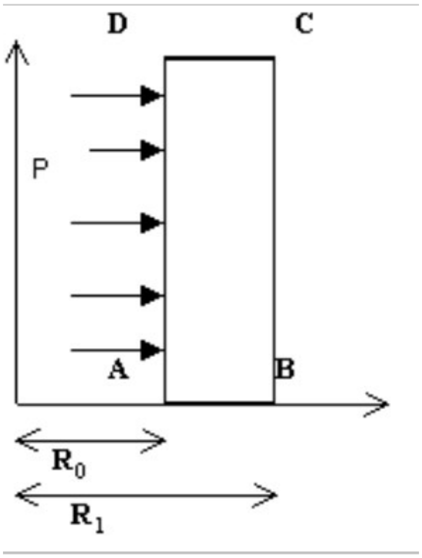
SSNA104 - Hollow roll subjected to a pressure, linear viscoelasticity

Summary:

This CAS-test makes it possible to validate the laws of LEMAITRE and LEMA_SEUIL established in *Code_Aster* in the case of linear viscoelastic behavior. The found results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



Dimensions of the cylinder:

R_0	$1\,m$
R_1	$2\,m$

Figure 1.1-a: Cut of the hollow roll and loading

1.2 Properties of materials

Young modulus: $E=1\,MPa$
Poisson's ratio: $\nu=0.3$

Law of LEMAITRE :

$$g(\sigma,\lambda,T)=\left(\frac{1}{K}\frac{\sigma}{\lambda^{\frac{1}{m}}}\right)^n \text{ with } \frac{1}{K}=1\text{ , }\frac{1}{m}=0\text{ , }n=1$$

Law LEMA_SEUIL :

$$g(\sigma,\lambda,T)=A\left(\frac{2}{\sqrt{3}}\sigma\right)\Phi \text{ with } A=\frac{\sqrt{3}}{2}\text{ , }\Phi=1 \text{ on all the grid}$$

$$S=10^{-10}$$

being given the value of the various parameters materials, the two laws are absolutely identical and can thus be compared with the same analytical solution.

1.3 Boundary conditions and loading

Boundary conditions:
The cylinder is blocked in DY on the sides $[AB]$ and $[CD]$.

Loading:
The cylinder is subjected to a pressure interns on $[DA] P0=1.\mathrm{E}-3\ MPa$

2 Reference solutions

2.1 Method of calculating used for the reference solutions

The whole of this demonstration can be read with more details in the document [bib1].

In the case of a linear viscoelastic isotropic material, one can describe the behavior in the course of time using two functions $I(t)$ and $K(t)$ so that strains and stresses can be written:

$$\varepsilon(t)=(I+K)*\frac{d\sigma(t)}{d\tau}-K*\frac{d(Tr(\sigma(t)))}{d\tau}\boldsymbol{I}_3$$

where I_3 indicate the matrix identity of row 3

and $*$ the product of convolution: $(f*g)(t)=\int_0^t f(t-\tau)\,g(\tau)\,d\tau$

One finds $I(t)=\frac{1}{E}+kt$, $K(t)=\frac{\nu}{E}+\frac{1}{2}kt$

The pressure is imposed P_0 at the moment $t=0$, the internal pressure is worth $p(t)=H(t)P_0$ where $H(t)=\begin{cases}0 & si \quad t-\tau<0\\ 1 & si \quad t-\tau\geq 0\end{cases}$ with in this case $\tau=0$

One uses the transform of Laplace Carson $f^{+}(n)=L(f(t))=n\int_0^{\infty} f(t)e^{-nt}dt$

From where $p^{+}=P_0$

The solution of the elastic problem are equivalent is:

$$\sigma^{+}=\begin{pmatrix}\gamma\left(1-\frac{r_1^2}{r^2}\right) & 0 & 0\\ 0 & \gamma\left(1+\frac{r_1^2}{r^2}\right) & 0\\ 0 & 0 & \sigma_Z^{+}\end{pmatrix} \text{ where } \gamma=\frac{P_0 r_0^2}{r_1^2-r_0^2}$$

One determines σ_Z^{+} by the condition on ε_Z^{+} data by the boundary conditions:

$$\varepsilon_Z^{+}=0=(I^{+}+K^{+})\sigma_Z^{+}-K^{+}(2\gamma+\sigma_Z^{+})=I^{+}\sigma_Z^{+}-2K^{+}\gamma$$

From where $\sigma_Z^{+}=\gamma\left(1+\frac{(2\nu-1)p}{p+Ek}\right)$.

One finds by the transform of opposite Laplace $\sigma_z(t)=\gamma(1-(1-2\nu)e^{-Eht})$, in the same way by applying the transform of Laplace reverses on σ_r and σ_θ , one finds

$$\sigma^+=\begin{pmatrix}\gamma\left(1-\frac{r_1^2}{r^2}\right) & 0 & 0\\ 0 & \gamma\left(1+\frac{r_1^2}{r^2}\right) & 0\\ 0 & 0 & \gamma(1-(1-2\nu)e^{-Eht})\end{pmatrix}$$

One from of deduced:

$$\dot{\varepsilon_V}=\begin{pmatrix}\frac{3}{2}k\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-\frac{r_1^2}{r^2}\right) & 0 & 0\\ 0 & \frac{3}{2}k\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-\frac{r_1^2}{r^2}\right) & 0\\ 0 & 0 & -k\gamma((1-2\nu)e^{-Eht})\end{pmatrix}$$

and while integrating with $\varepsilon_V(0)=0$;

$$\dot{\varepsilon_V}=\begin{pmatrix}\frac{3}{2}\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-k\frac{r_1^2}{r^2}t\right) & 0 & 0\\ 0 & \frac{3}{2}\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-k\frac{r_1^2}{r^2}t\right) & 0\\ 0 & 0 & -\gamma\frac{(1-2\nu)}{E}(1-e^{-Eht})\end{pmatrix}.$$

One from of deduced radial displacement

$$w(r,t)=r\gamma\left[\frac{1}{E}\left[(1+\nu)\frac{r_1^2}{r^2}+\frac{1-2\nu}{2}(3-(1-2\nu)e^{-Ekt})\right]+\frac{3}{2}k\frac{r_1^2}{r^2}t\right]$$

2.2 Results of reference

Displacement DX on the node B and constraints $SIXX$, $SIYY$ and $SIZZ$ in B

2.3 Uncertainty on the solution

0% : analytical solution

2.4 Bibliographical references

PH. BONNIERES: Two analytical solutions of axisymmetric problems in linear viscoelasticity and with unilateral contact, Note HI-71/8301

3 Modeling A

3.1 Characteristics of modeling

The problem is modelled in axisymetry.

3.2 Characteristics of the grid

1000 meshs `QUAD4`

3.3 Sizes tested and results

Identification	Moments	Reference
$DX(B)$	0.9	2.14498 E-3
$SIXX(B)$	0.9	0.0
$SIYY(B)$	0.9	2.7912 E-4
$SIZZ(B)$	0.9	6.66 E–4

4 Modeling B

4.1 Characteristics of modeling

The problem is modelled in axisymetry

4.2 Characteristics of the grid

1000 meshs QUAD4

4.3 Sizes tested and results

Identification	Moments	Reference
$DX(B)$	0.9	2.14498 E-3
$SIXX(B)$	0.9	0.0
$SIYY(B)$	0.9	2.7912 E-4
$SIZZ(B)$	0.9	6.66 E–4

5 Summary of the results

Results calculated by *Code_Aster* are in agreement with the analytical solutions but very strongly depend on the refinement of the grid.