Nonlocal modeling with gradient of deformation

Summary

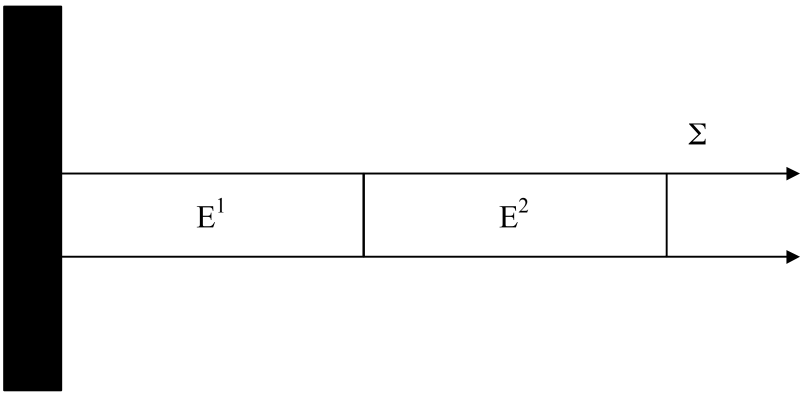
This document presents a model of delocalization of the laws of behavior per regularization of the deformation. It introduces an additional nodal variable: the regularized deformation, dependent on the local deformation by an equation of regularization of the least type square with penalization of the gradient than one solves simultaneously with the classical equilibrium equation. The regularized deformations are used for the calculation of the evolution of the internal variables (and not for the calculation of the constraints!). This method makes it possible to avoid certain problems involved in the digital processing of the local problems like the dependence the grid.

1 Nature of the formulation

In the presence of damage (softening), the local laws of behavior lead to a badly posed problem which numerically results in a localization of the deformations in a band thickness a mesh: with the limit, one breaks without dissipating energy.
There exist several extensions to the local models which make it possible to mitigate this problem of localization (relieving of the potential energy, enrichment of kinematics, theories with gradient, models nonlocal). This document deals with nonlocal model with gradient of deformations, modeling `*_GRAD_EPSI`, drifting of the model with gradient of equivalent deformation proposed by Peerlings and al. (1995). One introduces interactions between the material point and his space vicinity by regularizing the deformations thanks to an operator of delocalization. The regularized deformations are then used to evaluate the evolution of the internal variable.

It should be noted however that the constraints are calculated starting from the local deformations because the use of the deformations regularized in the calculation of the constraints would return to “too much regularizing” the problem, which would call into question the existence even of solutions. One is convinced some easily thanks to the following example:
Let us consider a bar made up of 2 different materials which have different Young moduli. One exerts on this bar a simple traction. The 2 elements being assembled in series, the constraint is equal in the two elements:

$$\sigma = E^1 \varepsilon^1 = E^2 \varepsilon^2 = \sum$$



On the interface between the two elements, the discontinuity of modulus Young thus imposes a discontinuity of the deformation. Let us consider now either the local deformation but a delocalized deformation. The classical operators of delocalization cause to return continues the deformation in the structure, which then generates obligatorily a discontinuity of constraint to the interface because of the difference of Young modulus, and this goes against the equilibrium equation.

The regularization of the deformations leads us to introduce a characteristic length definite by the operator `DEFI_MATERIAU` under the keyword factor `NOT ROOM` who conditions the width of the bands of localization. The scales thus are not defined any more by the digital processing of the problem but by a parameter material.

2 Limits of the local models

One initially proposes to illustrate the phenomenon of localization in the simple case of a bar subjected to a uniaxial traction.

One thus regards an assembly of identical elements assembled in series subjected to a traction as represented on [Figure 2-a].

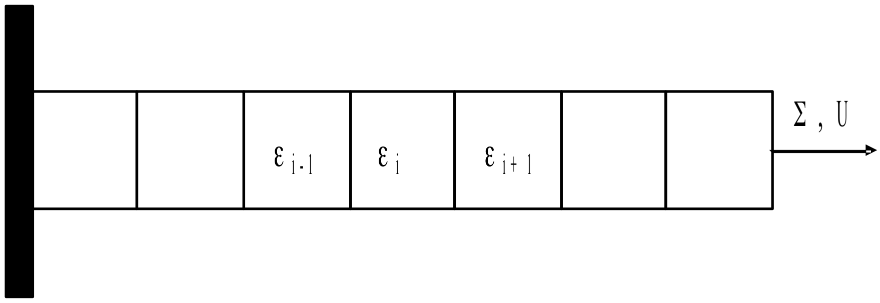


Figure 2-a: Assembly of identical elements assembled in series subjected to a tensile test

Each element obeys the same law of behavior of the elastic type endommageable with softening [Figure 2-b]. The state of material is described by two variables which are the deformation ε and the damage characterized by the scalar variable d . This variable is worth 0 when the material is healthy and grows up to 1 when it is completely damaged.
We will not enter here in detail of the equations governing such a behavior of material. Let us specify simply that these equations make it possible to describe the behavior of material completely. They indeed give us access to the constraints and the damage according to the rate of deformation, to see for example [R5.03.18].

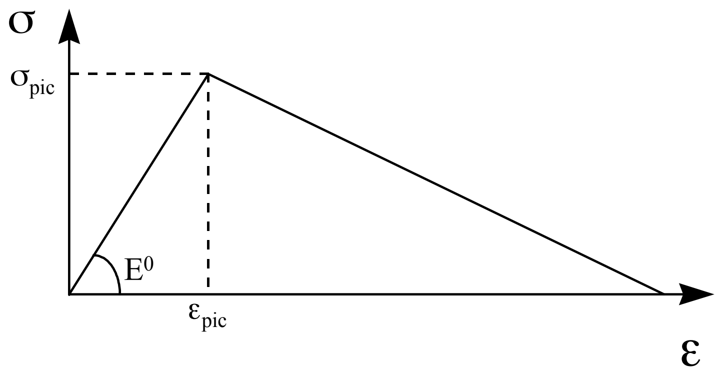


Figure 2-b: Law of behavior of material in uniaxial simple traction

The elements of the studied bar are assembled in series, which implies, because of the equilibrium equation of the structure, the equality of the constraint in all the elements:

$$\sigma_i = \Sigma$$

One can consequently study the total response of the assembly to a simple tensile test. This answer breaks up into two phases. In a first phase, the behavior of all the elements is elastic and the damage remains null. The answer of the structure thus exists and is single. The deformation is identical for all the elements and is worth:

$$\varepsilon_i = \frac{\Sigma}{E^0}$$

This phase continues as much as the peak of constraint is not reached. Microphone-heterogeneities of material imply light fluctuations of the field of elasticity between the various elements, which will involve the damage of a fastener before the others. The second phase starts when one of the elements which one notes A damage yourself. The constraint in the whole of the structure reached its maximum. While continuing traction, the constraint supported by the structure will decrease. The element A having passed the peak, it is in the lenitive phase of the behavior of material, which means that it will continue to damage itself during traction. The other elements did not reach the critical point, they thus simply will undergo an elastic discharge during the decrease of the constraint. This phase finishes when the element A is completely damaged. Finally, the damage as well as the deformation thus concentrated in only one element.

One then understands easily the digital consequences of the localization. The phenomenon describes previously on a simple sample will occur whatever the structure with a grid by finite elements. For reasons of stability, the localised solution tends to being selected. The damage and the deformation will concentrate in a band thickness an element and any refinement of the grid then will modify the total answer of the structure. One understands then well that it is impossible to describe the scale of the bands of localization, the length of the damaged band coming from the grid and not from a physical principle. Moreover, one gets a physically inadmissible result from an energy point of view. Indeed, the energy dissipated at the time of the damage will depend on the refinement of the grid, and one can even imagine the total rupture of a structure without energy expense if an extremely fine grid is considered.

3 Formulation with regularized deformations

3.1 Principle

One considers the state of material defined locally by the deformation ε and of the internal variables α . The data of the free potential energy $\varphi(\varepsilon, \alpha)$ allows to define the constraint σ .
In a general way, the law of behavior is given by the expression of the constraint and the law of evolution of the internal variables:

$$\sigma(\varepsilon, \alpha)$$
$$\dot{\alpha} = g(\dot{\varepsilon}, \varepsilon, \alpha)$$

The principle of the method of delocalization of the deformations is to use the deformations regularized in the law devolution of the internal variables:

$$\sigma(\varepsilon, \alpha)$$
$$\dot{\alpha} = g(\dot{\bar{\varepsilon}}, \bar{\varepsilon}, \alpha)$$

One thus understands the general information of the method which does not force to reconsider the integration of the law of behavior. It is indeed the same one as for the local model but while replacing ε by $\bar{\varepsilon}$. It is necessary nevertheless well to distinguish the calculation from the internal variables, which utilizes regularized deformations, of that of the constraints, which utilizes only the local deformations.

3.2 Choice of the operator of delocalization

The choice of the operator of regularization is purely arbitrary and is not based on any physical reasoning. One however may find it beneficial to choose an operator who is integrated easily and directly in `STAT_NON_LINE` by the finite element method. Thus, the use of an integral formulation, where the coupling between the finite elements on the level of the integration of the laws of behavior causes to increase considerably the bandwidth of the tangent matrix and to thus increase the number of operations to be carried out, is not judicious. The operator of regularization retained, proposed by Peerlings and al. (1995), employs a delocalization by least squares with penalization of the gradient:

$$\mathrm{R}(\varepsilon)=\min_{\bar{\varepsilon}}\int_{\Omega}\frac{1}{2}(\bar{\varepsilon}-\varepsilon)^2+\frac{1}{2}(L_c\nabla\bar{\varepsilon})^2 d\Omega$$

The term in gradient introduces the interaction between the material point and its vicinity and makes it possible to limit the strong concentration of gradient of deformations. To minimize such an integral amounts solving the following differential equation:

$$\bar{\varepsilon}-L_c^2\Delta\bar{\varepsilon}=\varepsilon$$

One sees appearing a major interest of the choice of this operator of regularization. The differential equation can be integrated classically by the finite element method, and this without introducing new not linearities. It is enough for that to introduce new nodal variables representing the generalized deformations.
There exists moreover a tangent matrix of reasonable bandwidth (compared to an integral formulation) but it should be noted that the tangent matrix is not symmetrical, as one will further see it by clarifying the tangent matrix.

Note:

The operator of delocalization was amended in the case of the laws of quasi-fragile damage (`ENDO_FRAGILE, MAZARS, ENDO_ISOT_BETON` and `ENDO_ORTH_BETON`), in order to try to solve the problem of inopportune widening of the band of damage when the material is ruined (cf [feeding-bottle 5]). An operator of relieving depending on the damage is introduced into the operator of regularization:

$$\bar{\varepsilon}=\mathbf{R}\,\varepsilon=\arg\min_{\bar{\varepsilon}}\int_{\Omega}\left[\frac{1}{2}\|\mathbf{A}^{relax}:(\bar{\varepsilon}-\varepsilon)\|^2+\frac{1}{2}\|L_c\vec{\nabla}\bar{\varepsilon}\|^2\right]d\Omega$$

where $\mathbf{A}^{relax}$ is a tensor of a nature 4 allowing to release the deformation regularized compared to the local deformation in the completely damaged directions. The tensor $\mathbf{A}^{relax}$ the identity is worth when the damage is not total. For the isotropic laws (`ENDO_FRAGILE, MAZARS, ENDO_ISOT_BETON`), $\mathbf{A}^{relax}$ becomes the null tensor of order 4 when the damage is total (one release in all the directions). In the case of the orthotropic law `ENDO_ORTH_BETON` , the tensor $\mathbf{A}^{relax}$ cancel yourself only in the completely damaged directions (cf [feeding-bottle 5]).

3.3 Variational formulation

In the model, two equations control the process of deformation, on the one hand the classical equilibrium equation and on the other hand the differential equation characterizing the regularization of the deformations. The integral formulation of our problem is the following one:

$$\forall v\in V^{ad} \qquad \int_{\Omega}(\nabla v)^C:\sigma\, d\Omega=\int_G v.T dG+\int_{\Omega} f\, dV$$

V^{ad} : space of acceptable displacements
T : forces imposed on the edge G

$$\forall \mathrm{e}\in[H^1(\Omega)]^6 \qquad \int_{\Omega}(e\,\bar{\varepsilon}+\nabla e.L_c^2\nabla\bar{\varepsilon})\,d\Omega=\int_{\Omega} e\,\varepsilon\, d\Omega$$

The limiting conditions for the generalized deformations are the natural conditions rising from the equation of regularization. They are of Neumann type:

$$\nabla \bar{\varepsilon} . n = 0$$

One indeed imposes no particular condition on the edge in the equation of regularization.

4 Discretization

4.1 Discretized equations

The equilibrium equation discretized between the external and interior forces is classical form (cf [R5.03.01]):

$$F_{\text{int}} + D^T \lambda = F_{ext}$$

with $F_{\text{int}} = \int_\Omega B^T \sigma \, d\Omega$ and $F_{ext} = \int_\Gamma N^T T \, d\Gamma$
(D^T : cf B^T of [R5.03.01])

where N are the functions of forms associated with the field with displacement and B the derivative of the functions of forms.

The differential equation on the regularized deformations is discretized in the same way:

$$K^{\varepsilon\varepsilon} \bar{\varepsilon} = F^{\varepsilon}$$

with $K^{\varepsilon\varepsilon} = \int_\Omega \left(\tilde{N}^T \tilde{N} + L_c^2 \tilde{B}^T \tilde{B} \right) d\Omega$ and $F^{\varepsilon} = \int_\Omega \tilde{N}^T \varepsilon \, d\Omega$

where $\tilde{N}$ are the functions of forms associated with the field with generalized deformations and $\tilde{B}$ the derivative of the functions of forms. It should be noted here that the functions of forms associated with the generalized deformations are different from the functions of forms associated with displacements.

The nodal residues associated with these two equations are the following ones:

$$F^u = F_{\text{int}} + D^T \lambda - F_{ext}$$
$$F^{\bar{\varepsilon}} = K^{\varepsilon\varepsilon} \bar{\varepsilon} - F^{\varepsilon}$$

The tangent matrix associated with the resolution of this system by the method of Newton is the following one:

$$K = \begin{pmatrix} \dfrac{\partial F^u}{\partial u} & \dfrac{\partial F^u}{\partial \bar{\varepsilon}} \\ \dfrac{\partial F^{\bar{\varepsilon}}}{\partial u} & \dfrac{\partial F^{\bar{\varepsilon}}}{\partial \bar{\varepsilon}} \end{pmatrix}$$

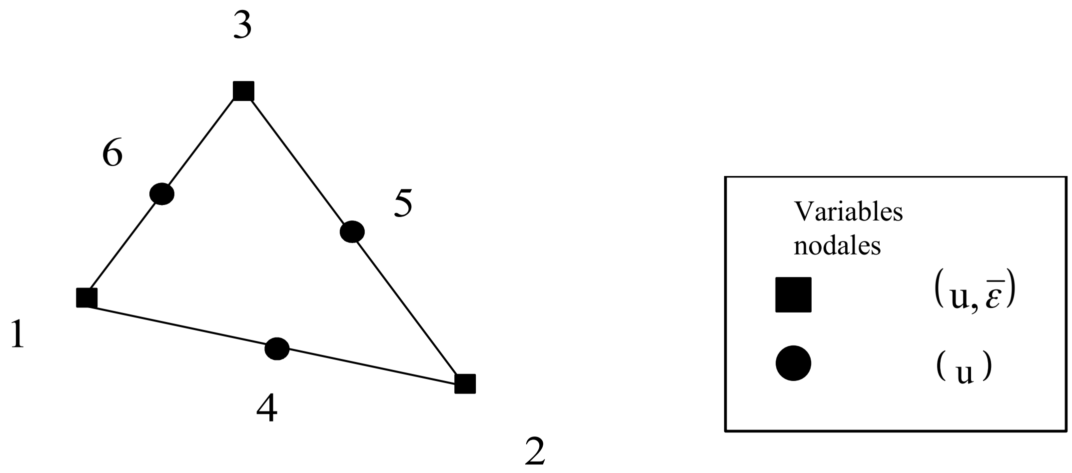
The various blocks of the tangent matrix are the following:

$$\frac{\partial F^u}{\partial u}\Big|_{i-1} = \int_\Omega B^T \frac{\partial \sigma}{\partial \varepsilon} B d\Omega$$
$$\frac{\partial F^u}{\partial \bar{\varepsilon}}\Big|_{i-1} = \int_\Omega B^T \frac{\partial \sigma}{\partial \bar{\varepsilon}} \tilde{N} d\Omega$$
$$\frac{\partial F^{\bar{\varepsilon}}}{\partial \bar{\varepsilon}}\Big|_{i-1} = \int_\Omega \left(\tilde{N}^T \tilde{N} + L_c^2 \tilde{B}^T \tilde{B} \right) d\Omega$$
$$\frac{\partial F^{\bar{\varepsilon}}}{\partial u}\Big|_{i-1} = \int_\Omega -\tilde{N}^T B d\Omega$$

It should be noted that the tangent matrix is not-symmetrical.

4.2 Choice of the finite elements

The introduction of new nodal variables forces to use new elements compatible with the new formulation. One is in the presence of two nodal unknown factors: displacements and regularized deformations. Deformation being the space derivative of a displacement, if functions of form are used P^2 for displacement, it is preferable to use functions of form P^1 for the deformations regularized for reasons of homogeneity. The quadratic elements, TRIA6 and QUAD8 for the 2D, TETRA10, PENTA15 and HEXA20 for the 3D, were developed. The components of displacement are assigned to all the nodes of the element whereas the components of the regularized deformations are affected only with the nodes tops. For more clearness, element TRIA6 is represented below:



4.3 Modelings available

These various elements are used in three types of modelings:

Calculation 2D in plane deformations:	`D_PLAN_GRAD_EPSI` (cf [U3.13.06])
Calculation 2D in plane constraints:	`C_PLAN_GRAD_EPSI` (cf [U3.13.06])
Calculation 3D:	`3D_GRAD_EPSI` (cf [U3.14.11])

The axisymmetric mode is not yet available.

5 Interface with the laws of behavior

The use of this method of delocalization requires the calculation of the following terms on the level of the law of behavior:

$$(\varepsilon, \bar{\varepsilon}) \Rightarrow \sigma, \alpha, \frac{\partial \sigma}{\partial \varepsilon}, \frac{\partial \sigma}{\partial \bar{\varepsilon}}$$

The last two terms are necessary only for the calculation of the tangent matrix.

6 Bibliography

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5) GODARD V.: Modeling of the anisotropic damage of the concrete with taking into account of the unilateral effect: Application to the simulation of the nuclear containment systems. Doctorate of the University Paris 6 (2005).

7 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
7	E.Lorentz EDFR & D /AMA	Initial text
8,4	V.Godard EDF-R&D/AMA	