# Nonlinear relations of behavior 1D

**Summary:**
This document describes the quantities calculated by the operator STAT_NON_LINE necessary to the implementation of the quasi static nonlinear algorithm describes in [R5.03.01] in the case as of elastoplastic or viscoplastic behaviors monodimensional. These behaviors are applicable to the elements of BAR, with the elements of beam and multifibre beams (direction axial only) and with the elements of concrete reinforcement (modeling GRID).

The behaviors described in this document are:

- the behavior of Von Mises with linear isotropic work hardening: VMIS_ISOT_LINE, and unspecified VMIS_ISOT_TRAC,
- the behavior of Von Mises with linear kinematic work hardening: VMIS_CINE_LINE,
- the behavior of Von Mises with linear, nonsymmetrical work hardening in traction and compression: with restoration of the center of the elastic range: VMIS_ASYM_LINE. This last was developed to model the action of the ground on the Cables with Gas Insulation,
- the behavior of PINTO-MENEGOTTO who allows to represent the uniaxial elastoplastic behavior of the reinforcements of the reinforced concrete. This model translates for it not linearity of the work hardening of the bars under cyclic loading and takes into account the Bauschinger effect. It makes it possible of more than simulate the buckling of the reinforcements in compression. This relation is available in *Code_Aster* for the elements of bar and the elements of grid,
- viscoplastic behaviors with effect of the irradiation: VISC_IRRA_LOG, GRAN_IRRA_LOG.
- the behavior of MAZARS in its version $1D$ . The version $1D$ model of MAZARS allows to give an account of the restoration of rigidity in the event of refermeture of the cracks.

The resolution is made in all the cases by a method of integration implicit as from the moment of preceding calculation, one calculates the stress field resulting from an increment of deformation, and the tangent behavior which makes it possible to build the tangent matrices.
One describes finally a method, similar to the method due to R.de Borst [R5.03.03] allowing to use all the behaviors available in 3D in the elements 1D.

# Contents

# 1 Use of the relations of behavior 1D

## 1.1 Relations of behavior 1D in *Code_Aster*

The relations treated in this document are:

| | |
|---|---|
| VMIS_ISOT_LINE | Von Mises with symmetrical linear isotropic work hardening |
| VMIS_ISOT_TRAC | Von Mises with unspecified isotropic work hardening |
| VMIS_CINE_LINE | Von Mises with symmetrical linear kinematic work hardening. |
| ECRO_CINE_1D | Von Mises with symmetrical linear kinematic work hardening. |
| GRILLE_ISOT_LINE | Von Mises with symmetrical linear isotropic work hardening |
| GRILLE_CINE_LINE | Von Mises with symmetrical linear kinematic work hardening |
| PINTO_MENEGOTTO | Behavior of the reinforced concrete reinforcements |
| GRILLE_PINTO_MEN | Behavior of the reinforced concrete reinforcements |
| VMIS_ASYM_LINE | Von Mises with asymmetrical linear work hardening and restoration |
| VISC_IRRA_LOG, GRAN_IRRA_LOG | Viscoplastic behaviour of the fuel assemblies: Models resulting from the tests REFLECTION and FLETANR |
| MAZARS | Behavior of MAZARS in its version $1D$ . |

These relations of behavior (incremental) are given in the operator STAT_NON_LINE [U4.51.03] under the keyword factor BEHAVIOR, by the keyword RELATION [U4.51.03]. They are valid only in small deformations. N describes for each relation of behavior the calculation of the stress field starting from an increment of deformation given (cf algorithm of Newton [R5.03.01]), the calculation of the forces **nodal** $R$ and of the tangent matrix $K_i^n$ .

## 1.2 General notations

All the quantities evaluated at the previous moment are subscripted by - .

Quantities evaluated at the moment $t+\Delta t$ are not subscripted.

The increments are indicated by $\Delta$ . One has as follows:

$$\mathbf{Q}=\mathbf{Q}(t+\Delta t)=\mathbf{Q}^-(t)+\Delta\,\mathbf{Q}=\mathbf{Q}^-+\Delta\,\mathbf{Q}$$

| | |
|---|---|
| $\boldsymbol{\sigma}$ | tensor of the constraints (in 1D, one is interested only in the single uniaxial nonworthless component). |
| $\tilde{\boldsymbol{\sigma}}$ | deviative operator: $\tilde{\boldsymbol{\sigma}}_{\mathbf{ij}}=\boldsymbol{\sigma}_{\mathbf{ij}}-\frac{1}{3}\boldsymbol{\sigma}_{kk}\boldsymbol{\delta}_{ij}$ . |
| $(\ )eq$ | equivalent value of Von Mises, equalizes in 1D with the absolute value |
| $\Delta\,\boldsymbol{\varepsilon}$ | increment of deformation. |
| $\mathbf{A}$ | tensor of elasticity, equal in 1D to the Young modulus E |
| $\lambda,\mu,E,K$ | moduli of the isotropic elasticity. |
| $\alpha$ | thermal dilation coefficient secant. |
| $T$ | temperature. |
| $(\ )_+$ | positive part. |
| $P$ | cumulated plastic deformation |
| $\varepsilon^p$ | plastic deformation |

## 1.3 Change of variables

Whatever the type of finite element which refers to a law of behavior 1D, it is necessary to carry out a change of variables to pass from the elementary quantities (efforts, displacements) to the constraints and deformations.

### 1.3.1 Calculation of the deformations (small deformations)

For each finite element of *Code_Aster*, in `STAT_NON_LINE`, the total algorithm (Newton) provides to the elementary routine, which integrates the behavior, an increase in the field of displacement.
For the elements of bar, one calculates the deformation (only one axial component) by:

$$\boldsymbol{\varepsilon}=\frac{u(l)-u(0)}{l},$$

and the increase in deformation by:

$$\boldsymbol{\Delta}\,\varepsilon=\frac{\Delta\,u(l)-\Delta\,u(0)}{l},$$

For the elements of grid (modelings `GRID` and `GRILLE_MEMBRANE`), one calculates the membrane deformation as for the elements of hulls DKT. Simply, only one direction corresponds physically to the directions of reinforcements. One thus finds oneself in the presence of a behavior 1D.
In addition, in small deformations, for all the models described in this document, one writes for any moment the partition of the deformations in the form of an elastic contribution, thermal dilation, and plastic deformation:

$$\boldsymbol{\varepsilon}(t)=\boldsymbol{\varepsilon}^{\mathbf{e}}(t)+\boldsymbol{\varepsilon}^{\mathbf{th}}(t)+\boldsymbol{\varepsilon}^{\mathbf{p}}(t) \text{ with}$$

$$\boldsymbol{\varepsilon}^{\mathbf{e}}(t)=\mathbf{A}^{-1}\big(T(t)\big)\boldsymbol{\sigma}(t)=\frac{1}{E(T)}\boldsymbol{\sigma}(t)$$

$$\boldsymbol{\varepsilon}^{\mathbf{th}}(t)=\boldsymbol{\alpha}\big(T(t)\big)\big(T(t)-T_{ref}\big)\mathbf{Id}$$

### 1.3.2 Calculation of the generalized efforts (forced integrated)

For integration of the behavior 1D, it is necessary to integrate the component of constraints obtained, to provide to the total algorithm (Newton) a vector containing the generalized efforts.
For the elements of bar, one calculates the effort (uniform in the element, by supposing that the section is constant) by: $N=S\,\sigma$,

and the vector forces nodal equivalent (as for the elements of beam, [R3.08.01]) by:

$$F=\begin{bmatrix}-N\\ N\end{bmatrix}$$

For the elements of `GRID` , one calculates the efforts as for the elements of hulls DKT (membrane efforts) by integration of the constraints in the thickness (only one sleeps and only one point of integration).

# 2 Behavior of Von-Put at linear isotropic work hardening: VMIS_ISOT_LINE or VMIS_ISOT_TRAC

## 2.1 Equations of the model VMIS_ISOT_LINE

They are the restriction of the behavior 3D [R5.03.02] on the uniaxial case:

$$\begin{cases} \dot{\varepsilon}^p = \frac{3}{2}\dot{p}\frac{\tilde{\sigma}}{\sigma_{eq}} = \dot{p}\frac{\sigma}{|\sigma|} \\ \frac{\sigma}{E} = \varepsilon - \varepsilon^p - \varepsilon^{th} \\ \sigma_{eq} - R(p) = |\sigma| - R(p) \le 0 \\ \begin{cases} \dot{p} = 0 \; si \; \sigma_{eq} - R(p) < 0 \\ \dot{p} \ge 0 \; si \; \sigma_{eq} - R(p) = 0 \end{cases} \end{cases}$$

with:

- $\dot{\varepsilon}^p$ speed of plastic deformation,
- $p$ cumulated plastic deformation,
- $\varepsilon^{th} = \alpha(T - T_{ref})$ thermal deformation,
- $R(p) = \frac{E E_T}{E - E_T} p + \sigma_y$ function of linear work hardening isotropic, or $R(p)$ well refine per pieces, deduced from the traction diagram.

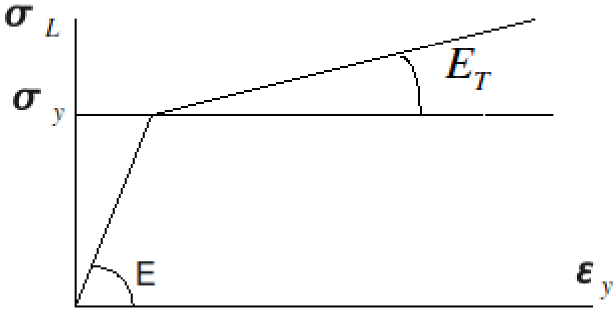
In the case VMIS_ISOT_LINE, the data of the material characteristics are those provided under the keyword factor ECRO_LINE or ECRO_LINE_FO of the operator DEFI_MATERIAU [U4.43.01].

/ ECRO_LINE = (D_SIGM_EPSI = $E_T$ , SY = $\sigma_y$ )
/ ECRO_LINE_FO = (D_SIGM_EPSI = $E_T$ , SY = $\sigma_y$ )

In the case VMIS_ISOT_TRAC, the data of the characteristics of materials are provided under the keyword factor TRACTION of the operator DEFI_MATERIAU [U4.43.01].

```
TRACTION = _F (SIGM = courbe_traction)
```

courbe_traction represent the traction diagram, point by point. The first point makes it possible to define the elastic limit $\sigma_y$ and it Young modulus $E$ [R5.03.02].




ECRO_LINE_FO corresponds if $E_T$ and $\sigma_y$ depend on the temperature and are then calculated for the temperature of the point of current Gauss. The Young modulus $E$ and the Poisson's ratio $\nu$ are those provided under the keywords factors ELAS or ELAS_FO. In this case the traction diagram is the following one:

$$\begin{cases} \sigma_L = E\,\varepsilon_L & si\,\varepsilon_L < \dfrac{\sigma_y}{E} \\ \sigma_L = \sigma_y + E_T\left(\varepsilon_L - \dfrac{\sigma_y}{E}\right) & si\,\varepsilon_L \geq \dfrac{\sigma_y}{E} \end{cases}$$

When the criterion is reached one a:

$\sigma_L - R(p) = 0$ , therefore $\sigma_L - R\left(\varepsilon_L - \dfrac{\sigma_L}{E}\right) = 0$ from where:

$$R(p) = \frac{E_T E}{E - E_T}\,p + \sigma_y = H\,p + \sigma_y$$

In the case of a traction diagram, the approach is identical to [R5.03.01].

## 2.2 Integration of the relation VMIS_ISOT_LINE

By direct implicit discretization of the relations of behavior, in a way similar to integration 3D [R5.03.02] one obtains:

$$\begin{cases} \left|\sigma^- + \Delta\sigma\right| - R(p^- + \Delta p) \leq 0 \\ E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) - \left(\sigma^- + \Delta\sigma\right) + \dfrac{E}{E^-}\sigma^- = E\,\Delta p\,\dfrac{\sigma^- + \Delta\sigma}{\left|\sigma^- + \Delta\sigma\right|} \\ \Delta p \;\geq\; 0 \;\; si \;\; \left|\sigma^- + \Delta\sigma\right| = R\left(p^- + \Delta p\right) \\ \Delta p \;=\; 0 \;\; si \;\; \left|\sigma^- + \Delta\sigma\right| < R\left(p^- + \Delta p\right) \end{cases}$$

Two cases arise:

- $\left|\sigma^- + \Delta\sigma\right| < R\left(p^- + \Delta p\right)$

  in this case $\Delta p = 0$ that is to say $\sigma \;=\; \mathrm{E}\left(\Delta\varepsilon - \Delta\varepsilon^{\mathrm{th}}\right) + \dfrac{\mathrm{E}}{\mathrm{E}^-}\sigma^-$

  thus $\left|\sigma^- \dfrac{\mathrm{E}}{\mathrm{E}^-} + \mathrm{E}\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)\right| < R(p^-)$

- $\left|\sigma^- + \Delta\sigma\right| = R(p^- + \Delta p)$

  in this case $\Delta p \geq 0$

  thus $\left|\dfrac{\sigma^- E}{E^-} + E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)\right| \geq R(p^-)$ .

One from of deduced the algorithm from resolution:

let us pose $\sigma^e = \dfrac{\mathrm{E}\,\sigma^-}{\mathrm{E}^-} + E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)$

if $\left|\sigma^e\right| \leq R(p^-)$ then $\Delta p \;=\; 0$ and $\Delta\sigma = \mathrm{E}\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)$

if $\left|\sigma^e\right| > R(p^-)$ then it is necessary to solve:

$$\sigma^e = \sigma^- + \Delta\sigma + E\Delta p \frac{\sigma^- + \Delta\sigma}{\left|\sigma^- + \Delta\sigma\right|}$$

$$\sigma^e = \left(1 + \frac{E\Delta p}{\left|\sigma^- + \Delta\sigma\right|}\right)\left(\sigma^- + \Delta\sigma\right)$$

thus by taking the absolute value:

$$\left|\sigma^e\right| = \left(1 + \frac{E\Delta p}{\left|\sigma^- + \Delta\sigma\right|}\right)\left(\sigma^- + \Delta\sigma\right)$$

maybe, while using $\begin{matrix}\left|\sigma^- + \Delta\sigma\right| = R\left(p^- + \Delta p\right) \\ \left|\sigma^e\right| = R\left(p^- + \Delta p\right) + E\Delta p\end{matrix}$.

One from of thus deduced:

- in the case of a linear work hardening: $\Delta p = \frac{\left|\sigma^e\right| - \left(\sigma_y + Hp^-\right)}{E+H}$
- and in the case of an unspecified work hardening, the curve $R(p)$ being refined per pieces, one solves the equation directly in $\Delta p$ : in the same way $E\Delta p + R\left(p^- + \Delta p\right) = \left|\sigma^e\right|$ that in 3D [R5.03.02].

Let us notice on the way that: $\frac{\sigma^e}{\left|\sigma^e\right|} = \frac{\sigma}{R(p)}$

then $\sigma = \left(\sigma^- + \Delta\sigma\right) = \frac{\sigma^e}{\left|\sigma^e\right|} R(p) = \frac{\sigma^e}{1 + \frac{E\Delta p}{R(p)}}$

Moreover, the option FULL_MECA allows to calculate the tangent matrix $\mathbf{K}_\mathbf{i}^\mathbf{n}$ with each iteration. The tangent operator who is used for building it is calculated directly on the preceding discretized system. One obtains directly:

if $\left|\sigma^e\right| > R\left(p^-\right)$ $\qquad \frac{\delta\sigma}{\delta\varepsilon} = E_T$

if not $\qquad \frac{\delta\sigma}{\delta\varepsilon} = E$

**Note:**

*The option RIGI_MECA_TANG who allows to calculate the tangent matrix $\mathbf{K}_\mathbf{i}^\mathbf{0}$ used in the phase of prediction of the algorithm of Newton, account of the indicator of plasticity takes at the previous moment:*

*if* $\chi = 1$ $\frac{\delta\sigma}{\delta\varepsilon} = E_T$ *if* $\chi = 0$ $\frac{\delta\sigma}{\delta\varepsilon} = E$

## 2.3 Internal variables

The relation of behavior VMIS_ISOT_LINE product two internal variables: $p$ and $\chi$

# 3 Behaviour of Von Mises, linear kinematic work hardening 1D: VMIS_CINE_LINE

## 3.1 Equation of the model VMIS_CINE_LINE

For reasons of performances the relation is written in 1D. They are the restriction of the behavior 3D ([R5.03.02] and [R5.03.16]) on the uniaxial case. The behavior 3D is written:

$$\boldsymbol{\sigma}=\mathbf{K}\left(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\varepsilon}^{\mathbf{th}}\right) \text{ with } \mathbf{K} \text{ operator of elasticity}$$

$$\mathbf{X}=C\,\boldsymbol{\varepsilon}^{\mathbf{p}}$$

$$F(\boldsymbol{\sigma},\mathbf{R},\mathbf{X})=(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{\mathbf{eq}}-\sigma_y \text{ with } \mathbf{A}_{\mathbf{eq}}=\sqrt{\frac{3}{2}\tilde{\mathbf{A}}\cdot\tilde{\mathbf{A}}}$$

$$\dot{\boldsymbol{\varepsilon}}^{\mathbf{p}}=\dot{p}\frac{\partial F}{\partial\boldsymbol{\sigma}}=\frac{3}{2}\dot{\mathbf{p}}\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{\mathbf{eq}}} \quad \begin{cases}\text{si } F<0 & \dot{p}=0\\ \text{si } F=0 & \dot{p}\geq 0\end{cases}$$

In the uniaxial case, the tensors are written:

$$\tilde{\boldsymbol{\sigma}}=\sigma\,\mathbf{D} \qquad \mathbf{X}=X\,\mathbf{D} \qquad \boldsymbol{\varepsilon}^{\mathbf{p}}=\frac{3}{2}\varepsilon^p\,\mathbf{D} \text{ with } \mathbf{D}=\begin{bmatrix}2/3 & & \\ & -1/3 & \\ & & -1/3\end{bmatrix}$$

As long as the loading is monotonous, the following relations immediately are obtained:

$$p=\varepsilon^p \qquad X=\frac{3}{2}C\,\varepsilon^p \qquad \sigma=\frac{3}{2}C\,\varepsilon^p+\sigma_y \qquad \sigma=F(\varepsilon)=\sigma_y+\frac{E\cdot E_T}{E-E_T}p$$

C is determined by: $C=\frac{2}{3}\frac{E\,E_T}{E-E_T}$ . One poses: $H=\frac{E\,E_T}{E-E_T}=\frac{3}{2}C$

The relation of behavior 1D is written then:

$$\left|\begin{array}{l}\left|\sigma^-+\Delta\sigma-X^--\Delta X\right|\\ E\,\Delta\varepsilon^p=E\left(\Delta\varepsilon-\Delta\varepsilon^{th}\right)-\left(\sigma^-+\Delta\sigma\right)+\frac{E}{E^-}\sigma^-\\ X=\frac{3}{2}C\,\varepsilon^p=H\,\varepsilon^p\\ \left|\sigma-X\right|-\sigma_y\leq 0\\ \begin{cases}\dot{p}=0\,si\left|\sigma-X\right|-\sigma_y<0\\ \dot{p}\geq 0\,si\left|\sigma-X\right|-\sigma_y=0\end{cases}\end{array}\right.$$

The data of the material characteristics are those provided under the keyword factor `ECRO_LINE` or `ECRO_LINE_FO` of the operator `DEFI_MATERIAU` [U4.43.01]:

/ ECRO_LINE = (D_SIGM_EPSI = $E_T$ , SY = $\sigma_y$ )

/ ECRO_LINE_FO = (D_SIGM_EPSI = $E_T$ , SY = $\sigma_y$ )

## 3.2 Integration of the relation VMIS_CINE_LINE

By direct implicit discretization of the relations of behavior, in a way similar to integration 3D ([R5.03.02] and [R5.03.16]) one obtains:

$$\begin{cases} \left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| - \sigma_y \le 0 \\ E\,\Delta\varepsilon^p = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) - \left(\sigma^- + \Delta\sigma\right) + \dfrac{E}{E^-}\sigma^- \\ \Delta\varepsilon^p = \Delta p \dfrac{\sigma^- + \Delta\sigma - X^- - \Delta X}{\left|\sigma^- + \Delta\sigma - X^- - \Delta X\right|} \\ \dfrac{X}{H} - \dfrac{X^-}{H^-} = \Delta\varepsilon^p \\ \Delta p \ge 0 \quad si \quad \left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| = \sigma_y \\ \Delta p = 0 \quad si \quad \left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| < \sigma_y \end{cases}$$

with $\Delta\varepsilon^{th} = \alpha\left(T - T_{ref}\right) - \alpha^-\left(T^- - T_{ref}\right)$

Two cases arise:

- $\left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| < \sigma_y$ in this case $\Delta p = 0$ that is to say $\sigma = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) + \dfrac{E}{E^-}\sigma^- - \dfrac{H}{H^-}X^-$ thus $\left|\sigma^- \dfrac{E}{E^-} - X^- \dfrac{H}{H^-} + E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)\right| < R(p^-)$,
- if not $\Delta p \ge 0$.

To simplify the writings one will pose: $\sigma^e = \dfrac{E}{E^-}\sigma^- - \dfrac{H}{H^-}X^- + E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)$.

One from of deduced the algorithm from resolution:

1) if $\left|\sigma^e\right| \le \sigma_y$ then $\Delta p = 0$, $X = X^-\dfrac{H}{H^-}$, $\sigma = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) + \dfrac{E}{E^-}\sigma^-$
2) if not it is necessary to solve:

$$\begin{cases} E\,\Delta\varepsilon^p = E(\Delta\varepsilon - \Delta\varepsilon^{th}) - \Delta\sigma = \sigma^e - \left(\sigma^- + \Delta\sigma\right) + X^-\dfrac{H}{H^-} \\ \Delta\varepsilon^p = \Delta p \dfrac{\sigma^- + \Delta\sigma - X^- - \Delta X}{\left|\sigma^- + \Delta\sigma - X^- - \Delta X\right|} = \Delta p \dfrac{\sigma - X}{\left|\sigma - X\right|} \\ X - \dfrac{H}{H^-}X^- = H\,\Delta\varepsilon^p \\ \left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| - \sigma_y = 0 \end{cases}$$

Let us notice that: $\dfrac{H}{H^-}X^- = X - H\,\Delta\varepsilon^p$.

One deduces then from the first equation: $\sigma^e = \sigma - X + (E + H)\Delta\varepsilon^p$

One thus obtains, while eliminating $\sigma - X$ second equation:

$$\Delta\varepsilon^p = \sigma^e \frac{\Delta p}{(E+H)\Delta p + \sigma_y}$$

While replacing $\Delta\varepsilon^p$ the relation enters $\sigma^e$ and $\sigma - X$, one obtains:

$$\sigma - X = \sigma^e \left( \frac{\sigma_y}{(E+H)\Delta p + \sigma_y} \right)$$

By taking the absolute value of the two members of the preceding equation, one finds $\Delta p$ :

$$(E+H)\Delta p + \sigma_y = \left|\sigma^e\right|$$

Once $\Delta p$ determined, one can calculate:

$$\Delta\varepsilon^p = \Delta p \frac{\sigma^e}{\left|\sigma^e\right|}$$

$$X = X^- + \Delta X = \frac{H X^-}{H^-} + H \Delta p \frac{\sigma^e}{\left|\sigma^e\right|}$$

and while using: $\frac{\sigma - X}{\sigma_y} = \frac{\sigma^e}{\left|\sigma^e\right|}$ , one obtains directly: $\sigma = \sigma_y \frac{\sigma^e}{\left|\sigma^e\right|} + X$

Moreover, the option FULL_MECA allows to calculate the tangent matrix $\mathbf{K_i^n}$ with each iteration. The tangent operator who is used for building it is calculated directly on the preceding discretized system. One obtains directly:

if $\left|\sigma^e\right| > R\left(p^-\right)$ $\quad \frac{\delta \partial}{\delta \varepsilon} = E_T$

if not $\quad \frac{\delta \sigma}{\delta \varepsilon} = E$

The option RIGI_MECA_TANG who allows to calculate the tangent matrix $\mathbf{K_i^0}$ used in the phase of prediction of the algorithm of Newton is obtained using the indicator of plasticity $\chi^-$ previous moment:

- if $\chi^- = 1$ then $\frac{\delta \sigma}{\delta \varepsilon} = E_T$
- if $\chi^- = 0$ then $\frac{\delta \sigma}{\delta \varepsilon} = E$

## 3.3 Internal variables

The relation of behavior VMIS_CINE_LINE product two internal variables: $X$ .et $\chi$

# 4 Behaviour of Von Mises, linear kinematic work hardening 1D: VMIS_CINE_GC

## 4.1 Equation of the model VMIS_CINE_GC

For reasons of performances the relation is also written in 1D for a use with finite elements of standard multifibre beam. The equations result from the restriction of the behavior 3D ([R5.03.02] and [R5.03.16]) on the uniaxial case.
The equations of the model are the same ones as those of the § 3.1.

The data of materials are those provided under the keyword factor ECRO_LINE of the operator DEFI_MATERIAU [U4.43.01] :

```
/ ECRO_LINE = _F(
        ♦ D_SIGM_EPSI   =  E_T                    [Reality]
        ♦ SY            =  σ_y                    [Reality]
       ◊ SIGM_LIM       = sigmlim                 [Reality]
       ◊  EPSI_LIM      = epsilim                 [Reality]
)
```

Operands SIGM_LIM and ESPI_LIM allow to define the terminals which correspond to the limiting states of service and ultimate, classically used at the time of study in civil engineer.

◊ SIGM_LIM = sigmlim
Definition of the ultimate stress.

◊ EPSI_LIM = epslim
Definition of the limiting deformation.

These terminals are obligatory when the behavior is used VMIS_CINE_GC (Cf [U4.51.11] non-linear Behaviors, [U4.42.07] DEFI_MATER_GC). In the other cases they are not taken into account.

## 4.2 Integration of the relation VMIS_CINE_GC

The method of integration identical to that is presented to the § 3.2.

## 4.3 Internal variables

Supported modeling is 1D, numbers of internal variables is of 6.

- $V1$ : This variable represents the constraint divided by the ultimate stress sigmlim.
- $V2$ : This variable represents the total deflection divided by the limiting deformation epslim.
- $V3$ : Kinematic work hardening: XCINXX. In 1D only a scalar is necessary.
- $V4$ : Plastic indicator: INDIPLAS. Indicate if the material exceeded the elastic criterion.
- $V5$ : nonrecoverable dissipation: DISSIP. During seismic calculations it can be useful for the user to know nonrecoverable dissipated energy. The variable DISSIP represent the nonrecoverable office plurality of energy. The nonrecoverable increment of energy is written in the form:

$$\Delta Eg=\frac{1}{2}\left(E^{+}\Delta\varepsilon-\left(\sigma^{+}-\sigma^{-}\right)\right)\Delta\varepsilon$$

- $V6$ : thermodynamic dissipation: DISSTHER. The thermodynamic increment of dissipation is written in the form: $\Delta Eg=\sigma_y\dot{p}$ .

# 5 Behavior of Von Mises with asymmetrical linear work hardening: VMIS_ASYM_LINE

## 5.1 Equations of the model VMIS_ASYM_LINE

### 5.1.1 Asymmetrical behaviour in traction and compression

It is a behavior uncoupled in traction and compression, built from VMIS_ASYM_LINE, but with elastic and different module limits of work hardening in traction and compression. We adopt an index $T$ for traction and $C$ for compression. Elastic behaviour in traction and compression identical and is characterized by the same Young modulus. There are two fields of isotropic work hardening defined by $R_T$ and $R_C$. The two fields are independent one of the other.

| | |
|---|---|
| $YT$ | elastic limit in traction. In absolute value. |
| $YC$ | elastic limit in compression. In absolute value. |
| $p_T$ | Variable interns in traction. Algebraic value. |
| $p_C$ | Variable interns in compression. Algebraic value. |
| $E_{TT}$ | Slope of work hardening in traction. |
| $E_{TC}$ | Slope of work hardening in compression. |

The equations of the model of behavior are:

$$\begin{cases} \dot{\varepsilon}^p = \dot{\varepsilon} - \mathrm{E}^{-1}\sigma - \dot{\varepsilon}^{th} \\ \dot{\varepsilon}^p = \dot{\varepsilon}^p_C + \dot{\varepsilon}^p_T \\ \dot{\varepsilon}^p_C = \dot{\varepsilon}_C \dfrac{\sigma}{|\sigma|} \\ \dot{\varepsilon}^p_C = \dot{\varepsilon}_T \dfrac{\sigma}{|\sigma|} \\ \sigma - R_T(p_T) \le 0 \\ -\sigma - R_C(p_C) \le 0 \end{cases} \quad \text{avec} \begin{cases} \dot{p}_C = 0 \quad si \quad -\sigma - R_C(p_C) < 0 \\ \dot{p}_C \ge 0 \quad si \quad -\sigma = R_C(p_C) \\ \dot{p}_T = 0 \quad si \quad \sigma - R_T(p_T) < 0 \\ \dot{p}_T \ge 0 \quad si \quad \sigma = T_T(p_T) \end{cases}$$

$\dot{\varepsilon}^p_C$ : speed of plastic deformation in compression,

$\dot{\varepsilon}^p_T$ : speed of plastic deformation in traction,

$\dot{\varepsilon}^{th}$ : thermal deformation of origin: $\varepsilon^{th} = \alpha(T - T_{ref})$

It is noticed that one cannot have simultaneously plasticization in traction and compression: that is to say $\dot{p}_C = 0$ , that is to say $\dot{p}_C = 0$ , that is to say both are worthless.

The data of the material characteristics are those provided under the keyword factor ECRO_ASYM_LINE of the operator DEFI_MATERIAU [U4.43.01].

ECRO_ASYM_LINE = _F (
DT_SIGM_EPSI = $E_{TT}$ , SY_T = $\sigma_{yT}$ ,
DC_SIGM_EPSI = $E_{TC}$ , SY_C = $\sigma_{yC}$ ,)

The Young modulus E is provided under the keywords factors ELAS or ELAS_FO.

One calculates the functions of work hardening by:

$$R_T(p)=\frac{E_{TT}E}{E-E_{TT}}p_T+\sigma_{yT}=H_T\cdot p_T+\sigma_{yT}$$
$$R_C(p)=\frac{E_{TC}E}{E-E_{TC}}p_C+\sigma_{yC}=H_C\cdot p_C+\sigma_{yC}$$

## 5.2 Integration of the behavior VMIS_ASYM_LINE

By direct implicit discretization of the asymmetrical relation of behavior, in a way similar to the preceding one, one obtains:

$$\begin{cases}\Delta\varepsilon^p=\Delta\varepsilon_T^p+\Delta\varepsilon_C^p\\ \Delta\varepsilon^p=\Delta\varepsilon-\Delta\varepsilon^{th}-\dfrac{\Delta\sigma}{E}\\ \Delta\varepsilon_T^p=\Delta p_T\dfrac{\sigma^-+\Delta\sigma}{\left|\sigma^-+\Delta\sigma\right|}\\ \Delta p_T\geq 0\quad si\quad (\sigma^-+\Delta\sigma)-R_T\left(\bar{p}_T+\Delta\bar{p}_T\right)\ \leq\ 0\\ \Delta p_T=0\quad si\quad \left(\sigma^-+\Delta\sigma\right)-R_T\left(\bar{p}_T+\Delta\bar{p}_T\right)\ <\ 0\\ \\ \Delta\varepsilon_C^p=\Delta p_C\dfrac{\sigma^-+\Delta\sigma}{\left|\sigma^-+\Delta\sigma\right|}\\ -\left(\sigma^-+\Delta\sigma\right)-R_C\left(\bar{p}_C+\Delta\bar{p}_C\right)\leq 0\\ \Delta p_C\geq 0\quad si\quad -\left(\sigma^-+\Delta\sigma\right)-R_C\left(\bar{p}_C+\Delta\bar{p}_C\right)\ =\ 0\\ \Delta p_C=0\quad si\quad -\left(\sigma^-+\Delta\sigma\right)-R_C\left(\bar{p}_C+\Delta\bar{p}_C\right)\ <\ 0\end{cases}$$

Integration is similar to that of `VMIS_ISOT_LINE` for each direction of traction and compression. It should well be seen that the centers of the fields of elasticity are data (calculated explicitly with the preceding step) for the incremental problem to solve.

Four cases arise:

- $\Delta\varepsilon-\Delta\varepsilon^{th}>0$ one poses $\sigma_T^e=\sigma^-+E\left(\Delta\varepsilon-\Delta\varepsilon^{th}\right)$
  - if $\sigma_T^e<R_T\left(\bar{p}_T\right)$ in this case $\Delta p_T\ =\ 0$ thus $\sigma=\sigma_T^e$ and $\dfrac{\delta\sigma}{\delta\varepsilon}\ =\ E$
  - if not: $\Delta p_T=\dfrac{\left|\sigma_T^e\right|-\left(\sigma_{yT}+H_T p_T^-\right)}{E+H_T}$ , $\Delta p_C=0$ $\quad\sigma=\dfrac{\sigma_T^e}{1+\dfrac{E\Delta p_T}{R_T\left(p_T\right)}}=\dfrac{\sigma_T^e}{\left|\sigma_T^e\right|}R_T\left(p_T\right)$ $\quad\dfrac{\delta\sigma}{\delta\varepsilon}=E_{TT}$
- $\Delta\varepsilon-\Delta\varepsilon^{th}<0$ one poses $\sigma_C^e=\sigma^-+E\left(\Delta\varepsilon-\Delta\varepsilon^{th}\right)$
  - if $-\sigma_C^e<R_C\left(\bar{p}_C\right)$ in this case $\Delta p_C\ =\ 0$ thus $\sigma=\sigma_C^e$ and $\dfrac{\delta\sigma}{\delta\varepsilon}\ =\ E$

◦ if not: $\Delta p_C = \dfrac{\left|\sigma_C^e\right| - \left(\sigma_{yC} + H_C p_C^{-}\right)}{E + H_C}$, $\Delta p_T = 0$

$$\sigma = \frac{\sigma_C^e}{1 + \dfrac{E \Delta p_C}{R_C(p_C)}} = \frac{\sigma_C^e}{\left|\sigma_C^e\right|} R_C(p_C)$$

$$\frac{\delta \sigma}{\delta \varepsilon} = E_{TC}$$

**Note:**

*The initial tangent matrix (option `RIGI_MECA_TANG`) is taken equal to the elastic matrix.*

## 5.3 Internal variables

The relation of behavior `VMIS_ASYM_LINE` product 2 internal variables: $p_C$ $p_T$.
It is not usable for the elements of grid.

# 6 Model of PINTO_MENEGOTTO

The model presented in this chapter describes the behavior 1D steels reinforcing of the reinforced concrete [feeding-bottle 1]. The law constitutive of these steels is made up of two distinct parts: the monotonous loading composed of three successive zones (linear elasticity, plastic stage and work hardening) and the cyclic loading whose analytical formulation was proposed by A. Giuffré and P. Pinto in 1973 [feeding-bottle 2] and was then developed by Mr. Menegotto [feeding-bottle 3].

During cycles, the way of loading between two points of inversion (semi-cycle) is described by an analytical curve of expression of the type $\sigma=f(\varepsilon)$ . The interest of this formulation is that the same equation controls the discharge and load diagrams (see for example the figures [Figure 6.1.1-a] and [Figure 6.1.2-a]). Parameters attached to the function $f$ are reactualized after each inversion of loading. The reactualization of these parameters depends on the way carried out in the plastic zone during the half - preceding cycle.

In addition, this model can treat the inelastic buckling of the bars (G. Monti and C. Nuti [feeding-bottle 4]). The introduction of new parameters into the equation of the curves then makes it possible to simulate the softening of the answer stress-strain in compression.

## 6.1 Formulation of the model

### 6.1.1 Monotonous loading

This chapter describes the first loading which the bar undergoes, i.e. the part preceding activation by the curve of Giuffré [Figure 6.1.1-a].
The monotonous traction diagram of steel is typically followed by the three following successive zones:

- The linear elasticity, defined by the Young modulus $E$ and elastic limit $\sigma_y$. $\sigma=E\varepsilon$ (zone 1, [Figure 6.1.1-a])
- The plastic stage, ranging between the limiting elastic strain $\varepsilon_y^0$ and deformation of work hardening $\varepsilon_h$, higher limit of the plate in deformation. During the stage the constraint remains constant. $\sigma=\sigma_y^0$ (zone 2, [Figure 6.1.1-a])
- Work hardening, following the traction diagram up to the ultimate point of constraint and deformation, $(\varepsilon_u,\sigma_u)$. This part is represented by a polynomial of the fourth degree: $\sigma=\sigma_u-(\sigma_u-\sigma_y^0)\left(\frac{\varepsilon_u-\varepsilon}{\varepsilon_u-\varepsilon_h}\right)^4$ (zone 2, [Figure 6.1.1-a])

The slope of work hardening (used thereafter, for the cyclic behavior) is defined here by: $E_h=\frac{\sigma_u-\sigma_y^0}{\varepsilon_u-\varepsilon_y^0}$ . It is the average slope of zones 2 and 3 of the following figure.

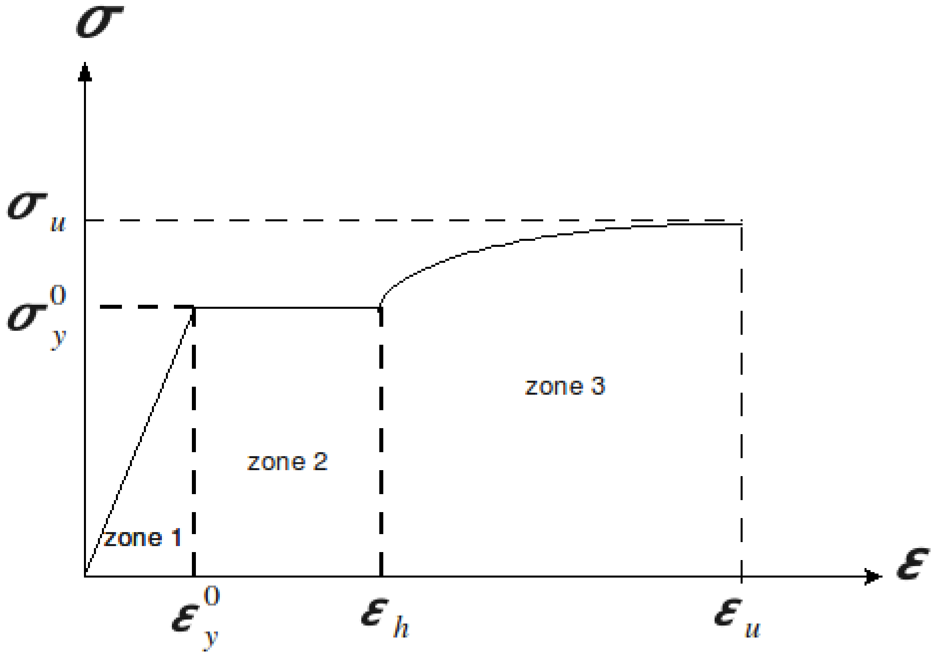



**Figure 6.1.1-a : Curve of behavior.**

### 6.1.2 Cyclic loading

One places oneself now if the bar undergoes a consecutive discharge with the first loading. Two cases arise then:

- the starting position is in the elastic zone. The discharge remains in this elastic case,
- the starting position is in the plastic zone ( $\varepsilon \geq \varepsilon_y^0$ ). The answer is first of all rubber band, then, for a certain value of the deformation, the discharge becomes nonlinear [Figure 6.1.2-a] (this is true for a discharge starting from zone 2 or of zone 3).

The relation which the deformation must satisfy so that the curve of Giuffré is activated is the following one:

$$\left| \varepsilon_{max} - \varepsilon \right| > \frac{\left|\varepsilon_y^0\right|}{3.0}$$ , with $\varepsilon_{max}$ maximum deformation reached in load.

As soon as one crossed this limit with the first discharge, it is the cyclic behavior (curve of Giuffré [Figure 6.1.2-a]) which is activated.

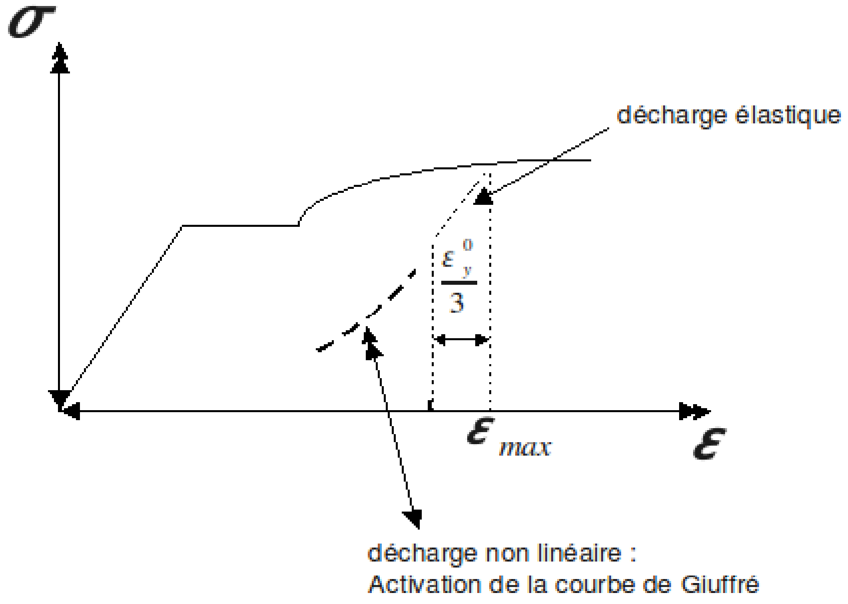



**Figure 6.1.2-a : Curve of behaviour with discharge.**

#### 6.1.2.1 Presentation of the nth semi-cycle

The shape of the curve of the nth semi-cycle depends on the plastic excursion carried out during the half - preceding cycle. One defines the following quantities [Figure 6.1.2.1-a]:

- $\sigma_y^n$ : Elastic limit of the nth semi-cycle. (Calculation clarified with [§ 5.1.2.2])
- $\sigma_r^{n-1}$ : Constraint at the last point of inversion (forced maximum attack with the n-1ième semi-cycle).
- $\varepsilon_r^{n-1}$ : Deformation at the last point of inversion (maximum deformation attack with the n-1ième semi-cycle).
- $\varepsilon_y^n$ : Deformation corresponding to $\sigma_y^n$: $\varepsilon_y^n=\varepsilon_r^{n-1}+\frac{\sigma_y^n-\sigma_r^{n-1}}{E}$
- $f(t)$ : Plastic excursion of the nth cycle

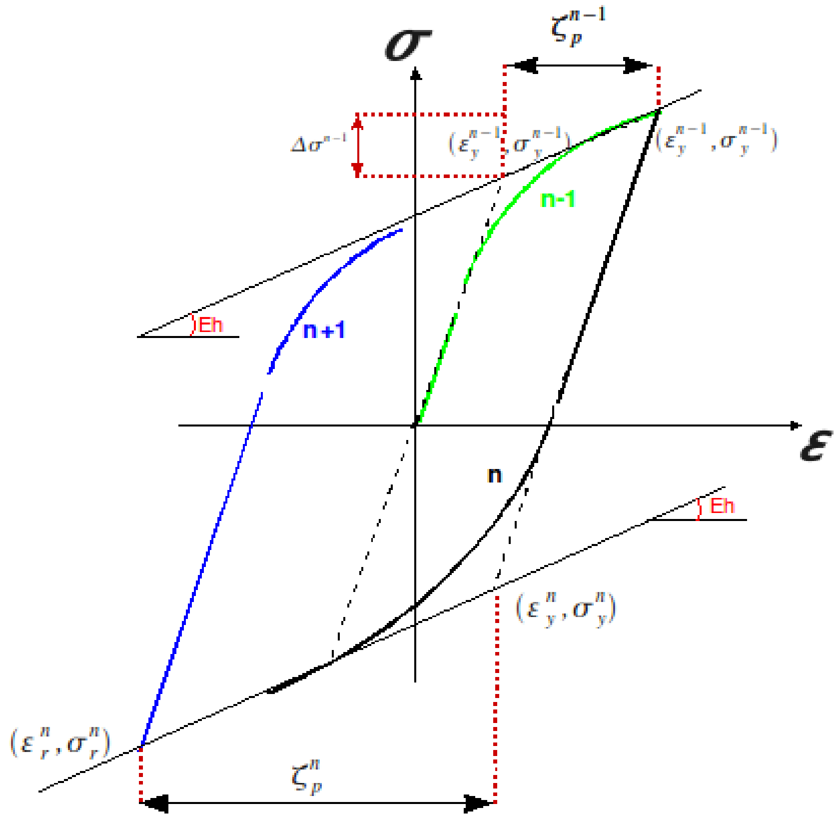



**Figure 6.1.2.1-a : Cyclic behavior.**

### 6.1.2.2 Law of work hardening

The model is based on a kinematic law of work hardening. The branches of the semi-cycles lie between two asymptotes of slope $E_h$ (asymptotic slope of work hardening).

One thus determines $\sigma_y^n$ in the following way: $\sigma_y^n=\sigma_y^{n-1}.\,sign\left(-\zeta_p^{n-1}\right)+\Delta\sigma^{n-1}$ where the function $sign(x)=-1$ if $x<0$ and $1$ if $x>0$ and where $\Delta\sigma^{n-1}$ of constraint of the preceding semi-cycle [Figure is the plastic increment 6.1.2.1-a] which is defined by: $\Delta\sigma^{n-1}=E_h\zeta_p^{n-1}$.

For each semi-cycle one thus determines $\sigma_y^n$ according to $\sigma_y^{n-1}$ and $\zeta_p^{n-1}$, one from of deduced, $\varepsilon_y^n$ then the following semi-cycle is calculated (by the law of behavior below). The maximum deformation (in absolute value) reached before changing direction will make it possible to calculate the plastic excursion $\zeta_p^n=\varepsilon_r^n-\varepsilon_y^n$.

### 6.1.2.3 Analytical description of the curves $\sigma=f(\varepsilon)$

The expression chosen in the model to follow the curves of loading is the following one:

$$\sigma^*=b\,\varepsilon^*+\left(\frac{1-b}{\left(1+\left(\varepsilon^*\right)^R\right)^{1/R}}\right)\varepsilon^*$$

With $b=\dfrac{E_h}{E}$ report of the slope of work hardening on the slope of elasticity.

$$\varepsilon^* = \frac{\varepsilon - \varepsilon_r^{n-1}}{\varepsilon_y^n - \varepsilon_r^{n-1}}$$

$$\sigma^* = \frac{\sigma - \sigma_r^{n-1}}{\sigma_y^n - \sigma_r^{n-1}}$$

$$\xi_p^{n-1} = \frac{\zeta_p^{n-1}}{\varepsilon_y^n - \varepsilon_r^{n-1}}$$

Size $R$ allows to describe the pace of the curve of the branches. It is function of the plastic way carried out during the preceding semi-cycle:

$$R(\xi) = R_0 - g(\xi) \text{ where } g(\xi) = \frac{A_1 . \xi}{A_2 + \xi}$$

Parameters $R_0, A_1$ and $A_2$ are constants without unit depending on the mechanical properties of steel. Their values are obtained in experiments and Menegotto [feeding-bottle 3] proposes:

$$R_0 = 20.0 \quad A_1 = 18.5 \quad A_2 = .015$$

### 6.1.3 Case of inelastic buckling

Monti and Nuti [feeding-bottle 4] show that for a relationship between the length $L$ and the diameter $D$ bar lower than 5, the curve of compression is identical to that of traction. On the other hand, when $L/D > 5$ a buckling of the bar is observed. In this case the curve of compression in the plastic zone has a lenitive behavior. The model available in *Code_Aster* allows to also describe this phenomenon.

One defines the following variables [Figure 6.1.3-a]:

- $E_0$ : Initial elastic Young modulus (correspondent with E without buckling).
- $b_c$ : Report of the slope of work hardening on the elastic slope in compression.
- $b_t$ : Report of the slope of work hardening on the elastic slope in traction (refill after compression with buckling).
- $E_r$ : Modulus Young reduced in traction (slope of the curve of refill after compression with buckling).

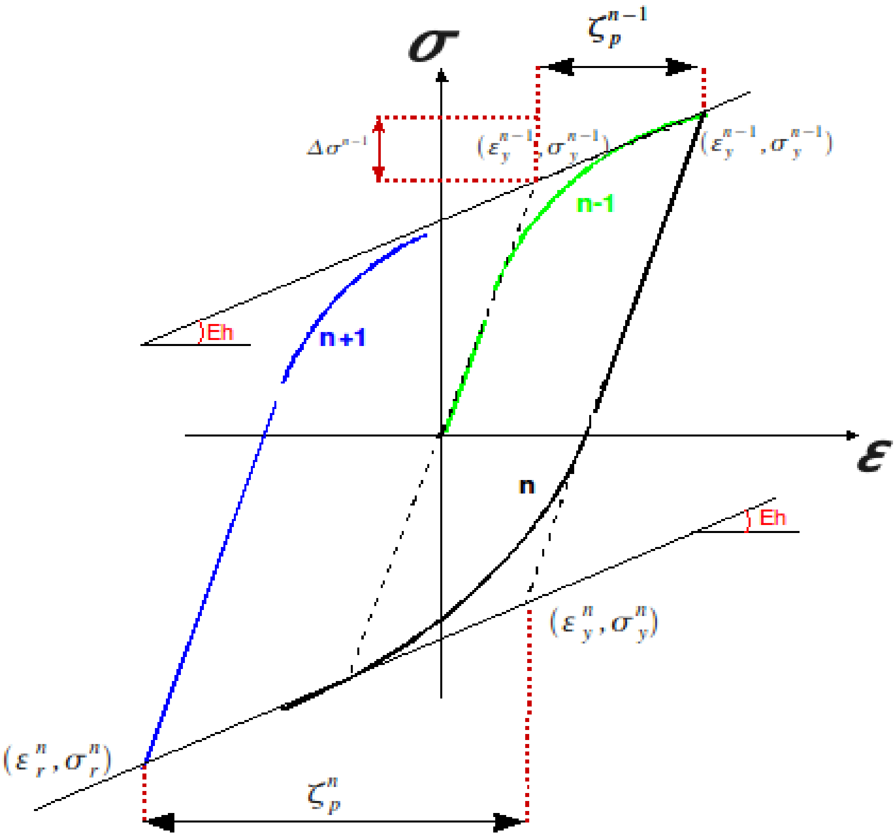



**Figure 6.1.3-a : Cyclic curve of behavior.**

### 6.1.3.1 Compression

A negative slope is introduced $b_c \times E$, where $b_c$ is defined by:

$$b_c = a(5.0 - L/D)\, e\left(b\zeta' \frac{E}{\sigma_y^0 - \sigma^\infty}\right)$$

With $\sigma_\infty = 4.0 \dfrac{\sigma_y^0}{L/D}$ and $\zeta' = \max\left(\left|\zeta_p^n\right|\right)$ the greatest plastic way carried out during the loading.

It is necessary then, as in the model without buckling, to determine $\sigma_y^n$. The method is identical, but a complementary constraint is added $\sigma_s^*$ in order to correctly position the curve compared to the asymptote [Figure 6.1.3-a].

$$\sigma_s^* = \gamma_s b E \frac{b - b_c}{1 - b_c}$$ where $\gamma_s$ is given by: $$\gamma_s = \frac{11.0 - L/D}{10\left(e^{cL/D} - 1.0\right)}$$

And one thus has: $\sigma_y^n = \left(\sigma_y^n\right)_{\text{sans flambage}} + \sigma_s^*$

This modifies also the value of $\varepsilon_y^n = \varepsilon_r^{n-1} + \dfrac{\sigma_y^n * \sigma_r^{n-1}}{E}$

### 6.1.3.2 Traction

At the time of the semi-cycle in traction according to one adopts a reduced Young modulus defines by:

$$E_r = E_0\left(a_5 + (1.0 - a_5)\, e^{\left(-a_6 \zeta_p^2\right)}\right) \text{ with } a_5 = 1.0 + (5.0 - L/D)/7.5$$

**Note:**

*Parameters $a, c$ and $a_6$ are constants (without unit) depend on the mechanical properties of steel and are in experiments given. Values adopted by Monti and Nuti [feeding-bottle 4] are:* $a=0.006 \quad c=0.500 \quad a_6=620.0$

## 6.2 Establishment in *Code_Aster*

This model is accessible in *Code_Aster* starting from the keyword `BEHAVIOR` (`RELATION = 'PINTO_MENEGOTTO'`) or (`RELATION = 'GRILLE_PINTO_MEN'`) order `STAT_NON_LINE` [U4.51.03]. The whole of the parameters of the model are given via the order `DEFI_MATERIAU` (keyword factor `PINTO_MENEGOTTO`) [U4.43.01]. One indexes the parameters here intervening in the model:

| Parameters of the model | Intervenes in | adopted value by default in *Aster* |
|---|---|---|
| $\sigma_y^0$ | First loading | – |
| $\varepsilon_u$ | First loading | – |
| $\sigma_u$ | First loading | – |
| $\varepsilon_h$ | First loading | – |
| $b=\frac{E_h}{E}$ | Cycles | If no value entered one takes the computed value with the first loading |
| $R_0$ | Cycles | 20 |
| $a_1$ | Cycles | 18.5 |
| $a_2$ | Cycles | 0.15 |
| $L/D$ | Cycles with buckling (if $L/D>5$ ) | 4 (to be by default except buckling) |
| $a_6$ | Buckling | 620 |
| $c$ | Buckling | 0.5 |
| $a$ | Buckling | 0,006 |

Parameters $R_0, a_1, a_2, a_6, c$ and $a$ depend on the mechanical properties of steel and are in experiments given. Adopted values by default in *Code_Aster* are those proposed in the literature [feeding-bottle 1].

One gives in [Figure 6.2-a] a comparison of the model following the value of $b=\frac{E_h}{E}$ for two values: $b=0.01$ and $b=0.001$

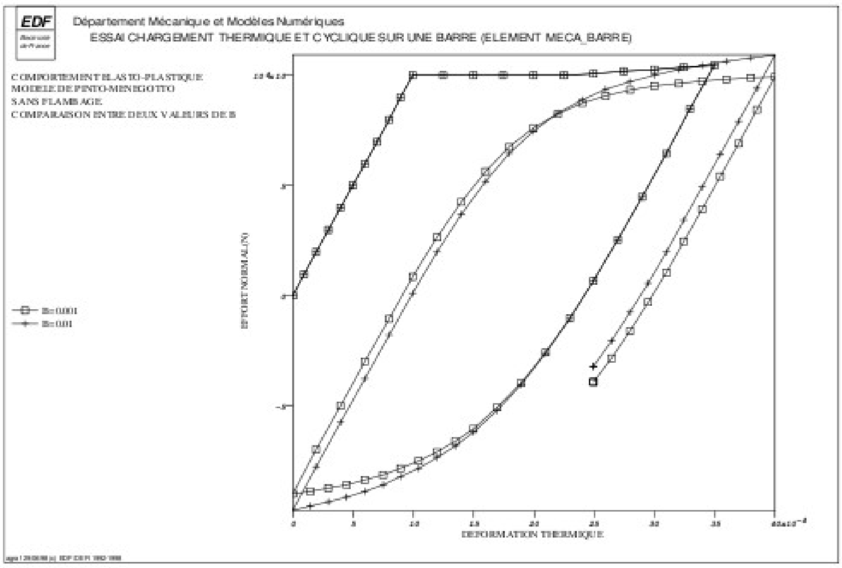



**Figure 6.2-a : Comparison of 2 sets of parameters.**

One gives in [Figure 6.2-b] a comparison of the model without buckling and the model and buckling.

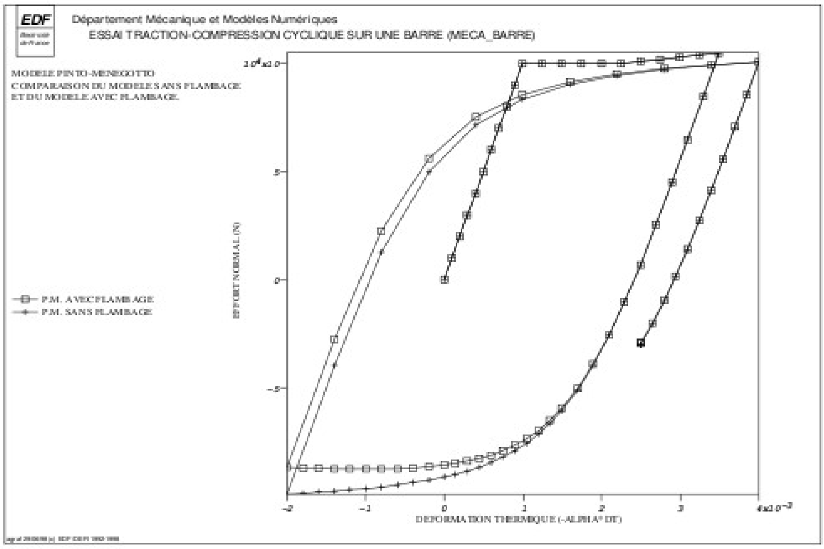



**Figure 6.2-b : Comparison with and without buckling.**

## 6.3 Internal variables

They 8, and are defined by:

$V1 = \varepsilon_r^{n-1}$

$V2 = \varepsilon,_r^n$

$V3 = \sigma_r^n$

$V4 = \varepsilon^- + \Delta\varepsilon - \alpha\left(T - T^-\right) V5 = \Delta\varepsilon + \alpha\left(T - T^-\right)$

$V6 =$ cycl = 0 si le comportement cyclique n'est pas activé
= 1 dans le cas contraire

$V7 = \chi$ = 0 si le pas de temps correspond à une évolution linéaire
= 1 dans le cas contraire (indicateur de plasticité

$V8 =$ indicateur de flambage

# 7 Behaviors VISC_IRRA_LOG and GRAN_IRRA_LOG

The model presented in this chapter describes the viscoplastic behaviors 1D VISC_IRRA_LOG and GRAN_IRRA_LOG (creep and growth under irradiation of the alloys M5 and Zircaloy-4) for the modeling of the fuel assemblies, and applicable to the elements of bars and multifibre beams.

## 7.1 Formulation of the model

The equations are the following ones:

$$\begin{cases} \dot{\varepsilon}^{vp} = \dot{\varepsilon}\dfrac{\sigma}{|\sigma|} \\ \dot{\varepsilon} = |\sigma| . \left( e^{\frac{-Q}{T}} \right) . \dot{\Phi} \left( \dfrac{A\omega}{1+\omega\Phi} + B \right) \\ \dfrac{\dot{\sigma}}{E} = \dot{\varepsilon} - \dot{\varepsilon}^{vp} - \dot{\varepsilon}^{g} - \dot{\varepsilon}^{th} \end{cases}$$

These relations are deduced from the creep tests HALIBUT and REFLECTION [8] for various values of neutron flux.

The coefficients are provided under the keyword VISC_IRRA_LOG or GRAN_IRRA_LOG of CHALLENGE_MATERIAL and $\Phi$ is the neutron fluence (integral flow compared to time).

$\varepsilon^{g}$ represent the deformation of growth under flow. She is taken into account only in the behavior GRAN_IRRA_LOG and expresses itself in the form:

$\varepsilon^{g}(t) = f(T, \Phi_t(x, y, z))$

**Note:**

*1) Neutron fluence $\Phi_t(x, y, z)$ express yourself obligatorily in $10^{20} n/cm^2$. By convention in DEFI_MATERIAU [U4.43.01], if the value provided under the keyword FLUX_PHI is equal to 1, it is the field of fluence which is used for the behavior. In the contrary case, the value provided in DEFI_MATERIAU is used as constant neutron flow.*

*2) It is a field with the nodes defined as variable of order in the order AFFE_MATERIAU.*

*3) Caution: The exposure field is incremental and corresponds to the history of irradiation (stored in internal variable – cf below) to which one adds the increment of the field of fluence coming from the variable of order.*

## 7.2 Internal variables

Three internal variables:

- $V1$ : cumulated viscoplastic deformation: $\varepsilon_p$ ;
- $V2$ : memorizing of the history of irradiation (fluence).
- $V3$ : deformation of growth: $\varepsilon^{g}$ .

## 7.3 Implicit integration

By direct implicit discretization of the relations of behavior, one obtains:

$$
\begin{cases}
\Delta\varepsilon^{vp}=\Delta p\dfrac{\sigma(t^-+\Delta t)}{|\sigma(t^-+\Delta t)|}\\
\Delta p=|\sigma(t^-+\Delta t)|\left(e^{\frac{-Q}{T}}\right).\left(\dfrac{A\omega}{1+\omega\Phi(t^-+\Delta t)}+B\right)\Delta\Phi\\
\dfrac{\sigma}{E}-\dfrac{\sigma^-}{E^-}=\Delta\varepsilon-\Delta\varepsilon^{vp}-\Delta\varepsilon^{g}-\Delta\varepsilon^{th}\\
\text{avec}\\
\Delta\varepsilon^{th}=\alpha(T)(T-T_{ref})-\alpha(T^-)(T-T_{ref})\\
\Delta\varepsilon^{g}=f(T^+,\Phi_t^+)-f(T^-,\Phi_t^-)
\end{cases}
$$

One can solve these equations explicitly while posing: $\sigma^e=\dfrac{E}{E^-}\sigma^-+E(\Delta\varepsilon-\Delta\varepsilon^g-\Delta\varepsilon^{th})$

then the system is reduced to: $\sigma=\sigma^e-E\,\sigma\left(e^{\frac{-Q}{T}}\right).\left(\dfrac{A\omega}{1+\omega\Phi}+B\right)\Delta\Phi$

thus the solution is obtained immediately: $$\sigma=\frac{\sigma^e}{1+E\left(e^{\frac{-Q}{T}}\right).\left(\dfrac{A\omega}{1+\omega\Phi}+B\right)\Delta\Phi}$$

and the tangent operator is written: $$\frac{\partial\sigma}{\partial\varepsilon}=\frac{E}{1+E\left(e^{\frac{-Q}{T}}\right).\left(\dfrac{A\omega}{1+\omega\Phi}+B\right)\Delta\Phi}$$

# 8 Model of MAZARS in 1D

## 8.1 Equations of the model

The objective of this modeling is to give an account of refermeture of the cracks. This model is used only with the multifibre beams. Equations presented in the document [R7.01.08] “Model of damage of MAZARS“are taken again and rewritten in $1D$.

$$\begin{cases}\sigma_{xx}=(1-D_t)E\langle\varepsilon_{xx}^e\rangle_+\\ \sigma_{xx}=(1-D_c)E\langle\varepsilon_{xx}^e\rangle_-\end{cases}$$ **[éq 8.1-1 ]**

with:

- $E$ : Young modulus,
- $D_t$ : the variable of damage in traction.
- $D_c$ : the variable of damage in compression.
- $\varepsilon_{xx}^e$ : elastic strain $\varepsilon_{xx}^e=\varepsilon-\varepsilon^{th}$
- $\varepsilon^{th}=\alpha(T-T_{ref})$ *:* thermal dilation

The only modification is to have a damage of traction and compression. Coupling $\alpha_t^\beta D_t+(1-\alpha_t)^\beta D_C$ do not exist any more. The damage remains always controlled by the extensions.

The damages of traction and compression are defined by the following equations if $\varepsilon_{eq}\geq\varepsilon_{d0}$ :

$$D_c=1-\frac{\varepsilon_{d0}(1-A_c)}{\varepsilon_{eq}}-\frac{A_c}{\exp[B_c(\varepsilon_{eq}-\varepsilon_{d0})]}\qquad D_c\in[0,1[$$ **[éq 8.1-2]**

$$D_t=1-\frac{\varepsilon_{d0}(1-A_t)}{\varepsilon_{eq}}-\frac{A_t}{\exp[B_t(\varepsilon_{eq}-\varepsilon_{d0})]}\qquad D_t\in[0,1[$$ **[éq 8.1-3]**

where $A_c, A_t, B_c, B_t, \varepsilon_{d0}$ are parameters materials to be identified.

The damage is controlled by the equivalent deformation $\varepsilon_{eq}$ . Lbe extensions are paramount in the phenomenon of cracking of the concrete, the introduced equivalent deformation is defined starting from the positive values of the deformations, that is to say:

$$\begin{cases} si\,\varepsilon_{xx}^e\geq 0\,alors\,\varepsilon_{eq}=|\varepsilon_{xx}^e|\\ si\,\varepsilon_{xx}^e\leq 0\,alors\,\varepsilon_{eq}=\sqrt{2}\,\nu\,|\varepsilon_{xx}^e|\end{cases}$$ **[éq 8.1-4]**

**Note:**

If $\varepsilon_{xx}^e\leq 0$ , in $1D$ the principal deformations in the other directions are $\varepsilon_{yy}^e=\varepsilon_{zz}^e=-\nu\,\varepsilon_{xx}^e$ . By using the formula $\varepsilon_{eq}=\sqrt{\langle\varepsilon_1\rangle_+^2+\langle\varepsilon_2\rangle_+^2+\langle\varepsilon_3\rangle_+^2}$ the preceding expression well is obtained.

The tangent matrix with for expression: $\dfrac{d\,\sigma_{xx}}{d\,\varepsilon_{xx}^e}=(1-\tilde{D})\,E-\dfrac{d\,\tilde{D}}{d\,\varepsilon_{xx}^e}\,E\,\varepsilon_{xx}^e$ with:

$$si\,\varepsilon_{xx}^e\geq 0\,et\,\varepsilon_{eq}\geq\varepsilon_{d0}\quad \frac{d\,\tilde{D}}{d\,\varepsilon_{xx}^e}=\frac{d\,D_t}{d\,\varepsilon_{xx}^e}=\left(\frac{\varepsilon_{d0}(1-A_t)}{\varepsilon_{eq}^2}+\frac{A_t B_t}{\exp[B_t(\varepsilon_{eq}-\varepsilon_{d0})]}\right)$$

$$si\,\varepsilon_{xx}^e<0\,et\,\varepsilon_{eq}\geq\varepsilon_{d0}\quad \frac{d\,\tilde{D}}{d\,\varepsilon_{xx}^e}=\frac{d\,D_c}{d\,\varepsilon_{xx}^e}=-\sqrt{2}\,\nu\left(\frac{\varepsilon_{d0}(1-A_c)}{\varepsilon_{eq}^2}+\frac{A_c B_c}{\exp[B_c(\varepsilon_{eq}-\varepsilon_{d0})]}\right)$$

The cases test [V6.02.120], [V6.02.119], [V5.02.130] put in work law of behavior of MAZARS in its version $1D$.

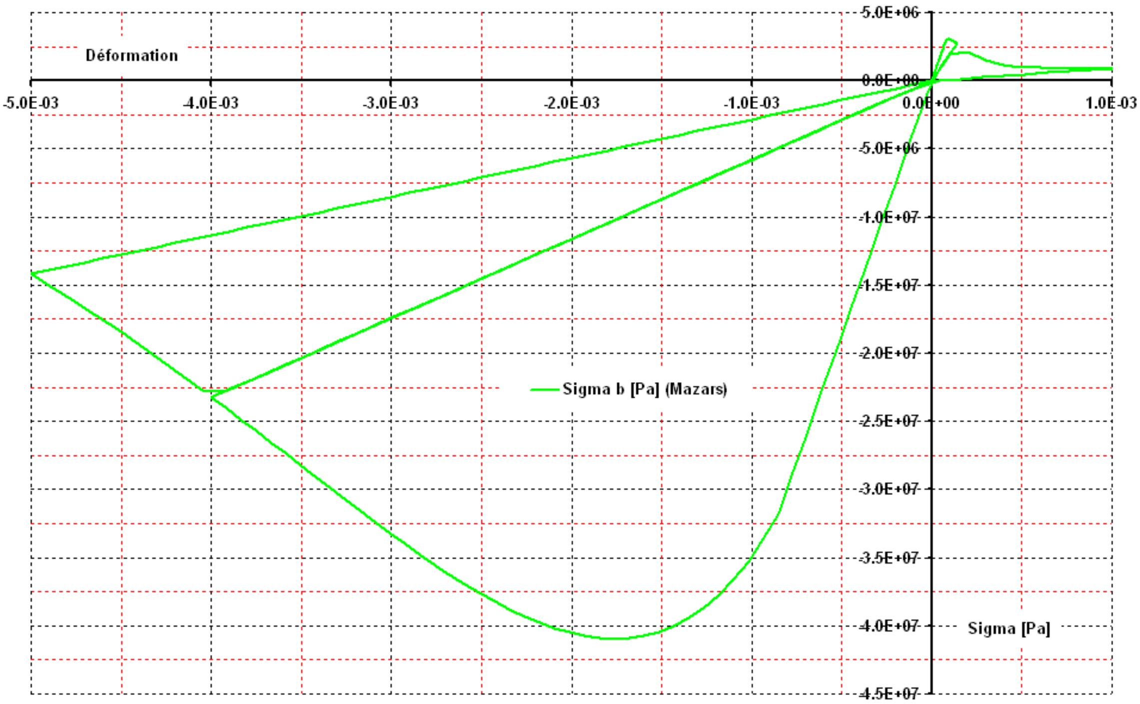



**Figure 8.1-a : Behavior of Mazars in its version $1D$ .**

## 8.2 Internal variables

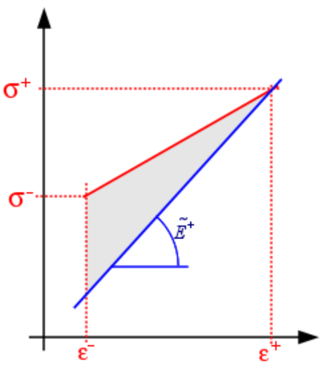
The law of behavior is written by uncoupling the damages from traction and of compression, the 2 damages are not any more of the internal variables [R7.01.08].

This law is dedicated to calculations of civil engineer. To facilitate interpretations of the results 2 variables are created to describe the state "limit" material concrete, in accordance with what this fact in the regulations of reinforced concrete calculation to the limiting states.

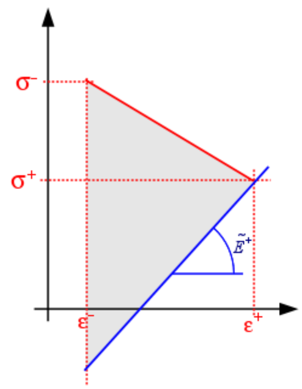
- The variable CRITSIG give information compared to the state of stress. This variable represents the constraint divided by the ultimate stress of the concrete given by the user SIGM_LIM.
- The variable CRITEPS give information compared to the state of deformation. This variable represents the equivalent deformation $\varepsilon_{eq}$ divided by the deformation limits given by the user using the key word EPSI_LIM.

Values of the ultimate stress SIGM_LIM and of the limiting deformation EPSI_LIM are modifiable by the user at the time of the definition of material: DEFI_MATERIAU [U4.43.01], DEFI_MATER_GC [U4.42.0 7].

The writing of the law of MAZARS does not allow to calculate an intrinsic dissipation with the model. But, during seismic calculations it can be useful for the user to know nonrecoverable dissipated energy. The variable DISSIP represent the nonrecoverable office plurality of energy. The nonrecoverable increment of energy is written $\Delta Eg=\frac{1}{2}\left(E(1-D^{+})\Delta\varepsilon-\left(\sigma^{+}-\sigma^{-}\right)\right)\Delta\varepsilon$ .



Material not-polishing substance



Lenitive material

Internal variables for the law of MAZARS in 1D :

- $V1$ CRITSIG : Criterion in constraint
- $V2$ CRITEPS : Criterion in deformation.
- $V3$ ENDO : Endommagement [R7.01.08].
- $V4$ EPSEQT : Equivalent deformation of traction
- $V5$ EPSEQC : Equivalent deformation of compression
- $V6$ RSIGMA : Report of tri-axialité.
- $V7$ TEMP_MAX Maximum temperature attack in material
- $V8$ DISSIP : Energy nonrecoverable.

# 9 Method to use in 1D all the behaviors 3D

As for the treatment of the plane constraints [R5.03.03], it is possible to profit for modelings 1D from the behaviors available in 3D. One extends for that the method due to R.de Borst to the case 1D, by treating this condition (unidimensional stress field) not with the level of the law of behavior but with the level of balance. One obtains thus during iterations of the algorithm of `STAT_NON_LINE` stress fields which tend towards an one-way field. One checks, with convergence of the total iterations of Newton, that the stress fields are indeed one-way, except for a precision, if not one continues the iterations. The method consists in breaking up the fields of strains and stresses into a purely one-way part (direction X) and a part relative to the other directions, and to carry out a static condensation by writing that the components of the constraints relative to the other directions are worthless. One does not consider in the tensors (order 2) only the diagonal terms, written in the form of vectors with 3 components. Direction X corresponds to the direction of the element (bars, multifibre beam) or to the direction of the reinforcements of grid. At one unspecified moment of the resolution of the incremental behavior, the tangent operator $D$ connect the increase in constraints to the increase in deformation by: $d\sigma=\left[\frac{\partial\sigma}{\partial\varepsilon}\right]d\varepsilon=D\,d\varepsilon$ that one rewrites:

$$\begin{bmatrix} d\sigma_x \\ d\sigma_y \\ d\sigma_z \end{bmatrix}=\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}\begin{bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\varepsilon_z \end{bmatrix}.$$

By writing these increases like the difference between the iterations $n$ and $n+1$ of Newton, one obtains:

$$d\sigma=\sigma^{n+1}-\sigma^{n}=\Delta\sigma^{n+1}-\Delta\sigma^{n}\ ,\ d\varepsilon=\Delta\varepsilon^{n+1}-\Delta\varepsilon^{n}$$

With convergence, this variation must tend towards zero.

By introducing the conditions $\sigma_y^{n+1}=0$ and $\sigma_z^{n+1}$ (one-way behavior), one obtains, for the iteration $n+1$ :

$$\begin{bmatrix} d\sigma_x \\ d\sigma_y \\ d\sigma_z \end{bmatrix}=\begin{pmatrix} \sigma_x^{n+1}-\sigma_x^{n} \\ \sigma_y^{n+1}-\sigma_y^{n} \\ \sigma_z^{n+1}-\sigma_z^{n} \end{pmatrix}=\begin{pmatrix} \sigma_x^{n+1}-\sigma_x^{n} \\ -\sigma_y^{n} \\ -\sigma_z^{n} \end{pmatrix}=\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}\begin{bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\varepsilon_z \end{bmatrix}$$

The two last equations make it possible to express $d\varepsilon_y$ and $d\varepsilon_z$ according to $d\varepsilon_x$ :

$$\begin{cases} d\varepsilon_y=\frac{1}{D_{22}}\left(-\sigma_y^{n}-D_{21}\,d\varepsilon_x-D_{23}\,d\varepsilon_z\right) \\ d\varepsilon_z=\frac{1}{D_{33}}\left(-\sigma_z^{n}-D_{31}\,d\varepsilon_x-D_{32}\,d\varepsilon_y\right) \end{cases}$$

that is to say

$$d\varepsilon_y=\frac{1}{\Delta}\left(-D_{33}\,\sigma_y^{n}+D_{23}\,\sigma_z^{n}+D_y\,d\varepsilon_x\right)$$

$$d\varepsilon_z=\frac{1}{\Delta}\left(-D_{32}\,\sigma_y^{n}+D_{22}\,\sigma_z^{n}+D_z\,d\varepsilon_x\right)$$

with $\Delta=D_{33}D_{22}-D_{23}D_{32},\quad D_y=D_{23}D_{31}-D_{21}D_{33},\quad D_z=D_{32}D_{21}-D_{31}D_{22}$

by deferring these expressions in the first equation, one obtains:

$$\sigma_x^{n+1}=\sigma_x^n+\left(D_{11}+\frac{D_{12}D_y+D_{13}D_z}{\Delta}\right)d\varepsilon_x+\frac{D_{12}D_{23}-D_{22}D_{13}}{\Delta}\sigma_z^n+\frac{D_{12}D_{32}-D_{12}D_{33}}{\Delta}\sigma_y^n$$

Balance with the iteration $n+1$ is written:

$$\int D^T\sigma^{n+1}dv=\int B^T\sigma_x^{n+1}dv=\int B^T\left(D_{11}+\frac{D_{12}D_y+D_{13}D_z}{\Delta}\right)d\varepsilon_x$$
$$+\int B^T\left(\sigma_x^n+\frac{D_{12}D_{23}-D_{22}D_{13}}{\Delta}\sigma_z^n+\frac{D_{12}D_{32}-D_{12}D_{33}}{\Delta}\sigma_y^n\right)dv$$
$$=K^n du^{n+1}+\int B^T\left(\sigma_x^n+\frac{D_{12}D_{23}-D_{22}D_{13}}{\Delta}\sigma_z^n+\frac{D_{12}D_{32}-D_{12}D_{33}}{\Delta}\sigma_y^n\right)dv$$

It is thus noted that the taking into account of the unidimensional behavior intervenes on two levels:

- in the tangent matrix, by the corrective term:

$$\int B^T\frac{D_{12}D_y+D_{13}D_z}{\Delta}B\ dv$$

- in the writing of the second member, by the corrective term:

$$\frac{\int B^T}{\Delta}\left((D_{12}D_{23}-D_{22}D_{13})\sigma_z^n+(D_{12}D_{32}-D_{12}D_{33})\sigma_y^n\right)dv$$

To implement this method, it is enough to calculate these corrective terms and to add them to the constraints and tangent matrix obtained of the resolution 3D of the behavior. For that it is necessary to store information of an iteration of Newton to the other, by the means of 4 additional internal variables. The stages of the resolution are:

- with the iteration $n+1$, the data are: $\Delta u^{n+1},\sigma^-,\alpha^-$ and 4 internal variables (calculated with the iteration $n$):

$$V1=\Delta\varepsilon_y^n+\frac{1}{\Delta}\left(D_{23}\sigma_z^n-D_{33}\sigma_y^n-D_y\Delta\varepsilon_x^n\right),\ V2=\frac{D_y}{\Delta}$$

$$V3=\Delta\varepsilon_z^n+\frac{1}{\Delta}\left(D_{32}\sigma_z^n-D_{22}\sigma_z^n-D_z\Delta\varepsilon_x^n\right),\ V4=\frac{D_z}{\Delta}$$

- before carrying out the integration of the behavior (carried out into axisymmetric) one calculates:

$$\Delta\varepsilon_y^{n+1}=\Delta\varepsilon_y^n+\frac{1}{\Delta}\left(-D_{33}\sigma_y^n+D_{23}\sigma_z^n+D_y d\varepsilon_x\right)$$
$$\Delta\varepsilon_z^{n+1}=\Delta\varepsilon_z^n+\frac{1}{\Delta}\left(-D_{32}\sigma_y^n+D_{22}\sigma_z^n+D_z d\varepsilon_x\right)$$

- the integration of the behavior provides the constraints $\sigma^{n+1}$ and the tangent operator $D$,
- one modifies the second member and the tangent matrix as indicated above,
- the new internal variables are stored and it is checked if $\left|\sigma_z^{n+1}\right|<\eta$ and $\left|\sigma_y^{n+1}\right|<\eta$, with $\eta=\xi\left|\sigma_x^{n+1}\right|$, $\xi=$ RESI_INTE_RELA

**Note:**

*The 4 additional internal variables are added after the internal variables of the law of behavior.*

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# 11 Description of the versions of the document

| **Version Aster** | **Author (S) Organization (S)** | **Description of the modifications** |
|---|---|---|
| 8.4 | J.M. PROIX, EDF-R&D/AMA<br>B. QUINNEZ, EDF-R&D/AMA<br>C. CHAVANT, EDF-R&D/AMA | Initial text |
| 10.2 | J-L.FLÉJOU | Correction formulas |
| 11.2 | J-L.FLÉJOU | Addition ECRO_LINE_1D |