Viscoplastic behavior with damage of CHABOCHE

Summary:

The viscoplastic model coupled with the isotropic damage of Chaboche (developed at the origin to successfully predict the lifetime and the cracking of the paddles of the modern turbojets) is used for calculations of prediction of the time of ruin of structures at high temperatures.

This model is established in *Code_Aster* under the name of VENDOCHAB ; the equations of speed are integrated numerically by an explicit diagram of Runge-Kutta of order 2 with automatic cutting under-not buildings according to an estimate of the error of integration (method of Runge-Kutta encased, confer [R5.03.14]), or by a method of integration implicit [R5.03.01].

Tests SSNV126 and SSNV183 validate the integration of this model. The document of validation [V6.04.126] provides the analytical solution for an isothermal uniaxial creep test.

Contents

Contents

1 Introduction

Calculations by finite elements carried out within the framework of the studies on the serious accidents of the nuclear reactors highlighted the need to use models of damage in order to envisage the ruin of a structure such as the tank subjected to the severe and complex thermal conditions (high temperatures going until fusion, high thermal gradients in space or time, etc) which would impose to him the corium [bib1].

The major interest of this choice lies in the fact that the value of the variable of damage to the rupture (or cracking) can be regarded as an intrinsic parameter of the material which is accessible, although that is difficult and delicate, by physical measurements (ultrasounds, diffraction X, etc). The criterion of rupture with the theory of the damage is then more “physical” that the criteria in maximum deformation used sometimes in viscoplastic calculations without damage or the criteria of damage not coupled (rule of addition of time actually passed under certain conditions (σ, T) divided by the time of ruin for these same conditions).

The model implemented in *Code_Aster* is a viscoplastic model of behaviour to viscosity-multiplicative work hardening coupled with the isotropic damage (model due to Chaboche [bib2]).

Nota bene:

One will find in the reference [bib3] a detailed description of the capacities of the model, a methodology for the identification of the parameters and the values of these parameters for steel 22 MoNiCr 3 7.

2 Formulation of the model

2.1 Tally theoretical

In this sub-chapter, one insists on the specificity of the law VENDOCHAB (i.e. the damage) compared to the usual viscoplastic models. For more details, one will refer to [bib2].

The theory of the damage describes the evolution of the phenomena between the virgin state and the macroscopic crack initiation in a material by means of a continuous variable (scalar or tensorial) describing the progressive deterioration of this material. Up this idea, due to Kachanov which was the first to use it to model the creep rupture of metals in uniaxial request, was taken in France in the Seventies by Lemaitre and Chaboche. The evolution of material of its virgin state in its damaged state is not always easy to distinguish from the phenomenon of deformation generally accompanying it and is due to several different mechanisms of which creep is part. The viscoplastic damage of creep corresponds to intergranular decoherences accompanying the deformations viscoplastic for metals with average temperatures and high.

To define what is this variable of damage, let us consider the surface S of one of the faces of an element of volume Ω located by its normal directed towards outside $\mathbf{n}$. On this section, the microscopic cracks and the cavities which constitute the damage leave traces of various forms. That is to say $\tilde{S}$ the effective resistant surface and S_D the total surface of the whole of the traces.

One a:

$$S_D = S - \tilde{S}$$

and one defines the variable of damage by:

$$D_n = \frac{S_D}{S}$$

D_n is the measurement of the local damage compared to the direction n . From a physical point of view, the variable of damage D_n is thus the surface relative of the cracks and cavities cut by the normal plan to the direction $\vec{n}$. From a mathematical point of view, while making tend S towards 0, the variable $D_{\vec{n}}$ is the surface density of discontinuities of the matter in the normal plan with n . $D_n = 0$ corresponds at the virgin state not damaged. $D_n = 1$ corresponds to the element of volume broken in two parts according to a normal plan to n .

The assumption of isotropy implies that the cracks and cavities are uniformly distributed in orientation in a point of material. In this case, the variable of damage becomes a scalar which does not depend any more orientation and is noted D . One a:

$$D = D_n \forall n$$

We will consider here only the isotropic variable of damage.

Total mechanical measurements (modification of the characteristics of elasticity, plasticity or viscoplasticity) are easier to interpret in term of variable of damage thanks to the concept of effective constraint introduced by Rabotnov. The effective constraint represents the constraint reported to the section which resists the efforts indeed. In the case of the isotropic damage, she is written:

$$\tilde{\sigma} = \frac{\sigma}{(1 - D)}$$

And one a:

1) $\tilde{\sigma} = \sigma$ for a virgin material
2) $\tilde{\sigma} \rightarrow +\infty$ at the instant of the failure

The principle of equivalence in deformation implies that any behavior with the deformation, unidimensional or three-dimensional of a damaged material is translated by the laws of behavior of the virgin material in which one replaces the usual constraint by the effective constraint.

One distinguishes 2 types of variables to characterize the medium:

Observable variables (measurable):

- the temperature T
- total deflection $\underline{\underline{\varepsilon}}$ who breaks up as indicated below:

$$\underline{\underline{\varepsilon}}=\underline{\underline{\varepsilon}}^{e}+\underline{\underline{\varepsilon}}^{vp}+\underline{\underline{\varepsilon}}^{th}$$

Internal variables:

- viscoplastic deformation $\underline{\underline{\varepsilon}}^{vp}$
- the isotropic variable of work hardening r
- the isotropic variable of damage D

That is to say $\Psi=\Psi\left(\underline{\underline{\varepsilon}},\underline{\underline{\varepsilon}}^{vp},T,r,D\right)$, the potential of state, the laws of state describing this potential are:

$$\begin{cases}\underline{\underline{\sigma}}=\rho\dfrac{\partial\Psi}{\partial\underline{\underline{\varepsilon}}}\\ R=-\rho\dfrac{\partial\Psi}{\partial r}\\ s=-\rho\dfrac{\partial\Psi}{\partial T}\\ Y=-\rho\dfrac{\partial\Psi}{\partial D}\end{cases}$$

According to the law of normality, one has, with Φ, potential of dissipation:

$$\begin{cases}\dot{\underline{\underline{\varepsilon}}}^{vp}=\dfrac{\partial\Phi}{\partial\underline{\underline{\sigma}}}\\ \dot{r}=\dfrac{\partial\Phi}{\partial R}\\ \dot{D}=\dfrac{\partial\Phi}{\partial Y}\end{cases}$$

The modeling of the work hardening and the damage of material is done via internal variables (or hidden). In the case of the model VENDOCHAB, internal variables introduced into *Code_Aster* are:

- $\underline{\underline{\varepsilon}}^{vp}$: tensor of the inelastic deformations
- p : cumulated plastic deformation
- r : variable of work hardening viscosity
- D : scalar variable of isotropic damage

2.2 Equations of the model

The equations of the models are written then:

$$
\begin{cases}
\underline{\underline{\varepsilon}}=\underline{\underline{\varepsilon}}^{e}+\underline{\underline{\varepsilon}}^{th}+\underline{\underline{\varepsilon}}^{vp}\\
\underline{\underline{\sigma}}=(1-D)\underline{\underline{\underline{\underline{\Lambda}}}}\,\underline{\underline{\varepsilon}}^{e}\\
\dot{\underline{\underline{\varepsilon}}}^{vp}=\frac{3}{2}\dot{p}\frac{\underline{\underline{\sigma}}'}{\sigma_{eq}}\\
\dot{p}=\frac{\dot{r}}{(1-D)}\\
\dot{r}=\left\langle\frac{\sigma_{eq}-\sigma_{y}(1-D)}{(1-D)K r^{1/M}}\right\rangle^{N}\\
\dot{D}=\left\langle\frac{\chi(\underline{\underline{\sigma}})}{A}\right\rangle^{R}(1-D)^{-k(\chi(\underline{\underline{\sigma}}))}
\end{cases}
$$

with:

$$\chi(\underline{\underline{\sigma}})=\alpha J_0(\underline{\underline{\sigma}})+\beta J_1(\underline{\underline{\sigma}})+(1-\alpha-\beta)J_2(\underline{\underline{\sigma}})$$

où : $J_0(\underline{\underline{\sigma}})$ est la contrainte principale maximale

$$J_1(\underline{\underline{\sigma}})=Tr(\underline{\underline{\sigma}})$$

$$J_2(\underline{\underline{\sigma}})=\sigma_{eq}=\sqrt{\frac{3}{2}\tilde{\sigma}'_{ij}\tilde{\sigma}'_{ij}}$$

$\langle x\rangle$ partie positive de x

where:

$\underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^{e}, \underline{\underline{\varepsilon}}^{th}$ and $\underline{\underline{\varepsilon}}^{vp}$	are respectively the deflections total, elastic, thermal and plastic,
$\underline{\underline{\underline{\underline{\Lambda}}}}=(\Lambda_{ijkl})$	is the elastic tensor of rigidity,
$\underline{\underline{\sigma}}'=\underline{\underline{\sigma}}-\frac{1}{3}Tr(\underline{\underline{\sigma}})\underline{\underline{Id}}$	is the deviatoric part of the tensor of the constraints,
p	is the cumulated plastic deformation,
r	is the variable of isotropic work hardening viscoplastic
D	is the scalar variable of isotropic damage

Nota bene:

The whole of the parameters of the model $\alpha, \beta, N, M, K, A, R$ et k can be functions of the temperature (in $°C$). k can be constant, depend on the temperature or on $\chi(\underline{\underline{\sigma}})$ (in MPa) and of the temperature.
In addition, it is seen that this model considers that it can exist a viscoplastic threshold σ_y who depends on the temperature.

It is seen that this model is reduced to the viscoplastic model of Lemaitre if it is considered that $D=0$ and if one neglects the equation of evolution of D. $M, N,$ et K are coefficients characteristic of the purely viscoplastic behavior of material.

The evolution of the damage is governed by a law with three parameters: A, R, et k. The equivalent constraint $\chi(\underline{\underline{\sigma}})$ allows to take account of a possible effect of the spherical part of the tensor of the constraints on the damage (a little as in the laws growth cavities at the base the models Gurson and Rousselier). The fact that the maximum principal constraint can play a part in $\chi(\underline{\underline{\sigma}})$ is difficult to imagine for materials as steel but returns the model plus general.

3 Calculation of the parameters material

The parameters of the law of behavior can be calculated starting from creep tests carried out for various levels of constraints and temperature. For that one uses a unidimensional law of behavior because the request of a cylindrical test-tube in traction can be modelled in dimension 1. The tensor of the constraints is reduced to its axial component.

$$\underline{\underline{\sigma}}=\begin{pmatrix}\sigma_0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \text{ and } \underline{\underline{\sigma}}'=\sigma_0\begin{pmatrix}\frac{2}{3} & 0 & 0\\ 0 & -\frac{1}{3} & 0\\ 0 & 0 & -\frac{1}{3}\end{pmatrix}$$

One thus has: $J_0(\underline{\underline{\sigma}})=J_1(\underline{\underline{\sigma}})=J_2(\underline{\underline{\sigma}})=\sigma_0$

$$\chi(\underline{\underline{\sigma}})=\sigma_0 \quad \forall(\alpha,\beta)$$

The system of equations to be solved is then:

$$\underline{\underline{\dot{\varepsilon}}}^{vp}=\dot{p}\begin{pmatrix}1 & 0 & 0\\ 0 & -\frac{1}{2} & 0\\ 0 & 0 & -\frac{1}{2}\end{pmatrix}$$

$$\dot{p}=\frac{\dot{r}}{(1-D)}$$

$$\dot{r}=\left\langle\frac{\sigma_0-\sigma_y(1-D)}{(1-D)K\,r^{\frac{1}{M}}}\right\rangle^N$$

$$\dot{D}=\left\langle\frac{\sigma_0}{A}\right\rangle^R(1-D)^{-k(\sigma_0)}$$

This system of equations is integrable, which makes it possible to have only one cumulated equation for the viscoplastic rate of deformation (that one can compare to the total deflection by neglecting the elastic strain).

One can then correlate this expression with the experimental data to adjust the coefficients, but the number of parameters and non-linearities make that difficult (moreover there is not unicity).
It is thus necessary to use a method of correlation calling on “physical” assumptions on the phenomenon of creep whose curve is represented hereafter.

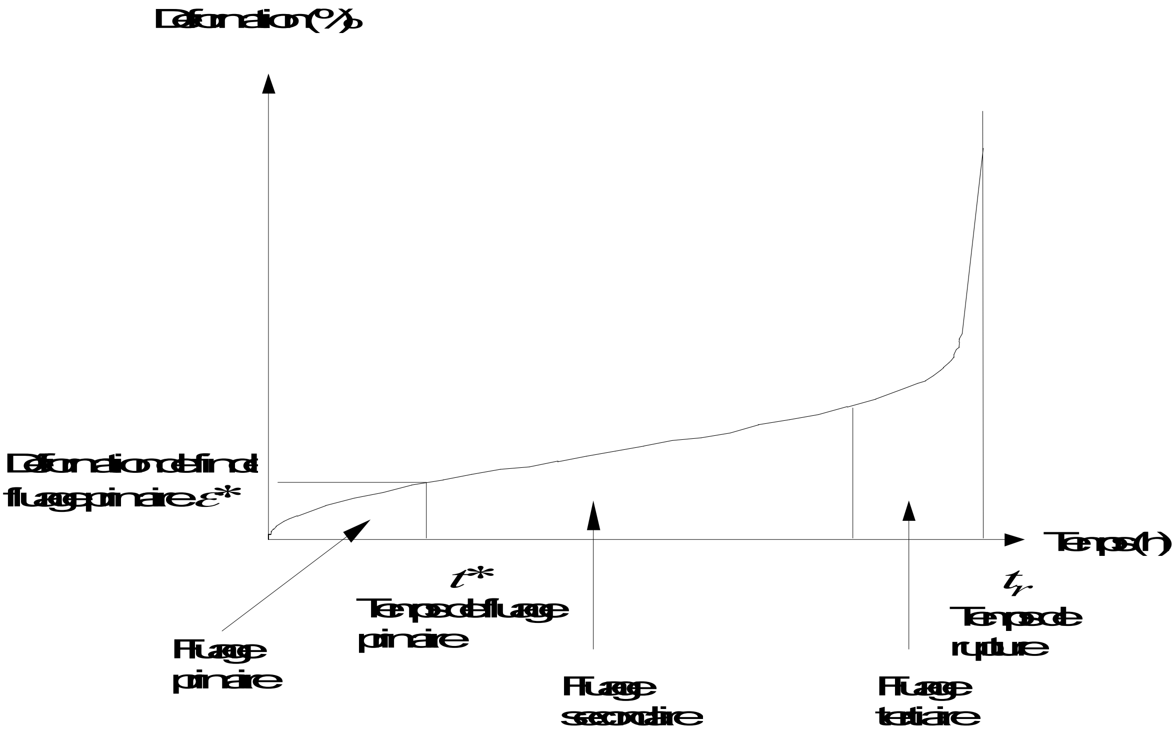


Figure 3- has: Various phases of creep on a curve of creep

The curve of deformation according to time obtained after a creep test breaks up into three parts:

- a part known as of primary education creep where the damage is negligible.
- a part known as of secondary creep where the speed of deformation is appreciably constant.
- a part known as of tertiary creep where work hardening is saturated and where phenomena of damage are dominating.

A method of calculating of the parameters using the experimental data (ε, t) (one also uses $(\dot{\varepsilon}, t)$ who from of deduced by a digital procedure) for various levels from constraints and various temperatures was elaborate at the ECA. It uses the expressions found in the case of higher a homogeneous and unidimensional constraint by making assumptions according to the part of the curve where the data are taken. For example, in the primary education phase of creep, one makes the assumption $D=0$ and in the secondary phase of creep, one uses the fact that $\dot{\varepsilon}$ is constant.

One will find in the references [bib3] and [bib4] the complete description and of the examples of calculations carried out on the German steel of tank 22 MoNiCr 3 7.

4 Establishment in *Code_Aster*

4.1 Algorithm of resolution

The algorithm used is of the total-room type.

The total iterations use the elastic matrix of rigidity calculated starting from the matrix of Hooke damaged:

$$\underline{\underline{\underline{\Delta}}}=(1-D)\,\underline{\underline{\underline{\Delta}}}^0$$

On the level as of local iterations (i.e. in each point of GAUSS), the digital integration of the equations of speed can be carried out either by an explicit diagram of Runge-Kutta of order 2 with automatic cutting under-not buildings according to an estimate of the error of integration (method of Runge-Kutta encased), or by an implicit scheme of Euler solved by a method of Newton. One will refer to the references [bib3] for all the details concerning the digital methods, and with [R5.03.14] for the explicit algorithms employed and their data-processing programming.

4.2 Implicit integration of the relation of behavior

For each total iteration of resolution of the variational problem of the balance and for each point of elementary integration, it is necessary to integrate the equations of the model described into [§3] to obtain the tensor of the constraints and to possibly calculate the operator of tangent behavior.
The problem written in a generic form at the moment T is made up by the four systems of nonlinear equations following:

$$\begin{cases} \underline{Rb}\left(\underline{\beta}, \underline{p}, \underline{\varepsilon}, \underline{\nu}_{etat}^{t-1}\right)=\underline{0} \\ \underline{Rp}\left(\underline{\beta}, \underline{p}, \underline{\varepsilon}, \underline{\nu}_{etat}^{t-1}\right)=\underline{0} \end{cases} \quad \text{éq 4.2-1}$$

with

$$\begin{cases} \underline{\Omega}\left(\underline{\underline{\sigma}}, \underline{\beta}, \underline{p}, \underline{\varepsilon}, \underline{\nu}_{etat}^{t-1}\right)=\underline{0} \\ \underline{\Theta}\left(\underline{\nu}_{etat}, \underline{\beta}, \underline{p}, \underline{\varepsilon}, \underline{\nu}_{etat}^{t-1}\right)=\underline{0} \end{cases} \quad \text{éq 4.2-2}$$

Rb is a system of six equations (six unknown factors) describing the unknown factors associated with the constraints. One notes $\underline{\beta}$ the vector 6 components of these unknown factors. The connection enters $\underline{\beta}$ and $\underline{\underline{\sigma}}$ is realized by means of the system of equations $\underline{\Omega}$ and the vector $\underline{p}$ contains the variables r and D.
Rp is a system of equations describing the internal unknown factors. One chooses a system of 2 equations with $\dot{D}$ and $\dot{r}$ like unknown factors interns. The evolution of the variables of state is described by the system equations $\underline{\Theta}$.

The implicit scheme of Euler is used and the algorithm is presented in the following form:

Initialization of the unknown factors of the discretized problem and recovery of the values of the variables of state obtained with the preceding step
iterations of the method of Newton (maximum number of pre iterations defined by the user): 1) Recovery of the values of the parameters intervening in the material law (the operator of elasticity) 2) Calculation of the criteria of constraint and their derivative compared to the constraints 3) Recovery of the values of the parameter K intervening in the evolution of the damage and its derivative 4) Calculation of the current price of the variables of state, equations describing the internal unknown factors and of the equations describing the constraints 5) Calculation of the derivative of the equations compared to the unknown factors 6) Resolution of the linear system $$\begin{bmatrix} \frac{\partial \mathbf{\underline{Rb}}^n}{\partial \underline{\beta}} & \frac{\partial \mathbf{\underline{Rb}}^n}{\partial \underline{p}} \\ \frac{\partial \mathbf{\underline{Rp}}^n}{\partial \underline{\beta}} & \frac{\partial \mathbf{\underline{Rp}}^n}{\partial \underline{p}} \end{bmatrix} \begin{bmatrix} \underline{d\beta} \\ \underline{dp} \end{bmatrix} = -\begin{bmatrix} \mathbf{\underline{Rb}}^n \\ \mathbf{\underline{Rp}}^n \end{bmatrix} \quad \text{éq 4.2-3}$$ • Test of convergence
Evaluation of the tangent operator

4.2.1 Implicit discretization of the equations of the model

Considering that an increment of time characterize a new state of the system [éq 4.2-1] and [éq 4.2-2] solved by an algorithm of Newton, one chooses to identify the state of one quantity at the previous moment by the exhibitor $^{T-1}$ whereas its state running is noted without exhibitor. Thus the variation of a quantity for the increment of time considered arises by $U=U^{t-1}+\Delta U=U^{t-1}+\Delta t\ \dot{U}(\theta\Delta t)$
For $\theta=0$, one obtains an explicit diagram and for $\theta=1$, a purely implicit diagram is obtained.

With these notations, the discretized form of the vectorial system is written:

$$\underline{Rb}\equiv\underline{\beta}-\left(1-\left(D^{t-1}+\Delta t\ \dot{D}\right)\right)\ \underline{\underline{\Delta}}\ \left(\underline{\varepsilon}-\underline{\varepsilon}_{th}-\left(\underline{\varepsilon}_{vp}^{t-1}+\Delta t\ \frac{3}{2}\ \dot{r}\ \frac{\underline{\sigma}'(\underline{\beta})}{\left(1-\left(D^{t-1}+\Delta t\ \dot{D}\right)\right)\sigma_{eq}(\underline{\beta})}\right)\right)=\underline{0}$$

éq 4.2.1-1

or more simply $\underline{Rb}\equiv\underline{\beta}-\left(1-\left(D^{t-1}+\Delta t\ \dot{D}\right)\right)\ \underline{\underline{\Delta}}\ \ \underline{\varepsilon}_{el}=\underline{0}$

$\underline{\Omega}\equiv\underline{\sigma}=\underline{\beta}$

where $\underline{\beta}$ is the vector 6 components from the tensor of the constraints $\underline{\underline{\sigma}}$.

$$\underline{Rp} \equiv \begin{cases} \dot{r} - \left\langle \dfrac{\sigma_{eq}(\underline{\beta}) - \sigma_y \left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)}{\left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)\ K\ \left(r^{t-1} + \Delta t\ \dot{r}\right)^{1/M}} \right\rangle^N = 0 \\ \dot{D} - \left\langle \dfrac{\chi(\underline{\beta})}{A} \right\rangle^R \left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)^{-k(\chi(\underline{\beta}))} = 0 \end{cases} \quad \text{éq 4.2.1-2}$$

The evolution of the variables of state is described by the system equations $\underline{\Theta}$:

$$\underline{\Theta} \equiv \begin{cases} D = D^{t-1} + \Delta t\ \dot{D} \\ \varepsilon_{vp} = \varepsilon_{vp}^{t-1} + \Delta t\ \dfrac{3}{2} \dfrac{\dot{r}}{\left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)} \dfrac{\underline{\sigma}'}{\sigma_{eq}} = \varepsilon_{vp}^{t-1} + \Delta t\ \dfrac{\dot{r}}{\left(1 - \left(D^{t-1} + \Delta t\ \dot{D}\right)\right)} \dot{\sigma}_{eq} \\ r = r^{t-1} + \Delta t\ \dot{r} \end{cases} \quad \text{éq 4.2.1-3}$$

where D , $\underline{\varepsilon}_{vp}$ and r are the variables of state whose history is preserved.

Deformations $\underline{\underline{\varepsilon}}$ and the variables of states are not unknown factors of the problem. These sizes will be filed with each increment of time converged to be re-used with the following increment.

4.2.2 Digital resolution

The resolution of the nonlinear system $\begin{cases} \underline{\mathbf{Rb}} = \underline{0} \\ \underline{\mathbf{Rp}} = \underline{0} \end{cases}$ use the method of Newton-Raphson associated with a technique with tangential approximation in order to seek the solutions in a field where the functions are correctly conditioned.

According to the algorithm of Newton-Raphson, one solves this system in an iterative way on the following sequence:

1) Initialization of the unknown factors
2) Search for a direction of descent by the resolution of the system [éq 4.2-3]
3) Test of convergence $err = \dfrac{\sum |\Delta x|}{\sum |x|}$

4.2.3 Operator of tangent behavior

The tangent operator is obtained by deriving the constraints compared to the total deflections according to the made up rules of derivation:

$$\frac{d\,\underline{\sigma}}{d\,\underline{\varepsilon}} = \frac{\partial \Sigma}{\partial \underline{\beta}} \frac{\partial \underline{\beta}}{\partial \underline{\varepsilon}} + \frac{\partial \Sigma}{\partial \underline{p}} \frac{\partial \underline{p}}{\partial \underline{\varepsilon}} + \frac{\partial \Sigma}{\partial \underline{\varepsilon}} \quad \text{éq 4.2.3-1}$$

where the function of constraints $\Sigma\left(\underline{\beta}, \underline{p}, \underline{\varepsilon}, \underline{v_{etat}^{t-1}}\right) = \underline{\beta}$. The derivative of the unknown factors compared to the total deflections are obtained by deriving the system [éq 4.2-1] that is to say:

$$\begin{bmatrix} \frac{\partial \underline{Rb}}{\partial \underline{\beta}} & \frac{\partial \underline{Rb}}{\partial \underline{p}} \\ \frac{\partial \underline{Rp}}{\partial \underline{\beta}} & \frac{\partial \underline{Rp}}{\partial \underline{p}} \end{bmatrix} \begin{bmatrix} \frac{\partial \underline{\beta}}{\partial \underline{\varepsilon}} \\ \frac{\partial \underline{p}}{\partial \underline{\varepsilon}} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \underline{Rb}}{\partial \underline{\varepsilon}} \\ \frac{\partial \underline{Rp}}{\partial \underline{\varepsilon}} \end{bmatrix}$$ **éq 4.2.3-2**

4.2.4 Typical case of the plane constraints

The elements 2D in plane constraints having to be usable for this model of behavior, one carries out an additional treatment in on-layer of the treatment general carried out in 3D.
A positive test on the case of the plane constraints means:

1) To the resolution of the system [éq 4.2-1], one adds the additional equations $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$
2) One modifies the tangent operator to assure energy balance.

4.3 Optimized implicit integration

If one does not take account of the trace of the constraints (obtained for zero values of the coefficients α *and* β *),* the function $\chi(\underline{\underline{\sigma}})=J_2(\underline{\underline{\sigma}})$
So moreover one a: $R=k$, then the equations of the model are reduced to:

$$\begin{cases}
\Delta\left(\varepsilon^e\right)=\left(\mathbf{\Delta}\,\varepsilon-\mathbf{\Delta}\,\varepsilon^{\mathbf{th}}-\mathbf{\Delta}\,\varepsilon^{\mathbf{vp}}\right)=\Delta\left(\dfrac{\Lambda^{-1}\sigma}{1-D}\right)=\left(\dfrac{\Lambda^{-1}\sigma}{1-D}\right)-\left(\dfrac{\Lambda^{-1}\sigma}{1-D^-}\right)^-\\
\Rightarrow\quad tr\left(\mathbf{\Delta}\,\varepsilon-\mathbf{\Delta}\,\varepsilon^{\mathbf{th}}\right)+\left(\dfrac{tr\,\sigma^-}{\left(3\lambda+2\mu\right)^-(1-D^-)}\right)=\dfrac{tr\left(\sigma\right)}{\left(3\lambda+2\mu\right)(1-D)}\\
\quad 2\mu\,\left(\Delta\varepsilon'-\mathbf{\Delta}\,\varepsilon^{\mathbf{vp}}\right)+2\mu\left(\dfrac{\sigma'}{2\mu(1-D^-)}\right)^-=\dfrac{\left(\sigma'\right)}{(1-D)}\\
\mathbf{\Delta}\,\varepsilon^{\mathbf{vp}}=\dfrac{3}{2}\dfrac{\Delta r}{(1-D)}\dfrac{\sigma'}{\sigma_{eq}}\\
\Delta r=\Delta t\left\langle\dfrac{\sigma_{eq}-\sigma_y(1-D)}{(1-D)K\,r^{1/M}}\right\rangle^N\\
\Delta D=\Delta t\left\langle\dfrac{\sigma_{eq}}{A\left(1-D\right)}\right\rangle^R
\end{cases}$$

$\Delta a=a-a^-$ Represent the variation of the quantity has between the current moment and the noted previous moment -

A pure implicit discretization here is chosen, i.e. that the terms in D, R which appear are selected at the moment of current calculation.
Note: one can write in an equivalent way the first equation in the form:

$$\left(\varepsilon^e\right)=\left(\varepsilon-\varepsilon^{\mathbf{th}}-\varepsilon^{\mathbf{vp}}\right)=\left(\frac{\Lambda^{-1}\sigma}{1-D}\right)\Rightarrow\left(\frac{\sigma'}{1-D}\right)=2\mu\left(\varepsilon-\mathbf{\Delta}\,\varepsilon^{\mathbf{th}}-\mathbf{\Delta}\,\varepsilon^{\mathbf{vp}}\right)=2\mu\left(\varepsilon^--\varepsilon^{p-}+\Delta\varepsilon'-\mathbf{\Delta}\,\varepsilon^{\mathbf{vp}}\right)$$

$$\text{soit }\left(\varepsilon^e\right)=2\mu\left(\varepsilon^{e-}+\Delta\varepsilon'-\mathbf{\Delta}\,\varepsilon^{\mathbf{vp}}\right)=2\mu\left(\left(\frac{\sigma'}{\left(1-D\right)2\mu}\right)^-+\Delta\varepsilon'-\mathbf{\Delta}\,\varepsilon^{\mathbf{vp}}\right)$$

Then by eliminating the viscoplastic deformation, and by gathering the unknown terms in the equation in constraints:

$$
\begin{cases}
tr(\sigma)=(3\lambda+2\mu)\left[\left(\dfrac{tr\,\sigma}{(3\lambda+2\mu)(1-D)}\right)^{-}+tr\left(\boldsymbol{\Delta\varepsilon}-\boldsymbol{\Delta\varepsilon}^{th}\right)\right]\\
2\mu\left(\boldsymbol{\Delta\varepsilon}^{\mathrm{vp}}\right)=2\mu\left(\dfrac{\sigma'}{2\mu(1-D)}\right)^{-}+2\mu(\Delta\varepsilon')-\dfrac{\sigma'}{(1-D)}=s^{e}-\dfrac{\sigma'}{(1-D)}\\
\text{avec }\; s^{e}=2\mu\left(\dfrac{\sigma'}{2\mu(1-D)}\right)^{-}+2\mu(\Delta\varepsilon')\\
2\mu\,\boldsymbol{\Delta\varepsilon}^{\mathrm{vp}}=\dfrac{3}{2}2\mu\dfrac{\Delta r}{(1-D)}\dfrac{\sigma'}{\sigma_{eq}}=s^{e}-\dfrac{\sigma'}{(1-D)}\\
\dfrac{\Delta r}{\Delta t}=\left\langle\dfrac{\dfrac{\sigma_{eq}}{(1-D)}-\sigma_{y}}{K\,r^{1/M}}\right\rangle^{N}\Leftrightarrow K\left(\dfrac{\Delta r}{\Delta t}\right)^{1/N}r^{1/M}+\sigma_{y}=\dfrac{\sigma_{eq}}{(1-D)}\quad\text{si }\dfrac{\sigma_{eq}}{(1-D)}\geq\sigma_{y},\quad \Delta r=0\ \text{ sinon}\\
\Delta D=\Delta t\left\langle\dfrac{\sigma_{eq}}{A(1-D)}\right\rangle^{R}\Leftrightarrow\Delta D=\Delta t\left\langle\dfrac{1}{A}\left(K\left(\dfrac{\Delta r}{\Delta t}\right)^{1/N}r^{1/M}+\sigma_{y}\right)\right\rangle^{R}
\end{cases}
$$

In this case one can simplify this system of equations while bringing back oneself to only one equation in Δr, in the following way:

Δr solution of:
$$
\begin{cases}
K\left(\dfrac{\Delta r}{\Delta t}\right)^{1/N}r^{1/M}+\sigma_{Y}+\dfrac{3\mu\Delta r}{1-D}=s^{e}_{eq} & \text{si }\dfrac{\sigma_{eq}}{1-D}\geq\sigma_{Y},\\
\Delta r=0 & \text{sinon}
\end{cases}
$$

$$\frac{\sigma'}{1-D}\left(1+\frac{3\mu\Delta r}{\sigma_{eq}}\right)=s^{e}\Rightarrow\sigma_{eq}+3\mu\Delta\frac{r}{1-D}=s^{e}_{\mathrm{eq}}$$

where ΔD is a function of Δr defined by $\Delta D=\Delta t\left\langle\dfrac{1}{A}\left(K\left(\dfrac{\Delta r}{\Delta t}\right)^{1/N}r^{1/M}+\sigma_{Y}\right)\right\rangle^{R}$

Numerically, one multiplies this equation by the term in $\Delta t^{1/N}$ (to avoid the concern when the step of time tends towards 0), and by $1-D$:

$$\Delta r\ \text{ solution de :}$$
$$f(\Delta r)=(1-D)\left(K(\Delta r)^{1/N}r^{1/M}+\sigma_{y}(\Delta t)^{1/N}\right)+3\mu\Delta r(\Delta t)^{1/N}-(1-D)s^{e_{eq}}(\Delta t)^{1/N}=0$$

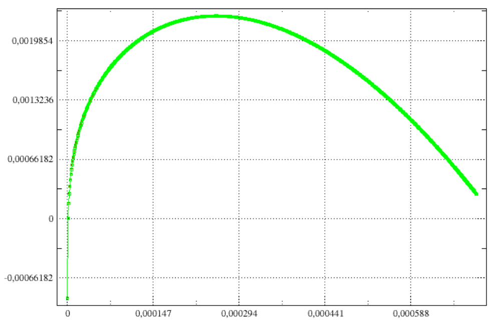
In fact, D cannot increase up to 1, but is limited to 0.99, which makes it possible to avoid singular tangent matrices.

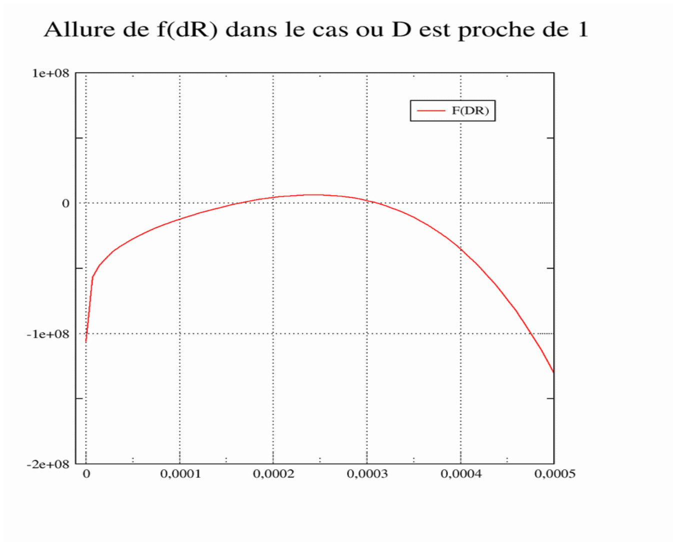
To find the solution of this equation, one uses a method of secant, after having obtained a framing. The function f is negative into 0 by construction. It is a question of seeking an upper limit for which f is positive.

$$\left\{\Delta r = -\frac{(1-D)}{3\mu}\left(K\left(\frac{\Delta r}{\Delta t}\right)^{1/N} r^{1/M} + \sigma_y\right) + \frac{(1-D)}{3\mu} s^{e\,eq} \le \frac{(1-D)}{3\mu} s^{e\,eq} \le \frac{1}{3\mu} s^{e\,eq}\right.$$

If $f\left(\frac{1}{3\mu} s^{e\,eq}\right) > 0$, then the framing is well defined. The solution is sought in an interval defined by the terminals: $[0, \frac{1}{3\mu} s^{e\,eq}]$.

If not, it is necessary to seek an upper limit making the function positive. However this function admits two solutions in general: on the following curve the function is represented $f(\Delta r)$ in two typical cases: for a situation where D is small, and for a situation where D is close to 1. It is noted that the function crosses the x-axis twice. The solution which one seeks is that which corresponds to Δr minimum.





With this intention, since $f\left(\frac{1}{3\mu} s^{e\,eq}\right) < 0$, one calculates the derivative of f. If east is positive one increases the upper limit until finding a value such as f that is to say positive. If the derivative is negative, one decreases the value of the terminal to obtain $f > 0$. One applies then the method of cord.

The precision chosen for the resolution is of $\mathrm{RESI\_INTE\_RELA} * f(0)$.

Once the value of Δr obtained, there is immediately that of ΔD, then the diverter of the constraints is calculated by:

$$\sigma' = (1-D)\left(1 + \frac{3\mu \Delta r}{\sigma_{eq}}\right)^{-1} s^e$$

and traces it by:

$$tr(\Delta\varepsilon - \Delta\varepsilon^{\mathrm{th}}) + \left(\frac{tr\,\sigma^-}{(3\lambda+2\mu)^-(1-D^-)}\right) = \frac{tr(\sigma)}{(3\lambda+2\mu)(1-D)}$$

5 Significance of the internal variables

Internal variables of the model at the points of Gauss (keyword `VARI_ELGA`) are accessible by:

1) $V1=\varepsilon_{vp}^{11}$
2) $V2=\varepsilon_{vp}^{22}$
3) $V3=\varepsilon_{vp}^{33}$
4) $V4=\varepsilon_{vp}^{12}$
5) $V5=\varepsilon_{vp}^{13}$
6) $V6=\varepsilon_{vp}^{23}$
7) $V7=p$, cumulated plastic deformation
8) $V8=r$, the variable of isotropic work hardening viscoplastic
9) $V9=D$, the variable of damage
10) $V10=ind$, indicator being worth 0 if the current point remained elastic with the current step, 1 if not.

6 Bibliography

- P. MONGABURE, A. SIRVENT, MR. DESMET: “RUPTHER: test 1”, report DMT/95/367
- LEMAITRE J., CHABOCHE J.L.: “Mechanical of solid materials”, ED. Dunod (1985)
- P. DECEIVED, J. - R. SCHNEITER: “Introduction of a coupled model of viscoplastic damage into Aster” - Progress report of phase 4 of the CERD RNE 533, note EDF/MTC to be appeared
- H. JAMET: “Determination of the parameters of the viscoplastic law of behavior coupled with the damage for steel 20MnMoNI55. Application to calculation by finite elements of a pipe of the primary education circuit”, CEA/DRN/DMT/SEMT/LAMS/DMT/95/406 notes

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
6.4	P.DUPAS	Initial text
8.4	J.M.PROIX, 0.DIARD	Modifications into implicit
9.4	J.M.PROIX	Modification of the keywords and internal variables
10,1	J.M.PROIX	Improvement of the search for the solution for the cases where the damage is strong: new algorithm of framing and method of secant.