_1z_8}{z_1}+\frac{z_9\sigma_1}{z_7+z_2\theta\Delta p} \quad \text{éq 4.4-11}$$

$$y_5=\frac{A_1a_2z_8}{z_1}-\frac{z_9a_6}{\tau_{eq}(z_7+z_2\theta\Delta p)} \quad \text{éq 4.4-12}$$

$$z_1=1+A_1\theta\Delta pD_1(f+A_np)\exp\left(\frac{\tau_m}{\sigma_1}\right) \quad \text{éq 4.4-13}$$

$$z_2=3\mu+R_{vp} \quad \text{éq 4.4-14}$$

$$z_3=K(f+A_np)z_1-A_1\sigma_1(1-f) \quad \text{éq 4.4-15}$$

$$z_4=R_{vp}\theta\Delta p-\tau_{eq} \quad \text{éq 4.4-16}$$

$$z_5=\tau_{eq}+3\mu\ \theta\Delta p \quad \text{éq 4.4-17}$$

$$z_6=D_1\exp\left(\frac{\tau_m}{\sigma_1}\right) \quad \text{éq 4.4-18}$$

$$z_7=z_6\sigma_1(f+A_np) \quad \text{éq 4.4-19}$$

$$z_8=\frac{1-f}{1-f-A_np} \quad \text{éq 4.4-20}$$

$$z_9=\frac{A_n}{1-f-A_np} \quad \text{éq 4.4-21}$$

$$x_1=z_3z_6(z_7+z_2\theta\Delta p)+z_1z_2\sigma_1-x_3 \quad \text{éq 4.4-22}$$

$$x_2=-z_3z_6\theta\Delta p(z_4+z_7)-z_1z_4\sigma_1+x_3\theta\Delta p \quad \text{éq 4.4-23}$$

$$x_3=A_nz_1z_6\sigma_1^2 \quad \text{éq 4.4-24}$$

$$R_{vp}=\frac{dR(p)}{dp}+\frac{1}{\theta\Delta t}\frac{dS(\Delta p/\Delta t)}{d\dot{p}} \quad \text{éq 4.4-25}$$

For the plastic model with nucleation of cavities ‘ROUSSELIER_PR‘and for the viscoplastic model without nucleation ‘ROUSSELIER_VISC‘, the corresponding simplified equations are obtained starting from the equations above while posing respectively $R_{vp}=dR(p)/dp$ and $A_n=0$.

5 Bibliography

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6 Features and checking

The laws of behavior described here are checked by the following tests:

For ROUSS_VISC :

CENTE01	CENTENARY. Validation of POST_ELEM option WEIBULL	[V1.02.001]
SSNP117	Model of Rousselier in 2D - DP	[V6.03.117]

For ROUSS_PR :

CENTE01	CENTENARY. Validation of POST_ELEM option WEIBULL	[V1.02.001]
SSNP117	Model of Rousselier in 2D - DP	[V6.03.117]
SSNV103	Tensile test shearing models of Rousselier	[V6.04.103]

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
6	*G. ROUSSELIER, R. MASSON, G. BARBER (EDF-R&D/MMC)*	Initial text
10	R.BARGELLINI (EDF-R&D/AMA)	Modification on the level of the algorithm discretized for taking into account of the changes of temperature