Law of damage of a fragile elastic material

Summary:

This document describes the model of elastic behavior fragile ENDO_FRAGILE available in statics and dynamics. The damage is modelled in a scalar way; the loadings in compression and traction are not distinguished. In addition to the local model, the nonlocal formulation with regularized deformation is also supported to control the phenomena of localization. The nonlocal formulation with gradient of damage is replaced by the law ENDO_SCALAIRE [R5.03.25]

1 Scope of application

The law ENDO_FRAGILE aim at modelling in the manner a simplest possible fragile elastic behavior. The material is elastic isotropic. Its rigidity can decrease in an irreversible way when the deformation energy becomes important, without distinguishing traction from compression. This loss of rigidity is measured by a variable interns scalar called damage which evolves of 0 (healthy material) to 1 (completely damaged material, i.e. without rigidity). Moreover, the constraint cannot exceed a threshold which also decrease with the level of damage to reach 0 when the material is completely damaged. One will refer to [bib1] for a description of this kind of phenomenology.

The property of the reduction of the threshold in constraint with the level of damage is called softening and generally involves a loss of ellipticity of the equations of the problem. It results from it a localization from the deformations and damage in bands of which the thickness is directly controlled by the size of the finite elements. To mitigate this deficiency of the model a nonlocal formulation is proposed, it is based on a regularization of the deformations and activated by modeling *_GRAD_EPSI [R5.04.02]. The width of the bands of localization is controlled by a parameter material, indicated in the operator DEFI_MATERIAU under the keyword LONG_CARA keyword factor NON_LOCAL [U4.43.01]. However, obtaining a physical problem posed again well is obtained only at the cost of one important overcost in time calculation. In addition, it should well be noticed that only the relations of behavior are deteriorated and not equilibrium equations. Consequently, the constraints preserve their usual direction. For the modeling based on the introduction of the gradient of the damage and activated by modeling *_GRAD_VARI [R5.04.01], to refer to the law ENDO_SCALAIRE [R5.03.25].

Lastly, that one activates or not these nonlocal formulations, the softening character of the behavior also involves the appearance of instabilities, physics or parasites, which result in snap - backs on the total answer and return the piloting of the essential loading in statics. The piloting of the type PRED_ELAS [R5.03.80] then seems the mode of control of the level of the most suitable loading.

2 Local law of behavior

2.1 Relations of behavior

The state of material is characterized by the deformation $\boldsymbol{\varepsilon}$ and the damage d understood enters 0 and 1. The relation stress-strain is elastic, rigidity is affected in a linear way by the damage:

$$\boldsymbol{\sigma}=(1-d)\,\boldsymbol{E}:\boldsymbol{\varepsilon} \qquad \text{éq 2.1-1}$$

with E the tensor of Hooke. In addition, the evolution of the damage, always increasing, is controlled by the following function threshold:

$$f(\boldsymbol{\varepsilon},d)=\frac{1}{2}\boldsymbol{\varepsilon}:\boldsymbol{E}:\boldsymbol{\varepsilon}-k(d) \quad \text{where} \quad k(d)=w^y\left(\frac{1+\gamma}{1+\gamma-d}\right)^2 \qquad \text{éq 2.1-2}$$

Coefficients w^y and γ, both positive, is parameters of the model. The condition of coherence then determines completely the rate of damage $\dot{d}$:

$$f(\boldsymbol{\varepsilon},d)\le 0 \quad \dot{d}\ge 0 \quad \dot{d}\; f(\boldsymbol{\varepsilon},d)=0 \qquad \text{éq 2.1-3}$$

The equations [éq 2.1-1] with [éq 2.1-3] are enough to entirely describe the law of behavior ENDO_FRAGILE, indeed very simple. One can also notice that it forms part of the formalism suggested by Marigo [bib2].

2.2 Identification of the parameters of the model

The parameters of this law of behavior are four. On the one hand, the Young modulus E and the Poisson's ratio ν who determine the tensor of Hooke by:

$$\mathbf{E}^{-1}\cdot\boldsymbol{\sigma}=\frac{1+\nu}{E}\boldsymbol{\sigma}-\frac{\nu}{E}(\mathrm{tr}\,\boldsymbol{\sigma})\,\mathbf{Id}$$ **éq 2.2-1**

In addition, w^y and γ who define the lenitive behavior. They are determined by a simple tensile test, cf [Figure 2.2-a]. To simplify the entry of the data of the model, one informs not w^y and γ but directly the tangent module E^T and the constraint with the peak σ^y under the keyword factor ECRO_LINE or ECRO_LINE_FO of the operator DEFI_MATERIAU. As for E and ν, they are given classically under the keyword factor ELAS or ELAS_FO.
For whatever purpose it may serve, here also expressions of the deformation with rupture ε^R in this simple tensile test, as well as voluminal energy k^0 consumed to completely damage a material point, this last expression being valid whatever the history of loading:

$$\varepsilon^R=\left(\frac{1}{E}-\frac{1}{E^T}\right)\sigma^y \qquad k^0=\frac{1}{2}\left(\frac{1}{E}-\frac{1}{E^T}\right){\sigma^y}^2=\frac{1}{2}\varepsilon^R\sigma^y=w^y\,\frac{1+\gamma}{\gamma}$$ **éq 2.2-2**

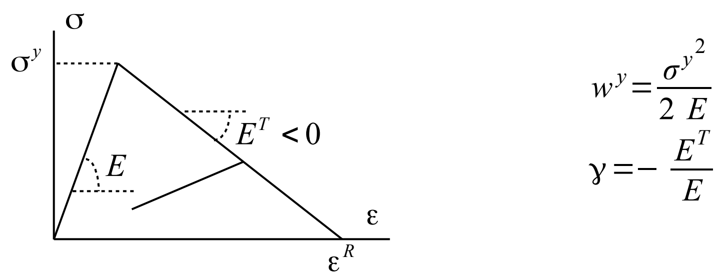

Figure 2.2-a: Simulation of a simple tensile test

2.3 Integration of the law of behavior

Temporal discretization of the equations [éq 2.1-1] with [éq 2.1-3] on a step of time $[t^- \; t]$ is realized by a diagram of implicit Euler. For any function of time q, one notes $q^-=q(t^-)$ and $q=q(t)$. To integrate in time the law of behavior then means to determine the state of constraint and damage solution of the following nonlinear system, where deformation $\boldsymbol{\varepsilon}$ and the state of material at the beginning of the step of time $(\boldsymbol{\varepsilon}^-, d^-)$ are given:

$$\boldsymbol{\sigma}=(1-d)\boldsymbol{E}.\boldsymbol{\varepsilon}$$ **éq 2.3-1**

$$f(\varepsilon,\, d)\leq 0 \quad d-d^-\geq 0 \quad (d-d^-)\, f(\varepsilon,\, d)=0$$ **éq 2.3-2**

A method of resolution was proposed by [bib3]. It starts by examining the solution without evolution of the damage (also called elastic test) then, if necessary, carries out a correction to check the condition of coherence. In this case, the existence and the unicity of the solution guarantee the good performance of the method. Let us consider the elastic test:

$$d=d^{-} \text{ solution if } f^{\mathrm{el}}(\varepsilon)=f(\varepsilon,\ d^{-})\leq 0 \quad \textbf{éq 2.3-3}$$

In the contrary case, the damage is obtained while solving $f(\varepsilon,\ d)=0$:

$$d=(1+\gamma)\left(1-\sqrt{\frac{w^{y}}{w}}\right) \text{ where } w=\frac{1}{2}\,\boldsymbol{\epsilon}\cdot E\cdot\boldsymbol{\epsilon} \quad \textbf{éq 2.3-4}$$

As for the constraint, it is given by [éq 2.3-1] in all the cases.
It still remains to be made sure that the damage does not exceed value 1. In fact, when $d=1$, the rigidity of the material point considered is cancelled. Insofar as no technique of suppression of finite elements "broken" is put in work (technical possibly delicate when the finite elements have several points of Gauss), of the worthless pivots can appear in the matrix of rigidity. This is why a digital threshold is introduced d_c beyond which one considers an elastic residual rigidity for the tangent matrix, equations of behavior remaining unchanged.

To preserve a reasonable conditioning of the matrix of rigidity, one chooses

$$d_c=1-10^{-5} \quad \textbf{éq 2.3-5}$$

An indicator χ, arranged in the second internal variable, the behavior specifies then during the step of current time:

- $\chi=0$ elastic behavior (deformation energy lower than the threshold)
- $\chi=1$ evolution of the damage
- $\chi=2$ (saturated damage) ($d=1$).

2.4 Description of the internal variables

The internal variables are two:

- $VI(1)$ damage d
- $VI(2)$ indicator χ

3 Formulation with gradient of damage

The formulation in gradient of damage is not available any more (starting from version 10.2 of Aster), to refer to the law of behavior `ENDO_SCALAIRE` [R5.03.25], which replaces `ENDO_FRAGILE` for modeling `GRAD_VARI` and this mainly for reasons of robustness of calculations.

4 Formulation with regularized deformation

4.1 Formulation continues in time

The approach with regularized deformation [R5.04.02] also makes it possible to control the phenomena of localization and for this reason seems an alternative to the formulation with gradient of damage. But unlike the latter, this formulation has the advantage of resorting to the standard algorithms for the nonlinear problems. Indeed, the only difference compared to the local law of behavior lies in the data of two deformations instead of one, the local deformation $\boldsymbol{\varepsilon}$ who intervenes in the relation stress-strain and the regularized deformation $\bar{\varepsilon}$ who controls the evolution of the damage. This one results from the local deformation by resolution of the system of partial derivative equations according to:

$$\begin{cases} \bar{\varepsilon} - L_c^2 \, \Delta \, \bar{\varepsilon} = \varepsilon & \text{dans la structure} \\ \nabla \, \bar{\varepsilon} \cdot \boldsymbol{n} = 0 & \text{sur le bord de normale } \boldsymbol{n} \end{cases}$$ **éq 4.1-1**

where the characteristic length L_c is again well informed under the keyword LONG_CARA of DEFI_MATERIAU. Finally, the relation of behavior is written in the following way, where the function threshold F was already defined in [éq 2.1-2]:

$$\boldsymbol{\sigma} = (1-d) \, \boldsymbol{E} \cdot \boldsymbol{\varepsilon}$$ **éq 4.1-2**

$$f(\bar{\varepsilon}, d) \le 0 \quad \dot{d} \ge 0 \quad \dot{d} \, f(\bar{\varepsilon}, d) = 0$$ **éq 4.1-3**

4.2 Integration of the law of behavior

One of the advanced advantages for the nonlocal formulation with regularized deformation is the little of modifications which it involves in the construction of the law of behavior. Indeed, the integration of the internal variables is completely controlled by the regularized deformation $\bar{\varepsilon}$. The expressions of the local law thus are found:

$$\begin{cases} \text{si} \quad f^{\text{el}}(\bar{\varepsilon}) = f(\bar{\varepsilon}, d^-) \le 0 & d = d^- \\ \text{si} \quad f^{\text{el}}(\bar{\varepsilon}) = f(\bar{\varepsilon}, d^-) > 0 & d = (1+\gamma) \left(1 - \sqrt{\dfrac{w^y}{\bar{w}}} \right) \end{cases} \quad \text{with} \quad \bar{w} = \frac{1}{2} \, \bar{\varepsilon} \cdot \boldsymbol{E} \cdot \bar{\varepsilon}$$ **éq 4.2-1**

The constraint is then obtained directly by the relation [éq 4.1-2]. Moreover, one introduces a critical damage [éq 2.3-5], as in the local case, to preserve a residual rigidity.

4.3 Internal variables

They is the same internal variables as for the local law:

- $VI(1)$ damage d
- $VI(2)$ indicator χ

5 Piloting by elastic prediction

The piloting of the type PRED_ELAS standard controls the intensity of the loading to satisfy a certain equation related to the value with the function threshold f^{el} during the elastic test [bib5]. Consequently, only the points where the damage is not saturated are taken into account. The algorithm which deals with this mode of piloting, cf [R5.03.80], requires the resolution of each one of these points of Gauss of the following scalar equation in which $\Delta\tau$ is a data and η the unknown factor:

$$f^{el}(\boldsymbol{\varepsilon}_{\text{impo}} + \eta \, \boldsymbol{\varepsilon}_{\text{pilo}}, a^-) = \Delta\tau$$ **éq 3-1**

Let us note that this equation is modified for piloting PRED_ELAS in ENDO_SCALAIRE in order to have the parameter $\Delta\tau$ who corresponds to the increment of damage which one seeks to obtain for at least a point of the structure. One then does not seek any more one parameter of piloting η who makes leave the criterion a value $\Delta\tau$ with the damage resulting from the step of previous time (cf Eq 3-1), but a parameter η who brings back for us on the criterion with a damage increased by $\Delta\tau$:

$$f^{el}(\boldsymbol{\varepsilon}_{\text{impo}}+\eta\,\boldsymbol{\varepsilon}_{\text{pilo}},a^{-})=\Delta\tau \quad\Rightarrow\quad f^{el}(\boldsymbol{\varepsilon}_{\text{impo}}+\eta\,\boldsymbol{\varepsilon}_{\text{pilo}},a^{-}+\Delta\tau)=0$$ **éq 3-2**

where Δt corresponds to the increment of time defined in the list of moments of calculation and COEF_MULT is the coefficient specified by the keyword COEF_MULT option PILOTING in the operator STAT_NON_LINE [U4.51.03].

6 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
5.0	E. LORENTZ EDF-R&D/AMA	Initial text
10.0	K. KAZYMYRENKO EDF-R&D/AMA	Taking into account of law ENDO_SCALAIRE

7 Bibliography

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