

Modeling of the shocks and friction in transitory analysis by modal recombination

Summary:

This document describes the physical laws of contact with friction between structures and the modeling which is made by it in the transitory algorithm of analysis by modal recombination of *Code_Aster* DYNA_TRAN_MODAL [U4.54.03]. For the various linear connections not - of contact usable, one details the calculation of the sizes defining the conditions of contact.

The diagrams of used use are described in [R5.06.04].

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1 Introduction

The problems of shock with friction which interest EDF relate to for example the modeling of the tubular vibrations of structures maintained by supports with games, or separated by games weak and thus being able to make contact. The tubes of the steam generators, the pencils of the control rods, the assemblies of fuel are examples of structures which one wishes to model the vibrations.

The major consequence of the vibrations in the presence of game is to cause shocks as well as friction between the structure and its supports or the structures from where risks of wear. This document describes the type of non-linearities introduced by the presence of these games, as well as modeling used to take them into account in the algorithm of modal recombination.

2 Relations of contact between two structures

Two relations govern the contact between two structures:

- the relation of unilateral contact which expresses the non-interpenetrability between the solid bodies,
- the relation of friction which governs the variation of the tangential stresses in the contact. One will retain for these developments a simple relation: the law of friction of Coulomb.

2.1 Relation of unilateral contact

Are two structures Ω_1 and Ω_2 . One notes $d_N^{1/2}$ the normal distance enters the structures, $F_N^{1/2}$ the force of normal reaction of Ω_1 on Ω_2 .

The law of the action and the reaction imposes:

$$F_N^{2/1} = -F_N^{1/2} \quad \text{éq 2.1-1}$$

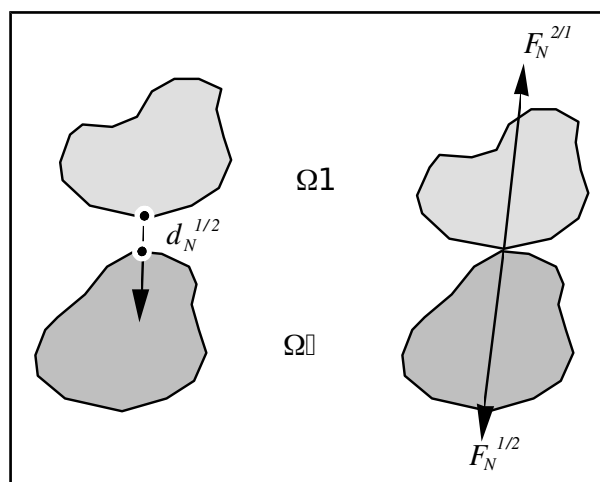


Figure 2.1-a: Normal distance and normal reaction

The conditions of unilateral contact, still called conditions of Signorini [bib5], are expressed in the following way:

$$d_N^{1/2} \geq 0 \quad , \quad F_N^{1/2} \geq 0 \quad , \quad d_N^{1/2} \cdot F_N^{1/2} = 0 \quad \text{et} \quad F_N^{1/2} = -F_N^{1/2} \quad \text{éq 2.1-2}$$

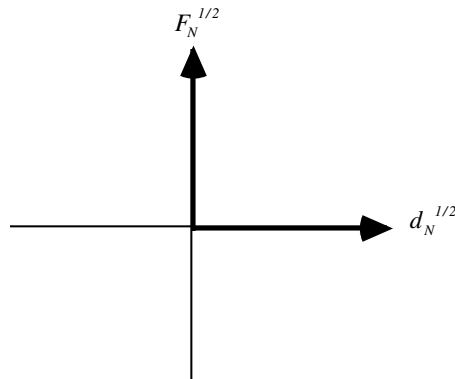


Figure 2.1-b: Graph of the relation of unilateral contact

This graph translates a relation force-displacement which is not differentiable. It is thus not usable in a simple way in a dynamic calculation algorithm.

If one restricts the study with the case of a tubular structure in the presence of an indeformable support, one notes d_n ($d_n = d_N^{1/2}$) the normal distance to the support, and F_n reaction of this last (attention! $F_n = F_N^{2/1} = -F_N^{1/2}$ to see diagram below).

The expression of the conditions of normal contact, expressing the limitation of displacements due to the support is worth:

$$d_n \geq 0 \quad , \quad F_n \leq 0 \quad , \quad d_n \cdot F_n = 0$$

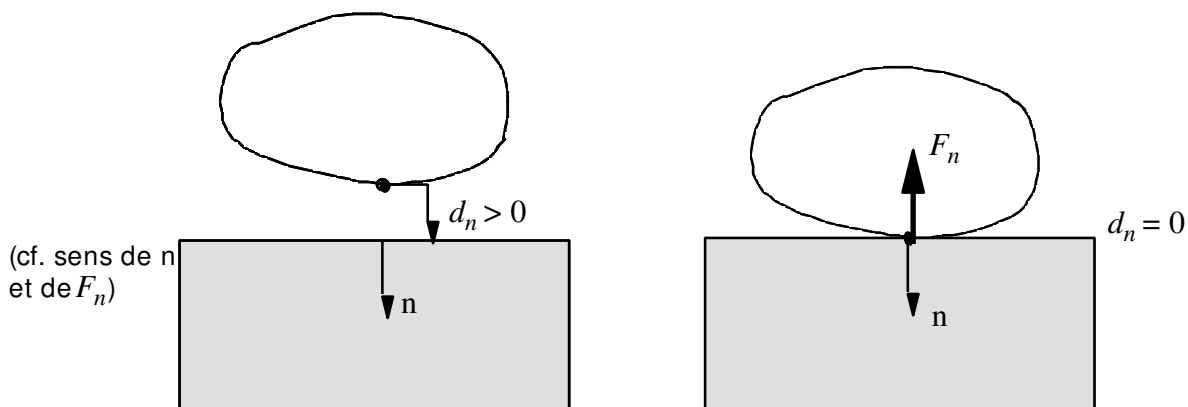


Figure 2.1-c: Normal distance and normal reaction enter a structure and a support

2.2 Law of friction of Coulomb

The law of Coulomb expresses a tangential limitation of effort $\mathbf{F}_T^{1/2}$ of tangential reaction of Ω_1 on Ω_2 . That is to say $\dot{\mathbf{u}}_T^{1/2}$ the relative speed of Ω_1 compared to Ω_2 in a point of contact and is μ the coefficient of friction of Coulomb, one has [bib5]:

$$s = \|\mathbf{F}_T^{1/2}\| - \mu \cdot F_N^{1/2} \leq 0, \quad \exists \lambda \dot{\mathbf{u}}_T^{1/2} = \lambda \mathbf{F}_T^{1/2}, \quad \lambda \leq 0, \quad \lambda \cdot s = 0 \quad \text{éq 2.2-1}$$

and the law of the action and the reaction:

$$\mathbf{F}_T^{2/1} = -\mathbf{F}_T^{1/2} \quad \text{éq 2.2-2}$$

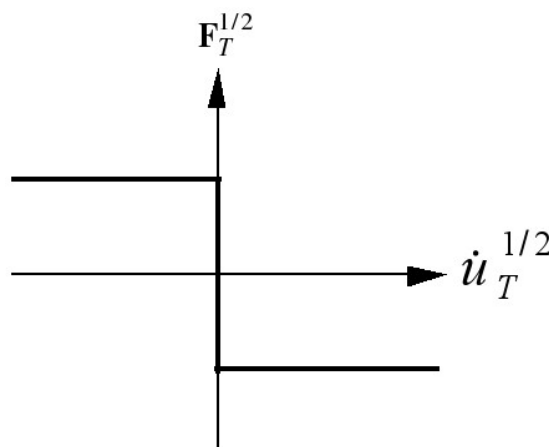


Figure 2.2-a: Graph of the law of friction of Coulomb

The graph of the law of Coulomb is also nondifferentiable and is thus not simple to use in a dynamic algorithm.

If one restricts the study with the case of a tubular structure in the presence of an indeformable support, only tangential stress $\mathbf{F}_T^{2/1} = \mathbf{F}_T$ is used, the law of friction expresses itself in the following way:

$$s = \|\mathbf{F}_T\| - \mu \cdot F_N \leq 0, \quad \exists \lambda \dot{\mathbf{u}}_T = \lambda \mathbf{F}_T, \quad \lambda \leq 0, \quad \lambda \cdot s = 0$$

A current extension of the law of Coulomb, resulting from the experiment, consists in having two coefficients of frictions: one for adherence, noted μ_s , the other for the slip, noted μ_d , with $\mu_s > \mu_d$. One has then in phase of adherence $\|\mathbf{F}_T\| \leq \mu_s \cdot F_N$ and in phase of slip $\|\mathbf{F}_T\| = \mu_d \cdot F_N$.

3 Approximate modeling of the relations of contact between 2 structures by penalization

3.1 Model of normal force of contact

The principle of the penalization applied to the graph of the figure [Figure 2.1-b] consists in introducing a univocal relation $F_N^{1/2} = f_\epsilon(d_N^{1/2})$ by means of a parameter ϵ . The graph of f_ϵ must tend towards the graph of Signorini when ϵ tends towards zero [bib6].

One of the possibilities consists in proposing a linear relation enters $d_N^{1/2}$ and $F_N^{1/2}$:

$$F_N^{1/2} = -\frac{1}{\epsilon} d_N^{1/2} \text{ si } d_N^{1/2} \leq 0 ; F_N^{1/2} = 0 \text{ sinon} \quad \text{éq 3.1-1}$$

If one notes $K_N = \frac{1}{\epsilon}$ called commonly “**stiffness of shock**“, one finds the classical relation, modelling an elastic shock:

$$F_N^{1/2} = -K_N \cdot d_N^{1/2} \quad \text{éq 3.1-2}$$

The approximate graph of the law of contact with penalization is the following:

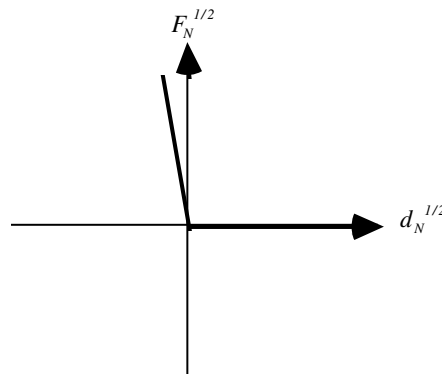


Figure 3.1-a: Graph of the relation of unilateral contact approached by penalization

To take account of a possible loss of energy in the shock, one introduces a “damping of shock” C_N . The expression of the normal force of contact is expressed then by:

$$F_N^{1/2} = -K_N \cdot d_N^{1/2} - C_N \cdot \dot{u}_N^{1/2} \quad \text{éq 3.1-3}$$

where $\dot{u}_N^{1/2}$ is the relative normal speed of Ω_1 compared to Ω_2 . To respect the relation of Signorini (not blocking), one must on the other hand check a posteriori that $F_N^{1/2}$ is positive or worthless. Only the positive part will thus be taken $\langle \cdot \rangle^+$ expression [éq 3.1-3]:

$$\begin{aligned} \langle x \rangle^+ &= x \text{ si } x \geq 0 \\ \langle x \rangle^+ &= 0 \text{ si } x < 0 \end{aligned}$$

The complete relation giving the normal force of contact which is retained for the algorithm is the following one:

$$\text{si } d_N^{1/2} \leq 0 F_N^{1/2} = \langle -K_N \cdot d_N^{1/2} - C_N \cdot \dot{u}_N^{1/2} \rangle^+ , \quad F_N^{1/2} = -F_N^{1/2}$$

$$\text{sinon } F_N^{1/2} = F_N^{1/2} = 0. \quad \text{éq 3.1-4}$$

3.2 Model of tangential force of contact

The graph describing the tangential force with law of Coulomb is not-differentiable for the phase of adherence ($\dot{u}_T^{1/2} = 0$). One thus introduces a univocal relation binding relative tangential displacement $d_T^{1/2}$ and the tangential force $F_T^{1/2} = f_\xi(d_T^{1/2})$ by means of a parameter ξ . The graph of f_ξ must tend towards the graph of Coulomb when ξ tends towards zero [bib6].

One of the possibilities consists in writing a linear relation enters $d_T^{1/2}$ and $F_T^{1/2}$:

$$F_T^{1/2} - F_T^{1/2^0} = -\frac{1}{\xi} \cdot (d_T^{1/2} - d_T^{1/2^0}) \quad \text{éq 3.2-1}$$

If one introduces a "tangential stiffness" $K_T = \frac{1}{\xi}$, the relation is obtained:

$$F_T^{1/2} = F_T^{1/2^0} - K_T \cdot (d_T^{1/2} - d_T^{1/2^0}) \quad \text{éq 3.2-2}$$

For digital reasons, related to the dissipation of parasitic vibrations [bib7] in phase of adherence, one is brought to add a "tangential damping" C_T in the expression of the tangential force. Its final expression is:

$$F_T^{1/2} = F_T^{1/2^0} - K_T \cdot (d_T^{1/2} - d_T^{1/2^0}) - C_T \cdot \dot{u}_T^{1/2} , \quad F_T^{2/1} = -F_T^{1/2} \quad \text{éq 3.2-3}$$

It is necessary moreover than this force checks the criterion of Coulomb, that is to say:

$$\|F_T^{1/2}\| \leq \mu \cdot F_N^{1/2} \text{ sinon on applique } F_T^{1/2} = -\mu \cdot F_N^{1/2} \cdot \frac{\dot{u}_T^{1/2}}{\|\dot{u}_T^{1/2}\|} , \quad F_T^{2/1} = -F_T^{1/2} \quad \text{éq 3.2-4}$$

The approximate graph of the law of friction of Coulomb modelled by penalization is the following:

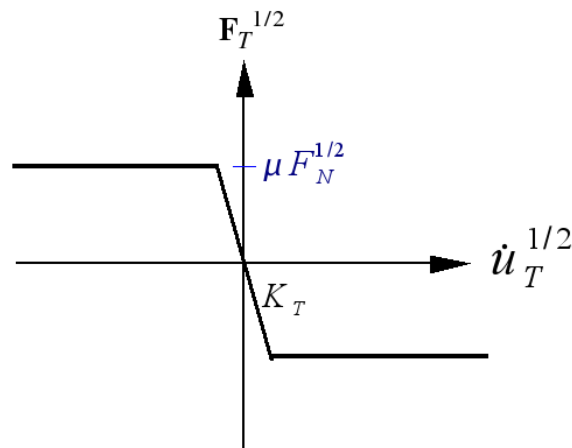


Figure 3.2-a: Graph of the law of friction approached by penalization

The case of the extension of the law of Coulomb with the distinction the adhesion coefficient enters μ_s and the coefficient of friction μ_d , the approximate graph of the law becomes:

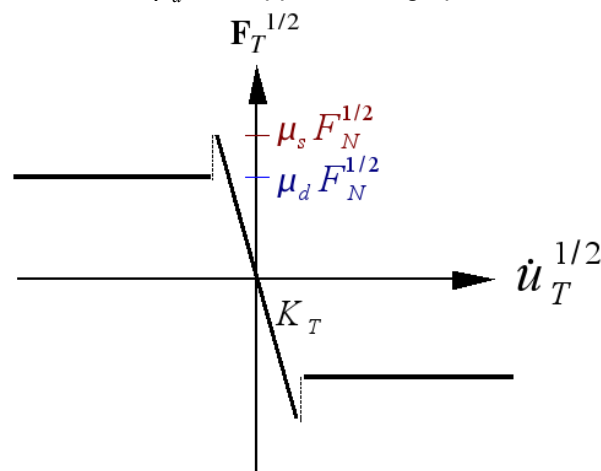


Figure 3.2-b: Graph of the alternative of the law of friction approached by penalization

4 Types of modelled connections of contact

As it was specified in the paragraph [§2.2], the developments presented here relate to the implementation of non-linear connections with unilateral contact and friction between 1 node and an obstacle or 2 nodes given.

The nodes in contact are supposed to belong to two slim structures of standard beam or to a beam and an indeformable obstacle. The nodes on which will carry the condition of contact are supposed to be carried by the average line of the beams.

4.1 Connections between a node and an indeformable obstacle

4.1.1 Connections of contact node on obstacle plan

One considers a slim structure represented by elements of type beam. Its displacement is limited in a point by the presence of an obstacle made up of two infinite half-planes in the direction Y (see [Figure 4.1.1-a]).

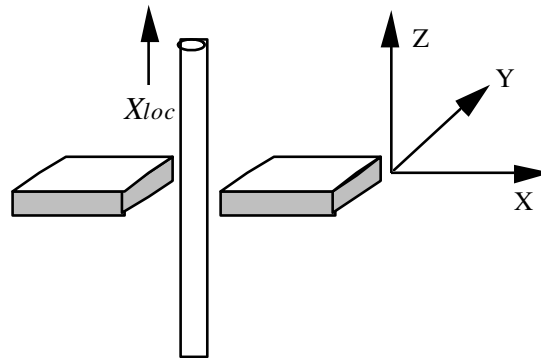


Figure 4.1.1-a: Slim structures with contact node on plan

To analyze the conditions of contact, one places oneself in the reference mark perpendicular to the axis X_{loc} , direction of neutral fibre or a generator of the beam. That is to say NOI , the node of the connection considered on the beam, geometry of the connection contact node on plan (called $PLAN_Y$ in *Code_Aster* [bib3]) is described on the figure below.

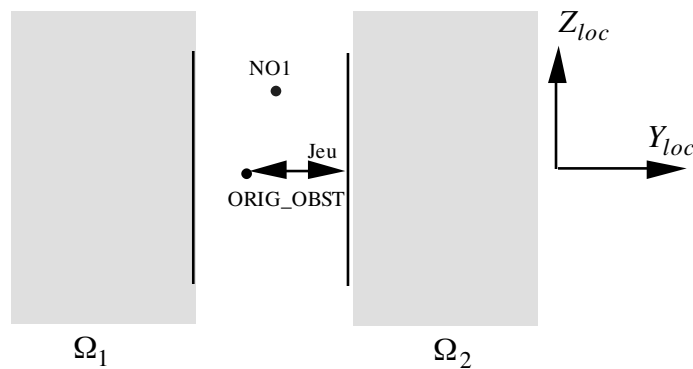


Figure 4.1.1-b: Geometry of the connection node on obstacle plan

Are $\begin{pmatrix} Y_{loc} \\ Z_{loc} \end{pmatrix}$ coordinates of the node NOI in the reference mark (Y_{loc}, Z_{loc}) , the origin of this reference mark is the point $ORIG_OBST$.

The normal distance d_N in this case, by neglecting rotations of the sections expresses itself then by:

$$d_N = -|Y_{loc}| + jeu \quad \text{éq 4.1.1-1}$$

The contact in this connection is judicious to take place whatever the shift in Z_{loc} between the two structures.

The normal vector \mathbf{n} in the reference mark (Y_{loc}, Z_{loc}) has as components:

$$\mathbf{n} = \begin{pmatrix} \text{signe}(Y_{loc}) \\ 0 \end{pmatrix} \quad \text{éq 4.1.1-2}$$

Other quantities $\dot{u}_N, F_N, \dot{u}_T, F_T$ are calculated in a general way as specified with [S3].

4.1.2 Connections of contact node on concave circular obstacle

One considers a hurred structure, represented by elements of type beam. Its displacement is limited in a point by the presence of an obstacle made up of a bored infinite plan of a circular hole (see figure below).

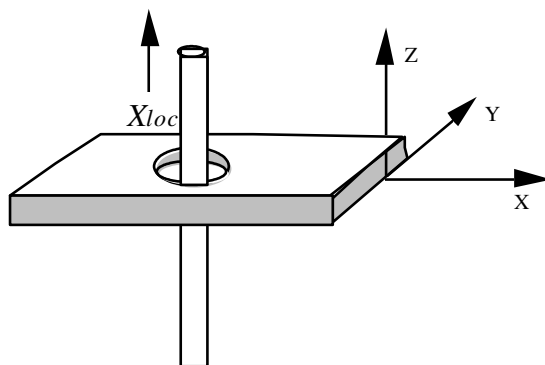


Figure 4.1.2-a: Slim structures with contact node on circular obstacle

To analyze the conditions of contact, one places oneself in the reference mark perpendicular to the axis X_{loc} , direction of neutral fibre or a generator of the beam. Are NOI , the node of the connection considered, geometry of the connection of contact node on circle (called `CIRCLE` in `Code_Aster` [bib3]) is described on the figure below.

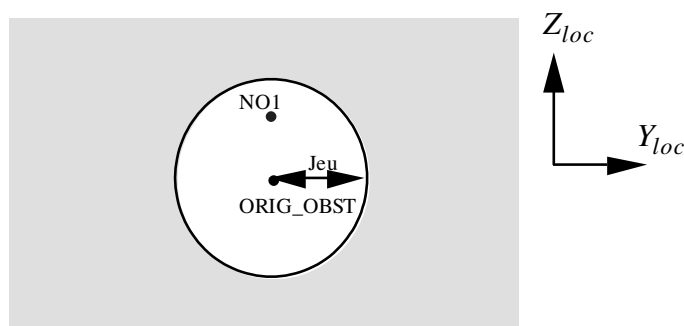


Figure 4.1.2-b: Geometry of the connection circular node obstacle

Are $\begin{pmatrix} Y_{loc} \\ Z_{loc} \end{pmatrix}$ coordinates of the node NOI in the reference mark (Y_{loc}, Z_{loc}) , of origin $ORIG_OBST$.

The normal distance d_N , by neglecting rotations of the sections expresses itself then by:

$$d_N = -\sqrt{(Y_{loc} - Y_{ORIG_obst})^2 + (Z_{loc} - Z_{ORIG_obst})^2} + jeu$$

One poses like normal vector n the vector:

$$n = \frac{ORIG_obst - NOEUD 1}{\|ORIG_obst - NOEUD 1\|}$$

jeu is a strictly positive distance.

Other quantities \dot{u}_N , F_N , \dot{u}_T , F_T are calculated in a general way as specified with [§3].

4.1.3 Connections of contact node on concave obstacle discretized by segments

One considers a hurred structure, represented by elements of type beam. Its displacement is limited in a point by the presence of an obstacle made up of a bored infinite plan of a hole of concave form unspecified which can be discretized in polar coordinates by segments (see figure Ci - below).

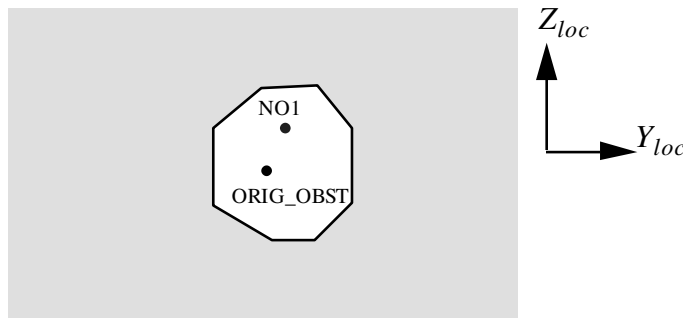
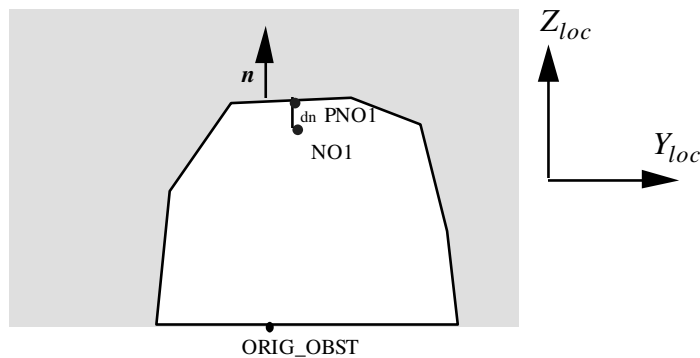


Figure 4.1.3-a: Geometry of the connection node on discretized concave obstacle

Are $\begin{pmatrix} Y_{loc} \\ Z_{loc} \end{pmatrix}$ coordinates of the node NOI in the reference mark (Y_{loc}, Z_{loc}) , of origin $ORIG_OBST$.

The facet of contact nearest to the node is searched NOI , the normal vector n is defined like the direct orthogonal vector with the facet:



That is to say PNO1 the projection of node NO1 at the facet, the normal distance d_N in this case is worth:

$$d_N = (NO1 - PNO1) \cdot n$$

Other quantities \dot{u}_N , F_N , \dot{u}_T , F_T are calculated in a general way as specified with [§3].

4.2 Connections between two nodes of two deformable structures

4.2.1 Connections of contact plan on plan

The contacts between assemblies fuel, on the level of the grids of mixture, constitute an example of contact plan on plan (see [Figure 4.2.1-a]).

One thus considers two hurled structures, being able to be modelled by beams of rectangular section on the level as of zones of contact.

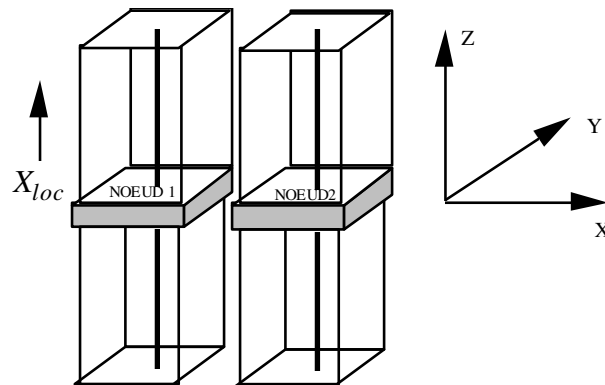


Figure 4.2.1-a: Slim structures with contact plan on plan

To analyze the conditions of contact, one places oneself in the reference mark perpendicular to the axis X_{loc} , direction of neutral fibre of the beams. Are $NO1$ and $NO2$, two nodes of the connection considered, geometry of the connection contact plan on plan (called BI_PLAN_Y in Code_Aster [bib3]) is described on the figure below.

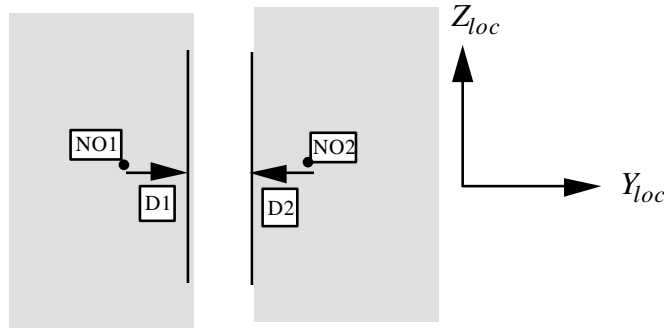


Figure 4.2.1-b: Geometry of the connection plan on plan

Are $\begin{Bmatrix} Y_{loc}^i \\ Z_{loc}^i \end{Bmatrix}$ coordinates of $NCEUDI$ in the reference mark (Y_{loc}, Z_{loc}) , of origin $ORIG_OBST$ ($ORIG_OBST$ can be provided by the user, by default $ORIG_OBST$ is selected like the medium of the nodes $NO1$, $NO2$).

The normal distance $d_N^{1/2}$ in this case, by neglecting rotations of the sections expresses itself then by:

$$d_N^{1/2} = |Y_{loc}^1 - Y_{loc}^2| - D_1 - D_2 \quad \text{éq 4.2.1-1}$$

D_1 and D_2 are strictly positive distances.

The contact in this connection is judicious to take place whatever the shift in Z_{loc} between the two structures.

The normal vector $n^{1/2}$ in the reference mark (Y_{loc}, Z_{loc}) has as components:

$$n^{1/2} = \begin{Bmatrix} \text{signe}(Y_{loc}^2 - Y_{loc}^1) \\ 0 \end{Bmatrix} \quad \text{éq 4.2.1-2}$$

Other quantities $\dot{u}_N^{1/2}$, $F_N^{1/2}$, $\dot{u}_T^{1/2}$, $F_T^{1/2}$ are calculated in a general way [§ 2.4].

4.2.2 Connections of contact rings on circle

If one considers now two cylinders of circular section, modelled by elements of beam. The connection of contact between two nodes of the average lines is supposed to take place between two circles as shown in the figure following:

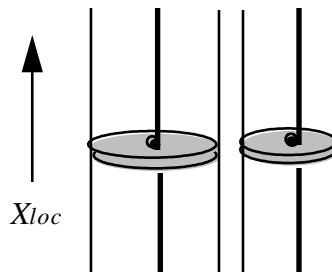


Figure 4.2.2-a: Slim structures with contact rings on circle

One places oneself in the reference mark perpendicular to the axis X_{loc} parallel with a generator of the cylinders. Are $NCEUD1$ and $NCEUD2$, the two nodes of the connection considered, the

geometry of the connection contact rings on circle (called BI_CERCLE in Code_Aster [bib3]) is described on geometry Ci - below:

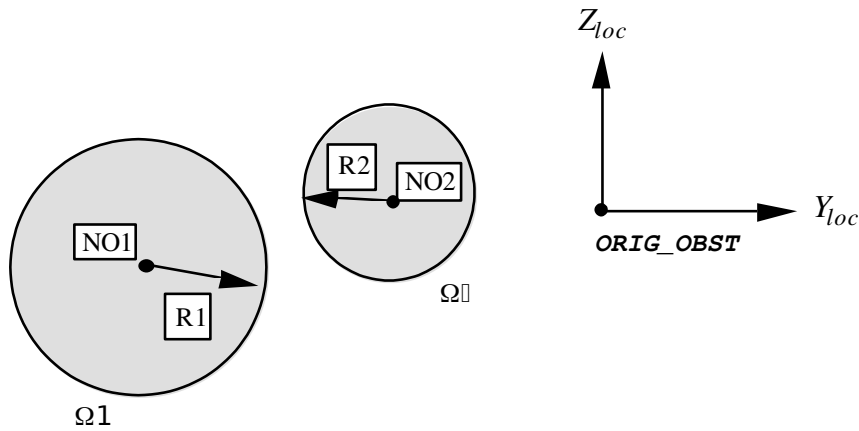


Figure 4.2.2-b: Geometry of the connection rings on circle

The normal distance $d_N^{1/2}$ has as an expression:

$$d_N^{1/2} = \sqrt{\left(Y_{loc}^1 - Y_{loc}^2\right)^2 + \left(Z_{loc}^1 - Z_{loc}^2\right)^2} - R_1 - R_2$$

One poses like normal vector of Ω_1 towards Ω_2 the vector:

$$n^{1/2} = \frac{NOEUD2 - NOEUD1}{\|NOEUD2 - NOEUD1\|}$$

5 Use of the localised non-linear forces of shock and friction in modal recombination

The non-linear forces expressed above are explicit functions of the position and speed of the nodes to which the conditions of contact relate.

One chooses to use the technique of pseudo-forces to solve the dynamic problem project. If the direct dynamic system is written:

$$M \ddot{X}_t + C \dot{X}_t + K X_t = F_{ext}(t) + F_{choc}(X_t, \dot{X}_t)$$

The technique of pseudo-forces consists in projecting on the basis of linear system and maintaining the forces non-linear with the second member.

The dynamic system project takes the shape:

$$\Phi^t M \Phi \ddot{\eta}_t + \Phi^t C \Phi \dot{\eta}_t + \Phi^t K \Phi \eta_t = \Phi^t \Phi_{ext}(t) + \Phi^t \Phi_{choc}(\Phi \eta_t, \Phi \dot{\eta}_t)$$

The problem project is integrated numerically by an explicit diagram.

Recommendations are given in [U2.06.04] for the choice of this base.

6 Precision on the use of the non-linearities of shock with friction

Non-linearities of shock between a structure and an obstacle or two structures were introduced into the algorithms of modal recombination of *Code_Aster* : an algorithm of Euler of order 1 and Devogelaere of order 4 [bib4] [R5.06.04].

These algorithms are used by the operator `DYNA_TRAN_MODAL` [bib1], [U4.54.03]. The type of connection of shock between the two nodes is specified by a specific order: `DEFI_OBSTACLE` [U4.21.07].

6.1 Definition of the type of connection of shock

The type of connection of shock is a generic concept, which does not comprise any physical information like a distance or unspecified dimension. The type of connection specifies simply the geometrical form of the connection considered.

Types of connection with shock with two nodes accepted by the order `DEFI_OBSTACLE` are described by the following keywords:

`PLAN_Y`, `PLAN_Z` or `CERCLE`
`BI_PLAN_Y`, `BI_PLAN_Z` or `BI_CERCLE` (see figure below).

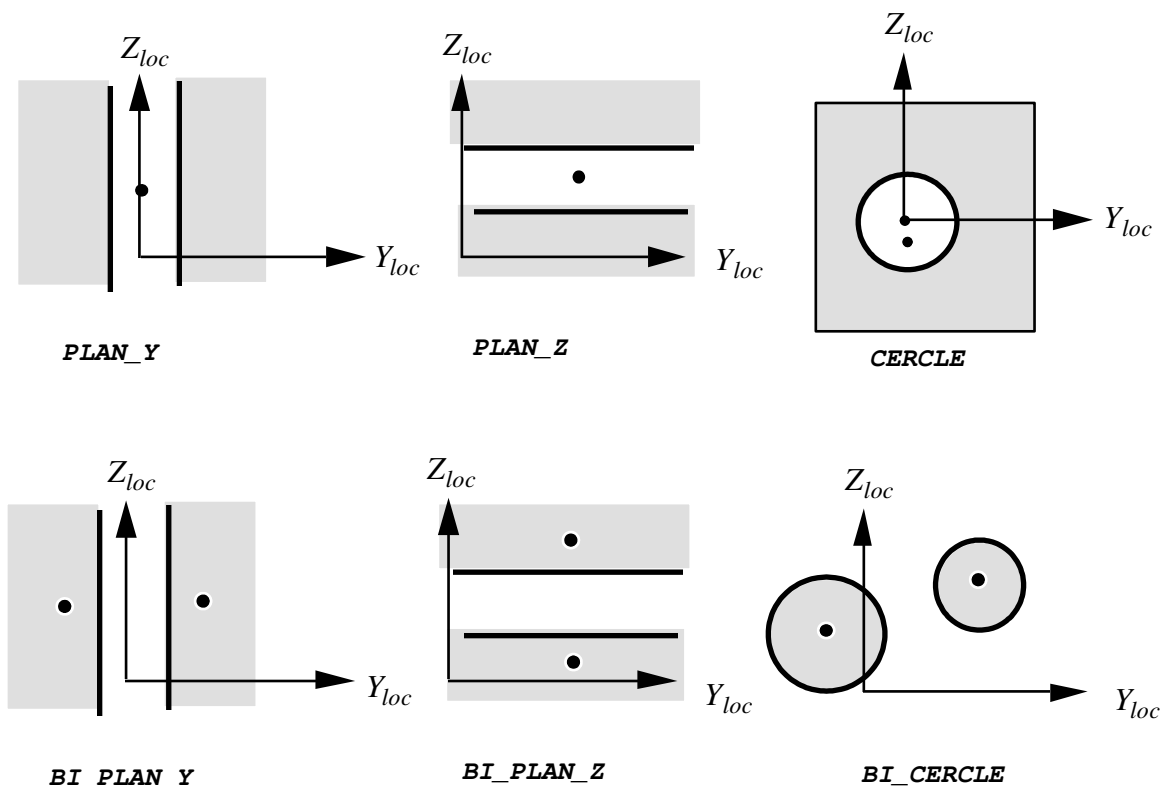


Figure 6.1-a: Geometries of the connections of shock

Prefix `BI_` specifies that it is about a connection with two nodes.

6.2 Definition of the local reference mark for the conditions of contact

The treated structures, being regarded as cylindrical slim (circular or rectangular section), are modelled by elements of beam. The contact is treated, as one saw with [§3.1] and [§3.2] in a plan perpendicular to the direction X_{loc} generator of the cylinders.

To define this change of reference mark completely, a local reference mark is introduced $(X_{loc}, Y_{loc}, Z_{loc})$.

The vector X_{loc} is the vector with 3 components provided behind the keyword `NORM_OBST`.

Using the first two nautical angles, one passes in a single way of the total reference mark (X, Y, Z) with a reference mark having X_{loc} like first basic vector (see [Figure 6.2-a] Ci - afterwards). The third rotation whose angle is provided behind the keyword `ANGL_VRIL` give a single correspondence between the principal reference mark and the local reference mark.

Foot-note:

the orientation of this local reference mark is important because it is in this reference mark that the conditions of contact are analyzed, and are provided the local positions of the nodes of shock.

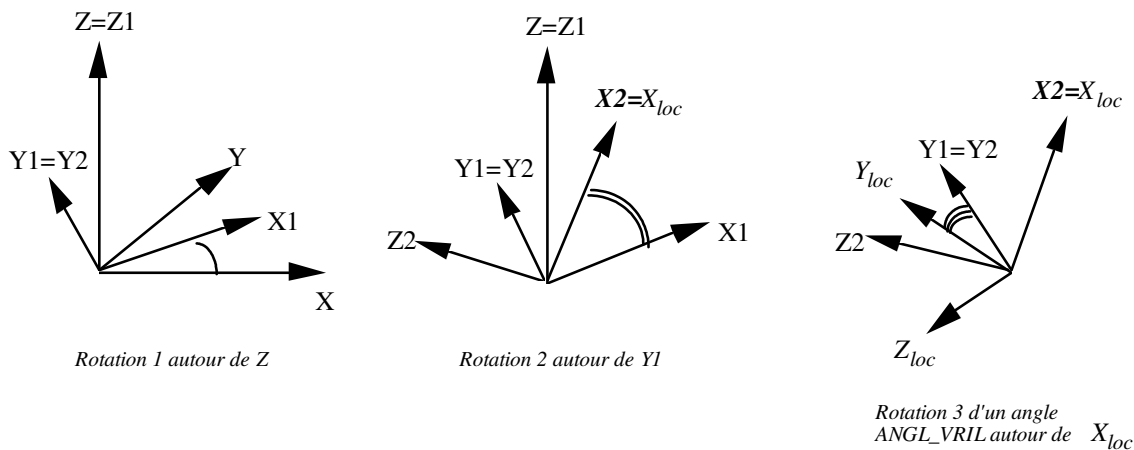


Figure 6.2-a: Rotations defining the local reference mark

The operand `ORIG_OBST` allows to define the origin of the local reference mark $(Orig, X_{loc}, Y_{loc}, Z_{loc})$. This operand is optional and in theory will not be used in the case of the shocks between two nodes. The code considers whereas the origin is located in the middle of the segment connecting the two nodes.

6.3 Definition of the nodes of the connections

One specifies, behind the keyword `NOEU_1` and `NOEU_2`, names of the two nodes of the structures on which will carry the conditions of shock. If it is about a connection between a node and an obstacle, only `NOEU_1` is well informed.

6.4 Definition of dimensions characteristic of the sections

The operand `GAME` is used for the conditions of contact between a node and an obstacle.

Operands `DIST_1` and `DIST_2` allow to specify dimensions characteristic of the sections of the structures surrounding the nodes of shock. In the case as of connections plan on plan, they are the thicknesses of matter surrounding the node of shock in the direction considered.

In the case of connections rings on circle, it acts of the rays of the sections surrounding the nodes of shock.

6.5 Definition of the parameters of contact

The parameters stiffnesses and damping of shock were introduced with the §3.1 and §3.2, one specifies the keywords here making it possible to define them for a given connection.

The operand `RIGI_NOR` is obligatory, it allows to give the value of normal stiffness of shock K_N .
The other operands are optional.

The operand `AMOR_NOR` allows to give the value of normal damping of shock C_N .

The operand `RIGI_TAN` allows to give the value of tangential stiffness K_T .

The operand `AMOR_TAN` allows to give the tangential value of damping of shock C_T .

The operand `COULOMB` allows to give the value of the coefficient of Coulomb.

Foot-note:

If a stiffness K_T is defined and that the keyword `AMOR_TAN` is absent, the code calculates a damping optimized in order to minimize the residual oscillations in adherence [bib7]:

$$C_T = 2 \cdot \sqrt{(k_i + K_T) \cdot m_i} - 2 \cdot x_i \cdot \sqrt{k_i \cdot m_i} ,$$

where I am the index of the dominating mode in the answer of the structure (the modal mass most important).

7 Bibliography

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8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
3	G. JACQUART (EDF/EP/AMV)	Initial text