
Law of behaviour of the assembly ASSE_CORN

Summary:

This document describes the nonlinear behaviour of the nonlinear assemblies of angles of pylons modelled by discrete elements `DIS_TR`. This law of behavior is affected on the discrete elements by means of the relation `ASSE_CORN` called by the operators of resolution of nonlinear problems `STAT_NON_LINE` [R5.03.01] or `DYNA_NON_LINE` [R5.05.05].

The law represents at the same time behaviour in traction of the assembly and the relation moment-rotation around the axis of the bolts perpendicular to the assembly. The other directions of loading present a linear elastic behavior described by classical characteristics of rigidity.

One distinguishes in the law from behavior two phases associated with two mechanisms: the first representing the friction and the slip of the bolts until the thrust, and the second representing the plasticization of the assembly until the ruin. The laws of the plastic type describing each one of these phases have even pace and have a concavity to their connection which makes convergence problematic and requires a particular digital processing in the options of calculation to which the iterative method of Newton appeals.

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1 Notations

| | |
|------------------|--|
| SLF | Surface Limits Friction |
| M_y | Moment in the assembly around the axis y |
| \bar{N}_1 | Effort limits slip of the assembly on the axis x |
| \bar{M}_1 | Moment limits slip of the assembly on the axis y |
| SLU | Ultimate Limiting surface |
| \bar{N}_2 | Ultimate limiting effort of the assembly on the axis x |
| \bar{M}_2 | Ultimate limiting moment of the assembly on the axis y |
| \bar{N} | Limiting effort |
| \bar{M} | Limiting moment |
| \bar{U}_1 | Displacement limits mechanism 1 on the axis x |
| $\bar{\theta}_1$ | Rotation limits mechanism 1 on the axis y |
| \bar{U}_2 | Displacement limits mechanism 2 on the axis x |
| $\bar{\theta}_2$ | Rotation limits mechanism 2 on the axis y |
| U | Displacement of the assembly on the axis x |
| θ | Rotation of the assembly on the axis y |
| n | Reduced effort $n = Nx / \bar{N}$ |
| m | Reduced moment $m = My / \bar{M}$ |
| U_r | Reduced displacement $U_r = U / \bar{U}$ |
| θ_r | Reduced rotation $\theta_r = \theta / \bar{\theta}$ |
| \bar{U} | Displacement limits on the axis x |
| $\bar{\theta}$ | Rotation limits on the axis y |
| $h(x)$ | Scalar function |
| a | Parameter of nonlinearity |
| \bar{d} | Constant scalar |
| \underline{d} | Vector reduced generalized displacement |
| \underline{f} | Vector reduced generalized effort |
| p | Variable interns scalar |
| feq | Effort generalized equivalent reduces scalar |
| F | Surface of loading |
| $R(x)$ | Scalar function $R(x) = h^{-1}(x)$ |
| \underline{D} | Vector generalized displacement |
| \underline{E} | Vector generalized effort |

| | |
|-------------------|---|
| $[\bar{D}]$ | Matrix displacement generalized limit |
| $[\bar{F}]$ | Matrix effort generalized limit |
| X^+ | Value of X at the moment $t + dt$ |
| X^- | Value of X at the moment t |
| e | Eccentricity of loading $e = M_y / N_x$ |
| e_r | Reduced eccentricity of loading $e_r = m/n$ |
| ε | Sign of n |
| $[]$ | Matrix |
| $\{\}$ | Vector column |
| $\langle \rangle$ | Vector line |
| K_o | Tangent operator at the moment t |
| K_n | Tangent operator at the moment $t + dt$ |
| K_{or}, K_{nr} | Reduced tangent operators |

2 Physical model of the one-way behaviour of the assembly

The assembly of an angle on the wing of another or a plate (bracket or splice plate) by bolts is schematized by [Figure 2-a].

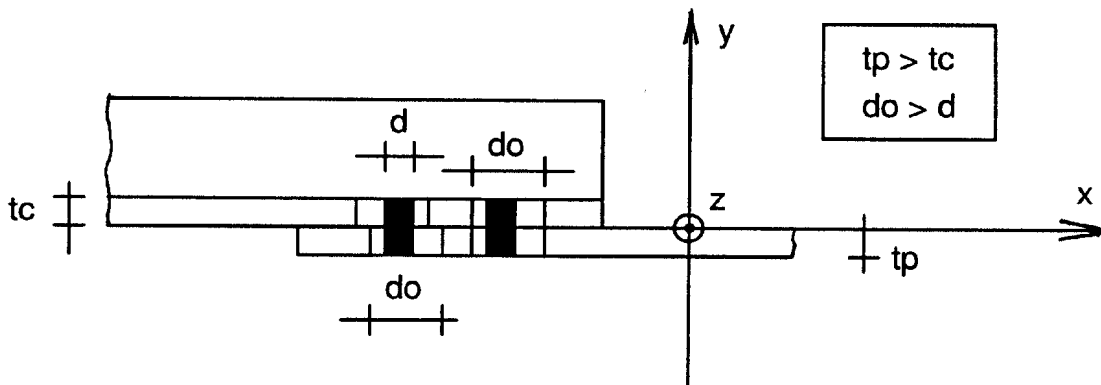


Figure 2-a: local reference mark of the connection; the axis x is confused with the axis of the bar and centers it y is confused with the axis of the bolts

The one-way behaviour of the assembly is modelled for the loading in traction or inflection.

The modeling selected of the one-way behavior in loading of the assembly subjected to a normal effort or a moment around y is represented by [Figure 2-b].

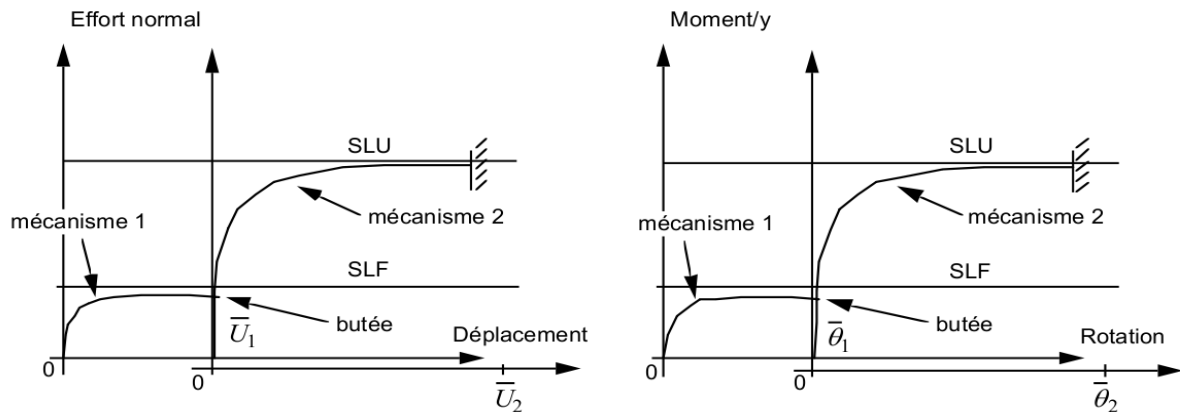


Figure 2-b: mechanisms of assembly in normal effort and moment

One distinguishes two phases of the behavior associated with two mechanisms:

- mechanism 1: friction and slip until the thrust (beginning of the shearing of the bolts).
- mechanism 2: plasticization of the assembly until the ruin by shearing of the bolts or tearing of the grips.

The limiting surface of friction (SLF) is the curve corresponding to the appearance of the slip in spaces $(N_x - U_x)$ and $(M_y - \theta_y)$. Friction is described by the law of Coulomb.

Ultimate limiting surface (SLU) is the curve corresponding to the ruin of the assembly in spaces $(N_x - U_x)$ and $(M_y - \theta_y)$. The ruin can be due, according to the design of the assembly, with the shearing of the bolts or the tearing of the grips.

The tests on the same geometry but with tightening torques of the different bolts show that the tangent stiffness of mechanism 2 at the point of thrust decreases when the SLF approaches the SLU.

This justifies the physical modeling retained for the assembly of the two mechanisms [Figure 2-b].

3 Relation of behavior of the mechanisms

The behavior of mechanisms 1 and 2 is similar. It is nonlinear between a rigid tangent initial behavior and an asymptotic limiting behavior.

It is described by two essential parameters: the parameter of nonlinearity and the parameter surface limit.

The thrust (mechanism 1) or ruins it (mechanism 2) are described by an associated kinematic criterion.

3.1 One-way behavior

We said to [§2] that the one-way behaviors in normal effort and moment around are similar there [Figure 2-b].

They can be described by the same relation if the adimensional sizes are used:

- 1) reduced forces: $n = \frac{N_x}{N}$ et $m = \frac{M_y}{M}$
- 2) reduced displacements: $U_r = \frac{U}{U}$ et $\theta_r = \frac{\theta}{\theta}$

[Figure 3.1-a] represents in adimensional form the one-way behavior. Analytically, it can be written (it is a choice):

$$U_r = h(n) \text{ ou } \theta_r = h(m)$$
$$\text{avec } h(x) = \frac{1}{\bar{d}} \frac{x^{a+1}}{1-x^a}$$
$$\bar{d} = \frac{\bar{n}^{a+1}}{1-\bar{n}^a}$$

a is the scalar parameter of nonlinearity. \bar{n} and a are identified on the one-way tests. \bar{n} who takes into account the variability of the tests generally takes the value 0.95.

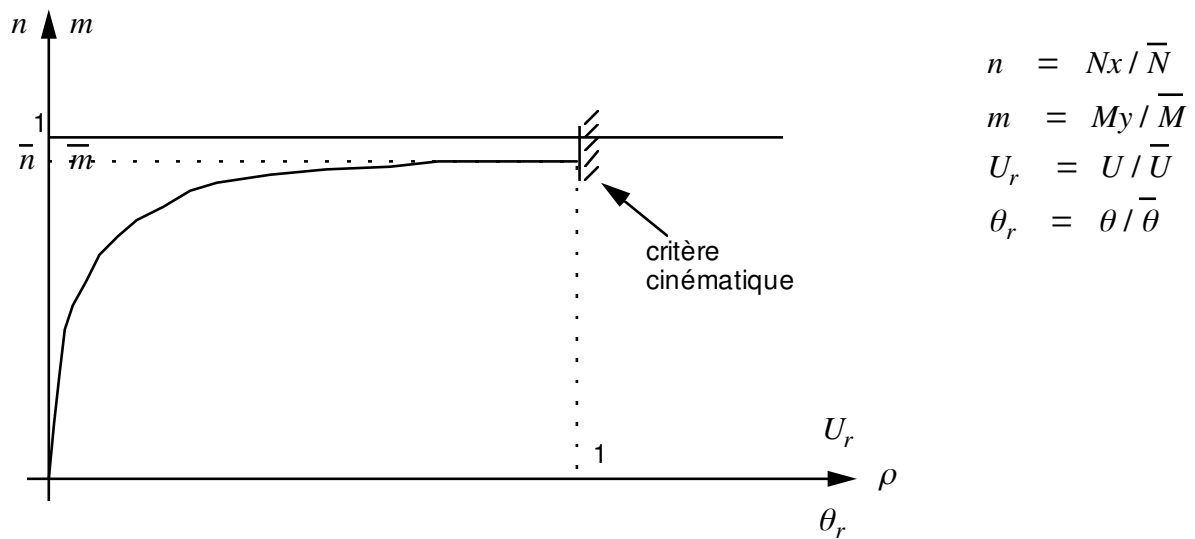


Figure 3.1-a: relation of behaviour of assembly

It is noticed that $h(\bar{n})=1$ or $h(\bar{m})=1$, i.e.: $U_r=1$ or $\theta_r=1$, or: $U=\bar{U}$ or $\theta=\bar{\theta}$.

The one-way kinematic criterion is thus checked for $n=\bar{n}$ or $m=\bar{m}$.

3.2 Incremental two-dimensional behavior

The coupling in extreme cases is defined by limiting surface:

$$\left(\frac{N_x}{\bar{N}}\right)^2 + \left(\frac{M_y}{\bar{M}}\right)^2 = 1$$

The one-way behavior in reduced variables is described by the relation of [§3.1]:

$$\underline{d} = h(\underline{f})$$

where \underline{d} is the vector reduced displacements $\begin{pmatrix} U_r \\ \theta_r \end{pmatrix}$
 \underline{f} is the vector reduced forces $\begin{pmatrix} n \\ m \end{pmatrix}$

Into two-dimensional behavior, the isotropy is translated by a model with an internal variable **scalar** p such as:

$$p = h(fe q) \text{ en chargement}$$

where $fe q$ is the equivalent reduced force (scalar).

$fe q$ is defined such as:

$$\underline{E} = fe q * \underline{E}^*$$

where \underline{E} is the point running of loading $\begin{pmatrix} N_x \\ M_y \end{pmatrix}$

\underline{E}^* is the limiting loading associated with $\underline{E} \begin{pmatrix} \overline{N}_x^* \\ \overline{M}_y^* \end{pmatrix}$

The expression of $fe q$ results from the expression of limiting surface. Membership of \underline{E}^* on the limiting surface is written:

$$\left(\frac{\overline{N}_x^*}{\overline{N}} \right)^2 + \left(\frac{\overline{M}_y^*}{\overline{M}} \right)^2 = 1$$

By the definition of $fe q$, one can write:

$$\left(\frac{N_x}{fe q \overline{N}} \right)^2 + \left(\frac{M_y}{fe q \overline{M}} \right)^2 = 1$$

i.e. according to the reduced forces n and m :

$$\left(\frac{n}{fe q} \right)^2 + \left(\frac{m}{fe q} \right)^2 = 1$$

$$\text{from where } fe q = \sqrt{n^2 + m^2}$$

The surface of loading then is defined F , homothetic on limiting surface, by:

$$F : \quad fe q - R(p) = 0 \\ \text{où } R(p) = h^{-1}(p)$$

For a formalism similar to that of plasticity with isotropic work hardening [bib2], one obtains the relation of behavior continues expressed in reduced sizes:

$$\begin{aligned} \underline{d} &= p \frac{\partial F}{\partial \underline{f}} = p \frac{\underline{f}}{feq} \\ p &= 0 \quad \text{si } feq - R(p) < 0 \\ p &= h'(feq) \quad \text{si } feq - R(p) = 0 \end{aligned}$$

The relation of behavior of the rigid type - plastic without elasticity is written finally:

$$\begin{aligned} \underline{D} &= \frac{p}{feq} [\underline{D}] [\underline{F}]^{-1} \underline{E} \\ \text{où } \underline{D} &= \begin{pmatrix} U \\ \theta \end{pmatrix} \quad \text{et } \underline{E} = \begin{pmatrix} N_x \\ M_y \end{pmatrix} \\ [\underline{D}] &= \begin{bmatrix} \bar{U} & 0 \\ 0 & \bar{\theta} \end{bmatrix} \quad \text{et } [\underline{F}] = \begin{bmatrix} \bar{N} & 0 \\ 0 & \bar{M} \end{bmatrix} \end{aligned}$$

The relation of incremental behavior in reduced sizes is obtained by integration of the relation continues between t (variables -) and $t + dt$ (variables +).

In loading, Δp check $F = 0$ with $t + dt$:

$$feq^+ = R(p^+ + \Delta p) \quad \text{éq 2.2-1}$$

By introducing the relation of behavior,

$$\Delta \underline{d} = \Delta p \frac{\underline{f}^+}{feq^+} \quad \text{éq 2.2-2}$$

one deduces the value from Δp ,

$$\Delta p = \|\Delta \underline{d} \cdot \Delta \underline{d}\| = \sqrt{\Delta U_r^2 + \Delta \theta_r^2}$$

and one calculates the value of feq^+ by [éq 2.2-1]. The relation of behavior [éq 2.2-2] gives the reduced efforts:

$$\begin{aligned} n^+ &= \frac{\Delta U_r}{\Delta p} R(p^+ + \Delta p) \\ m^+ &= \frac{\Delta \theta_r}{\Delta p} R(p^+ + \Delta p) \end{aligned}$$

In unloading, $\Delta p = 0$ and one has by [éq 2.2-2]:

$$\Delta \underline{d} = 0$$

4 Establishment in Code_Aster

The relation of behavior ASSE_CORN is assigned to discrete elements of modeling DIS_TR with 2 confused nodes. This relation is called by the operators of resolution of nonlinear problems STAT_NON_LINE [R5.03.01] or DYNA_NON_LINE [R5.05.05].

The local axes of these elements X, there, Z are defined as on [Figure 2-a].

The integration of this relation of behaviour of the assemblies in the operator STAT_NON_LINE of Code_Aster require the formulation of the tangent operators K_o and K_n [bib3].

- 1) K_o is tangent rigidity at the beginning of the step of time, moments t .
- 2) K_n is tangent rigidity at the end of the step of time, moments $t+dt$.

The illustration of the operators K_o and K_n is given by [Figure 4-a].

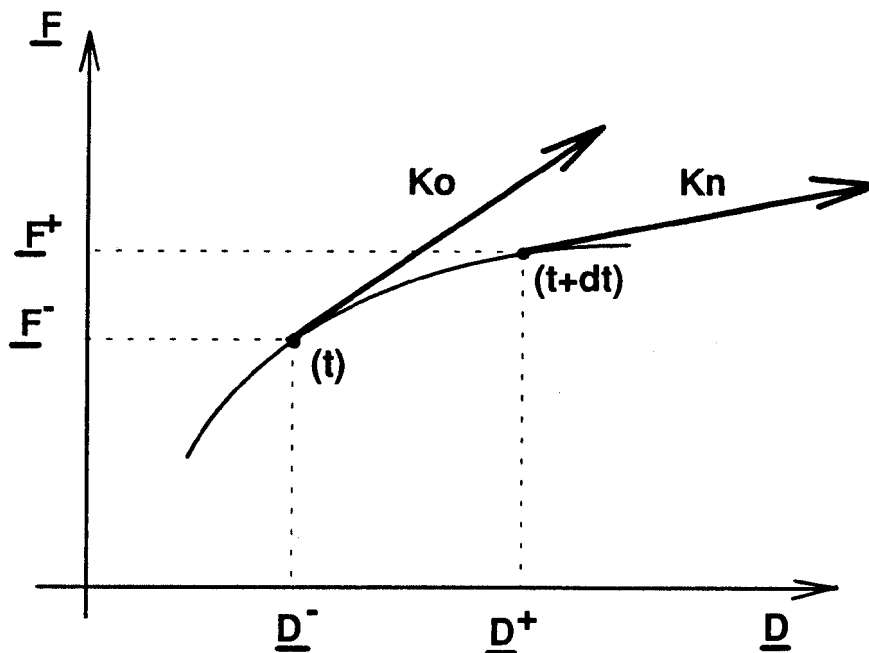


Figure 4-a: definition of the operators K_o and K_n

4.1 Formulation in sizes reduced in loading

4.1.1 Operator K_{nr}

We saw with [§3.2] that the relation of behavior is written:

$$f^+ = \frac{\Delta d}{\Delta p} R(p^+ + \Delta p)$$

$$\text{avec } \Delta p = \|\Delta \underline{d}\| = \sqrt{\Delta U_r^2 + \Delta \theta_r^2}$$

The operator K_{nr} is defined by:

$$K_{nr} = \left[\frac{\partial f_i}{\partial d_j} \right] \quad 1 \leq i, j \leq 2$$

He is written:

$$K_{nr} = \frac{\Delta p [Id] - [\Delta \underline{d}] \cdot \left\langle \frac{\partial \Delta p}{\partial \Delta d_j} \right\rangle^+}{\Delta p^2} R(p^+) + [\Delta \underline{d}] \cdot \left\langle \frac{\partial \Delta p}{\partial \Delta d_j} \right\rangle^+ \frac{R'(p^+)}{\Delta p}$$

Calculation gives then:

$$\left\langle \frac{\partial \Delta p}{\partial \Delta d_j} \right\rangle^+ = \left(\frac{\Delta U_r}{\Delta p} ; \frac{\Delta \theta_r}{\Delta p} \right)$$

$$[\Delta \underline{d}] \cdot \left\langle \frac{\partial \Delta p}{\partial \Delta d_j} \right\rangle^+ = \begin{bmatrix} \frac{\Delta U_r^2}{\Delta p} & \frac{\Delta U_r \Delta \theta_r}{\Delta p} \\ \frac{\Delta U_r \Delta \theta_r}{\Delta p} & \frac{\Delta \theta_r^2}{\Delta p} \end{bmatrix}$$

and with $\underline{a} = 1$ (only case currently treated), one a: $h(x) = \frac{1}{\bar{d}} \frac{x^2}{1-x}$

$$R(p) = h^{-1}(p) = \frac{1}{2} \left(-\bar{d} p + \sqrt{\bar{d}^2 p^2 + 4 \bar{d} p} \right)$$

$$R'(p) = \frac{1}{h'[R(p)]} = \frac{\bar{d} [1 - R(p)]^2}{R(p) [2 - R(p)]}$$

4.1.2 Operator K_{or}

For the elastoplastic behaviors, the operator K_o with $t=0$ is equal to the rigidity of the elastic structure. In our case, the tangent initial behavior is rigid. The operator K_{or} is defined then by the passage in the limit when p tends towards 0 of the operator K_{nr} . One obtains:

$$R'(p) = \lim_{p \rightarrow 0} \frac{R(p)}{p}$$

$$\text{d'où } K_{or} = \lim_{p \rightarrow 0} \frac{R(p)}{p} [Id]$$

However $R(p) < 1 \forall p$ and if one supposes that the user gives, for the first step of loading, of the values such as $\Delta p > 10^{-4}$, one can retain in practice:

$$K_{or_{t=0}} = \begin{bmatrix} 10^4 & 0 \\ 0 & 10^4 \end{bmatrix}$$

This value by default is modifiable (cf §5).
These remarks are illustrated by [Figure 4.1.2-a].

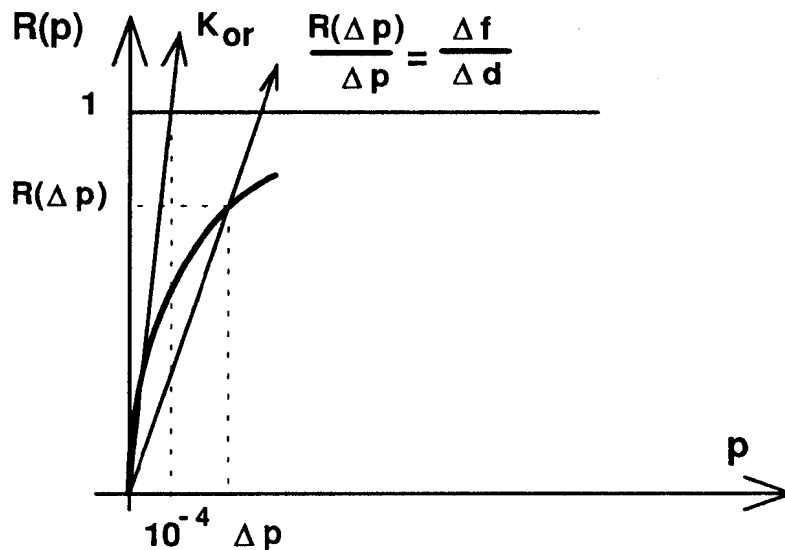


Figure 4.1.2-a: operator KB with $T = 0$

At the moment t running, the operator K_{or} is equal to the operator K_{nr} preceding step defined by [§4.1.1].

4.2 Formulation in sizes reduced in unloading

To avoid digital problems, one describes the behavior (rigid) in unloading by:

$$K_{or} = K_{nr} = K_{or_{t=0}}$$

4.3 Tangent operators K_n and K_o

The tangent operator K_n is written, starting from the tangent operator reduced calculated to the §4.1.

$$K_n = \left[\frac{\partial F_i}{\partial D_j} \right] \quad 1 \leq i, j \leq 6$$

with

$$\begin{aligned} \frac{\partial F_1}{\partial D_1} &= \frac{\partial N_x}{\partial U} = \frac{\partial n}{\partial U_r} \times \frac{\bar{N}}{\bar{U}} \\ \frac{\partial F_1}{\partial D_5} &= \frac{\partial N_x}{\partial \theta} = \frac{\partial n}{\partial \theta_r} \times \frac{\bar{N}}{\bar{\theta}} \\ \frac{\partial F_5}{\partial D_1} &= \frac{\partial M_y}{\partial U} = \frac{\partial m}{\partial U_r} \times \frac{\bar{M}}{\bar{U}} \\ \frac{\partial F_5}{\partial D_5} &= \frac{\partial M_y}{\partial \theta} = \frac{\partial m}{\partial \theta_r} \times \frac{\bar{M}}{\bar{\theta}} \\ \frac{\partial F_2}{\partial D_2} &= K_y \\ \frac{\partial F_3}{\partial D_3} &= K_z \\ \frac{\partial F_4}{\partial D_4} &= KR_x \\ \frac{\partial F_6}{\partial D_6} &= KR_z \end{aligned}$$

The other values are worthless.

The tangent operator K_o , with $t = 0$, are written:

$$K_o = \begin{bmatrix} 10^4 \frac{\bar{N}}{\bar{U}} & & & & & O \\ & K_y & & & & \\ & & K_z & & & \\ & & & K_{Rx} & & \\ O & & & & 10^4 \frac{\bar{M}}{\bar{\theta}} & \\ & & & & & K_{Rz} \end{bmatrix}$$

4.4 Digital processing of connection enters the mechanisms of the law of assembly

During the resolution of each step of loading by the iterative method of Newton, one must calculate with each iteration the tangent with the curve of balance force-displacement of the law of behavior. The problem is that connection between the mechanisms of the law of assembly, on the law of behavior, has a positive concavity (cf [Figure 2-b]) which makes convergence problematic when, during a step of loading, one passes from one mechanism to the other.

In the subroutine TE0041 who calculates, for each increment of load, the elementary matrix of tangential rigidity of a discrete finite element with 2 nodes having of the degrees of freedom in translation and in rotation, it proved to be necessary to converge, to calculate a secant stiffness directed of the initial state of worthless effort and displacement towards the state, at the end of the step of loading, constituted by the imposed effort and displacement corresponding on the curve of balance of the law of behavior. It was necessary for that, which was unusual on the level of this option, to know the internal number of iteration of the digital process calculating the step of loading, then to consider the effort imposed on the element at the end of this step.

Indeed, if one notes F^+ effort imposed on level of element (a priori unknown since only the assembled efforts are known), U^+ displacement corresponding on the curve of balance, and for the iteration i , respective values $U(i), F(i), K_s(i)$ displacement, effort and secant matrix – serving as tangent matrix – calculated at the end of the iteration, only as starter above mentioned subroutine is known U^i , and values at the beginning of the step of load $F(0)$ and $U(0)$, because one did not store the values with the preceding iteration $i-1$. In the expression of the residue calculated at the end of the iteration $i-1$: $F^+ - F(i-1) = K_s(i-1) \cdot (U(i) - U(i-1))$, one thus does not know any more but $U(i)$ with the iteration i , except in the typical case $i=1$ where one a:

$$F^+ - F(0) = K_s(0) \cdot (U(1) - U(0))$$

F^+ there is the only unknown value at the beginning and results from the others. One from of also deduced displacement U^+ at the end of the step according to the relation of balance:

$$p. [n, m] = R(p) \cdot \begin{bmatrix} \dot{U}_r \\ \dot{\Theta}_r \end{bmatrix}, \text{ from where secant stiffness } K_s(1) = F^+ / U^+.$$

The problem is that in this first iteration, displacement $U(1)$ imposed is different from final displacement to calculate U^+ in balance with F^+ from now on known (with the test of balance close to the step of preceding loading). Effort calculated at the end of this iteration $F(1)$ must thus be also different from F^+ and such as $F(1) = K_s(1) \cdot U(1)$ so that starting from the couple $U(1)$ and $F(1)$, one points with the secant $K_s(1)$ on the couple U^+ and F^+ . One thus obtains at the beginning of iteration 2 a displacement $U(2)$ very near to U^+ and one can then calculate by the relation of balance $F(2)$ very near also to F^+ as well as the secant stiffness $K_s(2) = F(2) / U(2)$.

If one converged exactly with the preceding step of load, 2 internal iterations are enough to converge exactly, if not one needs some additional iterations to satisfy the test with balance on the residue.

The method known as of "directed secant" is schematized on [Figure 4.3-a] where one has the following correspondences:

$$U^i = U(i) \\ K_t(U^i) = K_s(i)$$

for a law of behavior $LC(U^i) = F(i)$.

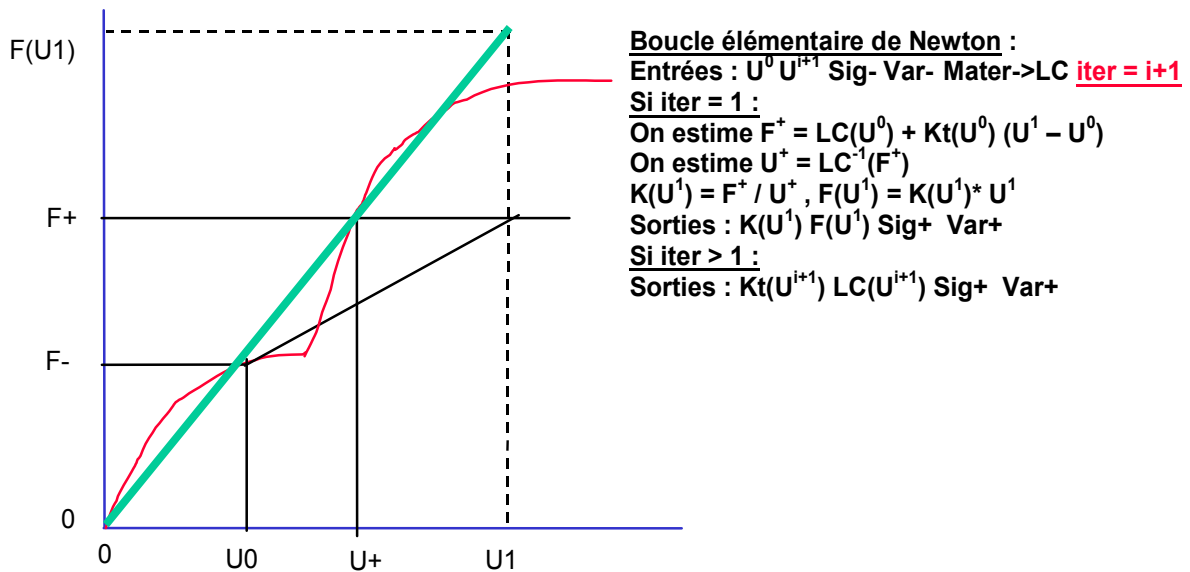


Figure 4.3-a: method of directed secant

One thus sees now why it was necessary in the option calculated by the above mentioned subroutine to know the internal number of iteration i in order to distinguish the typical case $i = 1$.
 Variables and parameters of the law of behavior

4.5 Variables of the law

The law of behavior comprises 7 internal variables per point of calculation:

- 1) $V1$ is displacement reduces equivalent p maximum reached out of mechanism 1,
- 2) $V2$ is displacement reduces equivalent p maximum reached out of mechanism 2,
- 3) $V3$ is an indicator which is worth 1 or 2 according to whether one is respectively on the limiting surface of mechanism 1 or 2, and 0 if one is under this limiting surface (after discharge for example),
- 4) $V4$ and $V5$ are respectively the maximum force and the moment reached out of mechanism 2 before discharge,
- 5) $V6$ and $V7$ are respectively displacement and the rotation origins of mechanism 1, which can be nonworthless when after discharge out of mechanism 2, the loading changes sign to pass by again out of mechanism 1.

4.6 parameters of the law

The parameters of the law of behavior entered like data under the keyword ASSE_CORN order DEFINI_MATERIAU [U4.43.01]:

- NU_1 : one enters behind this keyword the value of the parameter \bar{N}_1 mechanism 1,
- MU_1 : one enters behind this keyword the value of the parameter \bar{M}_1 mechanism 1,
- DXU_1 : one enters behind this keyword the value of the parameter \bar{U}_1 mechanism 1,
- DRYU_1 : one enters behind this keyword the value of the parameter $\bar{\theta}_1$ mechanism 1,
- C_1 : one enters behind this keyword the value common to the parameters \bar{n} and \bar{m} mechanism 1,

- NU_2 : one enters behind this keyword the value of the parameter \bar{N}_2 mechanism 2,
- MU_2 : one enters behind this keyword the value of the parameter \bar{M}_2 mechanism 2,
- DXU_2 : one enters behind this keyword the value of the parameter \bar{U}_2 mechanism 2,
- DRYU_2 : one enters behind this keyword the value of the parameter $\bar{\theta}_2$ mechanism 2,
- C_2 : one enters behind this keyword the value common to the parameters \bar{n} and \bar{m} mechanism 2,
- KY, KZ, KRX, KRZ take the values of the characteristics of linear behavior in the local directions “ respectively,
- RP_0 : one enters behind this keyword a possible value of K_{or} (10^4 by default).

Notice : The parameter a is not accessible to the user. He is fixed at the value $a=1$.

5 Bibliographical references

- 1) P. PENSERINI: “Modeling of the assemblies bolted in the webmasts” Notes EDF/R & D HM-77/93/287
- 2) P. PENSERINI: “Characterization and modeling of the behavior of the connections structure metal-foundation” Doctorate of the University Paris 6.1991
- 3) J.P. LEFEBVRE, P. MIALON: “Quasi-static nonlinear algorithm of Code_Aster” EDF note/R & D HI-75/7832

6 Description of the versions of the document

| Version Aster | Author (S) Organization (S) | Description of the modifications |
|---------------|---|----------------------------------|
| 6 | J.M.PROIX- R&D/AMA | Initial text |
| 8,4 | G. DEVESA, J.L. FLEJOU, P. PENSERINI EDF-R&D/AMA EDF/LME | |