Modeling of damping in dynamics linear

Summary:

The linear dynamic analyses of the structures subjected to imposed forces or movements require to add characteristics of mechanical cushioning to the characteristics of rigidity and mass of the model.

One has several classical modelings, applicable to all the types of finite elements available:

- the model of viscous damping,
- the model of damping hysteretic (known as also “structural damping”)
  for the harmonic analysis of viscoelastic materials.

For the analyses using the methods of dynamic response by modal recombination, with a modal base of real clean modes, it is possible to introduce modal damping coefficients.

Contents

1 Concept of mechanical cushioning

1.1 Models of damping

The movement of the structures subjected to forces or movements imposed, variable in the course of time, depends, in particular of the properties of damping, i.e. of the dissipation of energy in materials constitutive of the structure and the connections of the various elements of structure between them and with the surrounding medium.

The physical phenomena intervening in this dissipation of energy are many frictions, shocks, viscosity and plasticity, vibratory radiation with the supports.

The models of behavior representing these phenomena are often badly known and it is difficult to explicitly describe them at the elementary level. This is why the most used models are the simple models which make it possible to reproduce on a macroscopic scale the principal effects on the structures [bib1] [bib2]. Those currently available in *Code_Aster* are:

- viscous damping: dissipated energy proportional to the speed of the movement,
- damping hysteretic (known as also "structural damping"): dissipated energy proportional to displacement such as the force of damping of sign is opposed to that speed.

Let us note that the damping of Coulomb, which corresponds to a damping of friction for which dissipated energy is proportional by the strength of normal reaction to the direction of displacement requires to model the contact, which leaves the strictly linear framework. The nonlinear operators can take it into account in all its general information [R5.03.50 & R5.03, 52] while the transitory operator of resolution into modal can model the friction of Coulomb within the framework of specific contacts [R5.06.03].

The values of the parameters of these models are deduced, when they are available, from experimental results. At the stage of the design, one limits oneself to the use of guiding values.

1.2 General standards to characterize damping [bib1]

1.2.1 Loss ratio

The loss ratio η is an adimensional coefficient characteristic of the shock absorber effect defined as the report of the energy dissipated during a cycle in the maximum potential energy multiplied by 2π :

$$\eta=\frac{E_{d\,\text{par cycle}}}{2\pi\, E_{p\,\max}} \qquad \text{éq 1.2-1}$$

1.2.2 Reduced damping

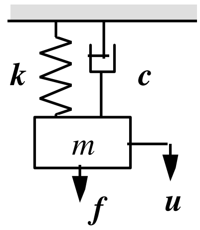
By definition reduced damping is equal to half of the loss ratio

$$\xi=\frac{\eta}{2} \qquad \text{éq 1.2-2}$$

2 Model of viscous damping

2.1 Physical definition of viscous damping

The classical cushioning devices (for example, by rolling of a viscous fluid through the openings of a piston pulled by the vibratory movement) deliver forces proportional to the speed of the movement and opposite sign. During a cycle, the work of these forces is positive: it is viscous damping.



For a simple oscillator of rigidity k , of mass m and of viscous damping c , the external force applied balance three components: elastic back pulling force ku , force of damping $c\dot{u}$ and inertial force $m\ddot{u}$ from where the dynamic equation moving absolute:

$$m\ddot{u}+c\dot{u}+ku=f \qquad \text{éq 2.1-1}$$

For this model of viscous damping the energy dissipated during a cycle of pulsation ω is proportional to the vibratory speed $-\omega u_0 \sin(\omega t)$ associated with displacement $u_0 \cos(\omega t)$:

$$E_{d\,\text{par cycle}}=\int_0^{2\pi} -c\,\omega\, u_0 \sin\omega t\, d\left(u_0\cos\omega t\right)=\pi\, c\,\omega\, u_0^2$$

and potential energy for a sinusoidal displacement $u_0 \cos\omega t$ is:

$$E_{p\max}=\int_{\pi/2}^{0} k u_0 \cos\omega t\, d\left(u_0 \cos\omega t\right)=\frac{1}{2} k u_0^2$$

For a cycle of pulsation ω and of sinusoidal displacement $u_0 \cos\omega t$, the loss ratio is proportional to the frequency of the movement:

$$\eta=\frac{c\,\omega}{k} \qquad \text{éq 2.1-2}$$

2.2 Harmonic oscillator with viscous damping

Classical analysis of the model not deadened associated with the equation [éq. 2.1-1], put in the form $\left(k-m\omega^2\right)u=0$ we gives $\omega_0=\sqrt{\frac{k}{m}}$ the own pulsation.

Critical damping from which the differential equation [éq 2.1-1] does not have any more an oscillating solution is given by the formulas $c_{critique}=2\sqrt{km}=2\mathrm{m}\,\omega_0=\frac{2\mathrm{k}}{\omega_0}$ what makes it possible to give a digital interpretation of the reduced damping, which is often expressed expressed as a percentage critical damping:

$$\xi=\frac{\eta}{2}=\frac{c}{c_{critique}}=\frac{c}{2\mathrm{m}\,\omega_0} \qquad \text{éq 2.1-3}$$

2.2.1 Answer to releasing excitation

From a static deformation $u_{st}=\frac{f_0}{k}$, to release (release of the system) produced a free oscillatory movement $u_l(t)=u_0 e^{-\xi\omega_0 t}\cos\omega_0^{'} t$ who reveals the own pulsation of the deadened system $\omega'_0=\omega_0\sqrt{(1-\xi^2)}$.

In the course of time, the extreme amplitude (u_1, u_2) decrease at each period of $e^{-\xi\omega_0 T}=e^{-2\pi\xi}=e^{-\delta}$ where δ is the decrement logarithmic curve: $\delta=2\pi\xi$

2.2.2 Answer to a harmonic excitation

The answer to a harmonic excitation of the form $f(t)=f_0 e^{j\omega t}$ is written with a forced answer permanent particular solution $u(t)=u_0 e^{(j\omega t-\varphi)}$ who is written with the reduced pulsation $\lambda=\frac{\omega}{\omega_0}$ $\frac{ku_0}{f_0}=\frac{1}{1-\lambda^2+j2\xi\lambda}=H_v(j\omega)$ where $H_v(j\omega)$ is the complex transfer transfer function of a simple oscillator with viscous damping.

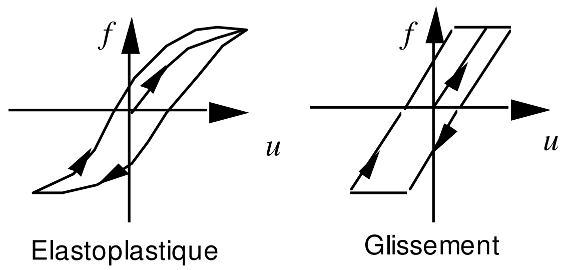
The module of the answer $\frac{u_0}{u_{st}}=\frac{ku_0}{f_0}=|H_v(j\omega)|=\frac{1}{\sqrt{(1-\lambda^2)^2+(2\xi\lambda)^2}}$ fact of appearing a dynamic amplification compared to the static answer u_{st} .

This amplification is maximum for $\lambda=\frac{\omega_0^{'}}{\omega_0}=\sqrt{(1-\xi^2)}$ and the value of maximum displacement gives $\frac{u_{0\max}}{u_{st}}=\frac{1}{2\xi\sqrt{(1-\xi^2)}}$. If vibratory speed is observed $\dot{u}(t)=j\omega u(t)$, the amplification vibratory speed is maximum for $\lambda=\frac{\omega_0}{\omega_0}=1$ and the maximum amplitude speed is $\dot{u}_{0\max}=\frac{1}{2\xi}=Q$, where Q is the mechanical analogy of the factor of overpressure of the electricians. These properties are at the origin of the methods of measurement of the characteristics of damping of the mechanical structures.

3 Model of damping hysteretic

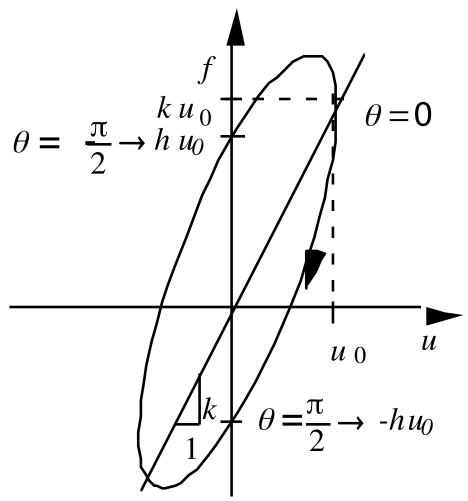
3.1 Physical definition of damping hysteretic

For a sinewave excitation applied to an elastoplastic structure or an elastic structure with friction, curved force-displacement reveals a positive work of the external force which corresponds to an energy dissipated in the structure, that one can at first approximation represent like below:



In both cases the loss ratio believes, in general with the amplitude of the cycle. For low values of the loss ratio (< 0.2), the form of the cycle does not have an appreciable effect on the movement and one can compare it to an ellipse [bib1].

In the typical case of a relation force-displacement whose cycle is of form elliptic, the expression of the loss ratio is simple. For a force applied F and a displacement $u=u_0\cos q$ the back pulling force is $ku_0\cos\theta$, k being the "classical" stiffness of the mechanical system, and damping forces it $-hu_0\sin\theta$, h being stiffness out of phase of $90°$, which leads to the relation of balance $F=ku_0\cos\theta-hu_0\sin\theta$.



Energies, dissipated during a maximum cycle and potential, are:

$$E_{d\,\text{par cycle}}=\int_0^{2\pi}-hu_0\sin\theta\, d\left(u_0\cos\theta\right)=\pi\, hu_0^2 \quad\text{and}\quad E_{pmax}=\int_{\pi/2}^{0}ku_0\cos\theta\, d\left(u_0\cos\theta\right)=\frac{ku_0^2}{2}$$

from where the loss ratio

$$\eta=\frac{\pi\, hu_0^2}{2\pi\frac{ku_0^2}{2}}=\frac{h}{k} \qquad \text{éq 3.1-1}$$

For a sinusoidal cycle $\theta=\omega t$, The damping coefficient hysteretic $\eta=\frac{h}{k}$ is independent of ω . It can be given starting from a test under harmonic cyclic loading.

3.2 Harmonic oscillator with damping hysteretic

The model of damping hysteretic is usable to treat the harmonic answers of structures with viscoelastic materials.

The energy dissipated by cycle in the form $E_{d\,\text{par cycle}}=\int_0^{2\pi}\sigma\, d\varepsilon$ allows to highlight a complex Young modulus E^* starting from the relation stress-strain of a viscoelastic material $\sigma=\sigma_0 e^{j\omega t}$ and $\varepsilon=\varepsilon_0 e^{j(\omega t-\varphi)}$ where σ_0 and ε_0 are the amplitudes and φ the phase:

$$E^*=\frac{\sigma}{\varepsilon}=\left(\frac{\sigma_0}{\varepsilon_0}\right)e^{j\varphi}=\left(\frac{\sigma_0}{\varepsilon_0}\right)(\cos\varphi+j\sin\varphi)$$

While noting $E_1=\left(\frac{\sigma_0}{\varepsilon_0}\right)\cos\varphi$ the real part and $E_2=\left(\frac{\sigma_0}{\varepsilon_0}\right)\sin\varphi$ the imaginary part one obtains $E^*=E_1+jE_2=E_1(1+j\eta)\,\text{avec}\,\eta=\frac{E_1}{E_2}=\text{tg}\,\varphi$, where j is also called loss angle.

The classical analysis of the equation [éq 2.1-1] has direction, with a model of damping hysteretic, only for one harmonic excitation $f(t)=f_0 e^{j\omega t}$ who leads to the equation

$$m\ddot{u}+k(1+j\eta)u=m\ddot{u}+(k+jh)u=f_0 e^{j\omega t} \qquad \text{éq 3.2-1}$$

where the real part of displacement u represent the displacement of the mass and $h=k\eta$.

As previously cf [§ 2.2], the harmonic answer can be written, with the reduced pulsation $\lambda=\frac{\omega}{\omega_0}$, in the form $\frac{ku_0}{f_0}=\frac{1}{1-\lambda^2+j\eta}=H_h(j\omega)$ where $H_h(j\omega)$ is the complex transfer transfer function of a simple oscillator with damping hysteretic.

The module of the answer $\frac{u_0}{u_{st}}=\frac{ku_0}{f_0}=|H_h(j\omega)|=\frac{1}{\sqrt{(1-\lambda^2)^2+\eta^2}}$ fact of appearing a dynamic amplification compared to the static answer, amplification which is maximum for $\lambda=1$ and the value of maximum displacement gives $\frac{u_{0\max}}{u_{st}}=\frac{1}{\eta}=\frac{1}{2\xi}$.

In conclusion, damping reduces associated with damping hysteretic is:

$$\xi=\frac{\eta}{2}=\frac{h}{2\text{k}}=\frac{h}{2\text{m}\,\omega_0^2} \qquad \text{éq 3.2-2}$$

4 Other models of damping

One does not treat models here representing the damping "added" by the confined motionless fluids or the fluids moving. One will refer to the documents [R4.07.xx], treating fluid coupling - structure.

5 Analysis of structure with damping

Modelings presented are not easily generalizable with the various analyses of structures cf [§1].

Note:

Two modelings do not have the same field of linear analysis:

- *viscous damping is usable in transitory or harmonic analysis,*
- *damping hysteretic is usable only in harmonic analysis.*

Options of modelings in *Code_Aster* allow the definition:

- of a total damping for the structure,
- the depreciation located on meshs or groups of meshs.

5.1 Total damping of the structure

In the absence of sufficient information on the components and connections creating a dissipation of energy, a current modeling consists in building a matrix of "total" damping.

5.1.1 Viscous damping proportional "total"

One places oneself within the framework of the classical equations of the dynamics of the linear structures:

$$\boldsymbol{M}\,\ddot{\boldsymbol{U}}+\boldsymbol{C}\,\dot{\boldsymbol{U}}+\boldsymbol{KU}=\boldsymbol{F}(t) \qquad \text{éq 5.1.1-1}$$

The concept of damping of RAYLEIGH makes it possible to define the matrix of damping C like linear combination of the matrices of rigidity and mass:

$$\boldsymbol{C}=\alpha\,\boldsymbol{K}+\beta\,\boldsymbol{M} \qquad \text{éq 5.1.1-2}$$

Advantages:

- easy to implement by using the operators `DEFI_MATERIAU` [U4.43.01 §3.1] and `ASSEMBLY (OPTION=' AMOR_MECA')` [U4.61.21]. One can also use the operator `COMB_MATR_ASSE` [U4.53.01], after having assembled the matrices of rigidity and mass with real coefficients;
- useful for the validation of algorithms of resolution;
- historically, its success is attached to the methods of analysis transient by modal recombination starting from a base of real clean modes.
  Properties of orthogonality of the real clean modes solutions of the problem to the eigenvalues $\left(\boldsymbol{K}-\omega^2\boldsymbol{M}\right)\varphi=0$ result in the simultaneous diagonalisation in the passage into modal coordinates generalized of $\varphi^T\boldsymbol{K}\,\varphi$ and $\varphi^T\boldsymbol{M}\,\varphi$.
  The damping of RAYLEIGH is a condition sufficient for diagonaliser $\varphi^T\boldsymbol{C}\,\varphi$.
  The system of modal equations $\ddot{\boldsymbol{q}}+\frac{\varphi^T\boldsymbol{C}\,\varphi}{\varphi^T\boldsymbol{M}\,\varphi}\dot{\boldsymbol{q}}+\omega^2\boldsymbol{q}=\frac{\varphi^T}{\varphi^T\boldsymbol{M}\,\varphi}\boldsymbol{F}(t)$ becomes diagonal then.

$$\ddot{\boldsymbol{q}}+2\,\xi\,\omega\,\dot{\boldsymbol{q}}+\omega^2 q=\frac{\varphi^T}{\varphi^T\boldsymbol{M}\,\varphi}\boldsymbol{F}(t) \qquad \text{éq 5.1.1-3}$$

Disadvantages:

- This modeling does not make it possible to represent the heterogeneity of the structure compared to damping.
- The damping actually introduced into the model strongly depends on the identification of the coefficients α and β Cf [§ 5.1.2].

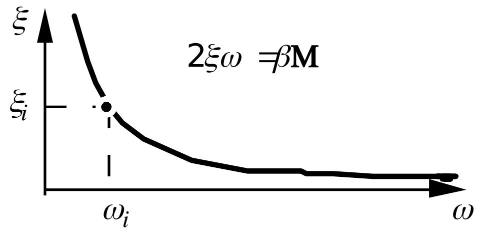
5.1.2 Influence of the damping coefficients proportional

Three simple cases of identification are presented here to illustrate, the effects induced by this modeling:

- damping proportional to the characteristics of inertia: $\alpha=0,\beta$

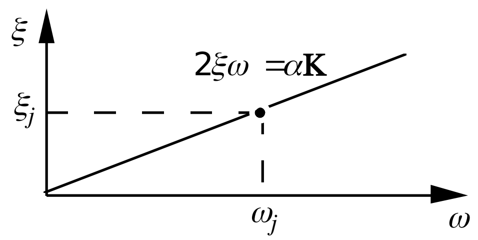
  This case was very much used of direct transitory resolution: if the matrix of mass is diagonal, that of damping is still and the saving space memory is obvious in it.

  The coefficient β can be identified with experimental reduced damping ξ_1 clean mode (φ_1,ω_1) who takes part more in the answer cf [éq. 2.1-1] from where $\beta=2\xi_1\omega_1$. For any other pulsation one obtains a reduced modal damping $\xi=\beta\dfrac{\omega_1}{\omega}$. High modes $\omega>>\omega_i$ will be deadened very little and the low frequency modes $\omega<\omega_1$ too much deadened.



- damping proportional to the characteristics of rigidity: α,β .

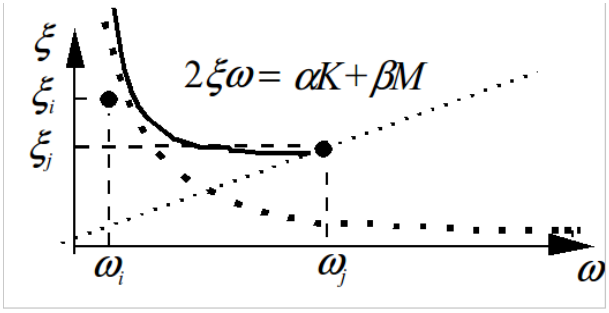
  The coefficient α can be identified, like previously from ξ_2 associated with the mode (φ_2,ω_2) from where $\alpha=2\xi_2\omega_2$. For any other pulsation one obtains a reduced modal damping $\xi=\alpha\dfrac{\omega}{\omega_2}$. High modes $\omega>>\omega_2$ are very deadened.



- damping proportional complete: $\alpha=\alpha,\beta=\beta$

  From an identification on two independent modes (φ_1,ω_1) and (φ_2,ω_2), one obtains for any other pulsation a reduced modal damping $\xi=\dfrac{1}{2}\left(\alpha\omega+\dfrac{\beta}{\omega}\right)=\xi_1\dfrac{\omega}{\omega_1}+\xi_2\dfrac{\omega_2}{\omega}$.

  In the interval $[\omega_1,\omega_2]$, the variation of reduced damping is weak and outwards one finds the addition of the preceding disadvantages: the modes external with the interval are deadened too much.



In none the preceding cases, one will be able to reproduce an assumption of equal modal damping for all the modes. Methods were imagined for tending towards this objective [bib1].

5.1.3 "Total" damping hysteretic

The generalization of the equation of the simple oscillator with damping hysteretic leads to the system of complex equations or $\boldsymbol{F}(\Omega)$ is a harmonic excitation.

$$\boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{K}(1 + j\eta)\boldsymbol{U} = \boldsymbol{F}(\Omega) \quad \text{Éq 5.1.3-1}$$

Knowing the matrix of real rigidity, it is possible to build a hysteretic matrix of damping $\boldsymbol{K}_h = j\eta\boldsymbol{K}$, with a "total" loss ratio η .

As previously of resolution by modal recombination, starting from a base of real clean modes, one obtains $\varphi^T\boldsymbol{M}\varphi\ddot{\boldsymbol{q}} + j\varphi^T\boldsymbol{K}_h\varphi\boldsymbol{q} + \varphi^T\boldsymbol{K}\varphi\boldsymbol{q} = \varphi^T F(t)$ where the hysteretic matrix of damping generalized is diagonal $\varphi^T\boldsymbol{K}_h\varphi = [\mathrm{diag}\,\eta\gamma_i]$, as the matrix of generalized rigidity $\varphi^T\boldsymbol{K}\varphi = [\mathrm{diag}\,\gamma_i]$.

According to the definition of reduced damping (cf [éq 1.2-2]), modal damping is constant for all the modes from where $\xi = \frac{\eta}{2}$

Advantages:

- easy to implement by using the operators DEFI_MATERIAU [U4.43.01 §3.1] and ASSEMBLY (OPTION=' RIGI_MECA_HYST') [U4.61.21]. One can also proceed by using the operator COMB_MATR_ASSE [U4.53.01], after having assembled the matrix of rigidity;
- very useful for the validation of algorithms of resolution;
- the damping actually introduced into the model is constant for all the modes of the structure, as asks it regulations of construction.

Disadvantages:

- this modeling is badly adapted for the industrial studies, because it does not make it possible to represent the heterogeneity of the structure compared to damping;
- only the harmonic analysis (in complex) is possible.

5.2 Damping localised

For the analyses requiring a modeling representing the heterogeneity of the structure, it is possible to affect characteristics of damping located on the meshs of the structure, in fact on elements of the model.

5.2.1 Elements shock absorbers

It is possible to apply elements discrete shock absorbers:

- on meshs POI1 : damping is related to the displacement (respectively speed) of the node support,
- on meshs SEG2 : damping is related to the relative displacement (respectively relative speed) of the two nodes connected.

The operator AFFE_CARA_ELEM [U4.24.01] allows to define for each discrete element:

- a matrix of damping of the viscous type $\boldsymbol{a}_{discret}$ whose terms are assigned to the various degrees of freedom of the nodes concerned; several modes of description of the matrix are available.
- a hysteretic loss ratio $\boldsymbol{\eta}_{discret}$ multiplier of the matrix of rigidity of the affected discrete element to the mesh support.

5.2.2 Affected damping with any type of finite element

The affected elastic material with any finite element can be defined with parameters of damping by the operator DEFI_MATERIAU [U4.23.01]:

- Viscous damping proportional with two parameters of RAYLEIGH α and β .

  AMOR_ALPHA = α

  AMOR_BETA = β

  For all the types of finite elements (of continuous, structural or discrete mediums), it is possible to calculate the real elementary matrices corresponding to the option of calculation 'AMOR_MECA', after having calculated the elementary matrices corresponding to the options of calculation 'RIGI_MECA' and 'MASS_MECA'.

  The elementary matrix of the element i affected of material α_j, β_j is then of the form:

  - for a finite element

  $$\boldsymbol{c}_{\mathbf{elem}\,i}=\alpha_j \boldsymbol{k}_{\mathbf{elem}\,i}+\beta_j \boldsymbol{m}_{\mathbf{elem}\,i}$$

  - for a discrete element

  $$\boldsymbol{c}_{\mathbf{elem}\,i}=\boldsymbol{a}_{\mathbf{discret}\,i}$$

- Damping hysteretic with a coefficient of $e\eta$

  AMOR_HYST = coeff

  For all the types of finite elements (of continuous, structural or discrete mediums), it is possible to calculate the complex elementary matrices corresponding to the option of calculation 'RIGI_MECA_HYST', after having calculated the elementary matrices corresponding to the options of calculation 'RIGI_MECA'.

  The elementary matrix of the element i affected of material α_j, β_j is then of the form:

  - for a finite element

  $$\boldsymbol{k}^*_{\mathbf{elem}\,i}=\boldsymbol{k}_{\mathbf{elem}\,i}\left(1+j\,\eta_j\right)$$

  - for a discrete element

  $$\boldsymbol{k}^*_{\mathbf{elem}\,i}=\boldsymbol{k}_{\mathbf{elem}\,i}\left(1+j\,\eta_{\mathrm{discret}\,i}\right)$$

5.2.3 Construction of the matrix of damping

The assembly of the elementary matrices of damping is obtained with the operator ASSE_MATRICE usual [U4.42.02] or by the macro order ASSEMBLY [U4.31.02]. One must use same classifications and the same mode of storage as for the matrices of rigidity and mass (operator NUME_DDL [U4.42.01]).

Note:

The matrix of damping obtained is nonproportional

$\boldsymbol{C} \neq \alpha \boldsymbol{K} + \beta \boldsymbol{M}$ or $\boldsymbol{K_h} \neq j \eta \boldsymbol{K}$

6 Use of the matrix of damping

6.1 Use of the matrix of viscous damping

6.1.1 Direct linear dynamic analysis

The matrix of viscous damping $\boldsymbol{C}$, whatever its mode of development and its character proportional or not proportional, is usable for the direct linear dynamic analysis (keyword MATR_AMOR) with the operators:

- of transitory analysis DYNA_LINE_TRAN [R5.05.02] and [U4.54.01]
- of harmonic analysis DYNA_LINE_HARM [R5.05.03] and [U4.54.02]

6.1.2 Dynamic analysis by modal recombination

For the analyses by modal recombination, one must project this matrix in the subspace defined by a unit φ real clean modes, obtained on the associated problem not deadened $\left(\boldsymbol{K}-\omega^2 \boldsymbol{M}\right)\varphi=0$.

This operation is possible with the macro order PROJ_BASE [U4.55.11] or with the operator PROJ_MATR_BASE [U4.55.01].

For the calculation of the dynamic response in force or imposed in modal space, one has following possibilities:

- use of the matrix of damping generalized $\boldsymbol{\varphi}^T \boldsymbol{C} \boldsymbol{\varphi}$:
  - in transitory analysis with the operator DYNA_TRAN_MODAL [R5.06.04] and [U4.54.03] and the keyword AMOR_GENE,
  - in seismic analysis by spectral method with the operator COMB_SISM_MODAL [R4.05.03] and [U4.54.04] and the keyword AMOR_GENE,
  - in harmonic analysis with the operator DYNA_LINE_HARM [R5.05.03] and [U4.54.02] and the keyword MATR_AMOR.

  Let us recall that in the case of heterogeneous damping (localised use of the options of damping), the matrix $\boldsymbol{\varphi}^T \boldsymbol{C} \boldsymbol{\varphi}$ is not diagonal.
- use of viscous modal damping by providing a straight line modal depreciation reduced for all the modes ξ or a list of values ξ_i .

  Several methods of identification of these coefficients are possible but there does not exist ordering of sytematic construction of the list of values. One can nevertheless quote the use of the assumption of BASILE $\left(2\xi_i\omega_i=\mathrm{diag}\dfrac{\boldsymbol{\varphi}^T \boldsymbol{C} \boldsymbol{\varphi}}{\boldsymbol{\varphi}^T \boldsymbol{M} \boldsymbol{\varphi}}\right)$, the use regulation RCC-G (or ETC-C) for the seismic analysis with damping of the ground, exploitation of experimental results, …
  - in transitory analysis with the operator DYNA_TRAN_MODAL [R5.06.04] [U4.54.03] and the keyword AMOR_REDUIT.

- in seismic analysis by spectral method with the operator COMB_SISM_MODAL [R4.05.03] [U4.54.04] and keywords AMOR or LIST_AMOR. An evolution is required to generalize the keyword AMOR_REDUIT.
- in harmonic analysis one **request for evolution** with the operator DYNA_LINE_HARM [R5.05.03] [U4.54.02] is deposited. It is not treated in version 3.6.

For the analyses by dynamic under-structuring, with the use of a modal base (RITZ bases) one will refer to [R4.06.03] and [R4.06.04].

6.2 Use of the complex matrix of rigidity

The complex matrix of rigidity $\boldsymbol{K}^{*}=\boldsymbol{K}+\boldsymbol{K}_{h}$, where $\boldsymbol{K}_{h}$ is an imaginary matrix (within the meaning of the complexes!), is usable for the direct harmonic analysis with the operator DYNA_LINE_HARM [R5.05.03] and [U4.54.02] and the keyword MATR_RIGI.

For the analyses by modal recombination, no functionality is currently available for the hysteretic use of the model of damping.

6.3 Complex modal analysis

The matrix of viscous damping $\boldsymbol{C}$ is essential for the modal analysis complexes with the operator whom deals with the quadratic problem with the eigenvalues [R5.01.02]: CALC_MODES [U4.52.02].

Let us recall that the complex clean modes allow a better adapted approach under investigation dynamic of the strongly deadened structures (reduced damping ξ > 20%). To date no tool of dynamic response by modal recombination using a base of complex clean modes is available in *Code_Aster*.

7 Bibliography

1) "Damping": Francoise GANTENBEIN and Michel LIVOLANT (CEA-DMT) in "Génie Parasismique" Work collective - Presses of the E.N.P.C. (1985)

2) "Damping in the structural analyses": Forum IPSI - Φ^2 ACE Volume XVIII N°2 (June 1994)

8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
3	*J.R. LEVESQUE (EDF-R&D/IMA/MMN)*	Initial text