SSNA115 – Wrenching of a rigid reinforcement with cohesive elements

Summary:

This case test has as an aim the digital study of the wrenching of a rigid reinforcement embedded in a hollow roll. Decoherence is described with three cohesive models.:

Modeling A : starting from elements with discontinuity interns with a cohesive law CZM_EXP (see documentation [R7.02.14]) by using modeling AXIS_ELDI.

Modeling B : starting from elements of joint with the cohesive law CZM_LIN_REG (see documentation [R7.02.11]) by using modeling AXIS_JOINT.

Modeling C : starting from elements of interface with the cohesive law CZM_TAC_MIX (see documentation [R7.02.11]) by using modeling AXIS_INTERFACE_S.

To validate the results we will be based on the analytical solution developed in [bib3]. The interested reader will be able to also refer to it for a thorough study of this case test.

1 Problem of reference

1.1 Geometry and loading

That is to say a hollow roll length L, of interior ray R_f and of external ray R. That is to say a rigid reinforcement of circular section of ray R_f embedded in its center. One notes Γ_i and Γ_e surfaces interior and external of the hollow roll (see [Figure 1.1-a]). The loading consists in applying, to the rigid reinforcement, a displacement $U^i e_z$ ($U^i > 0$) as well as a null displacement on the external edge Γ_e.

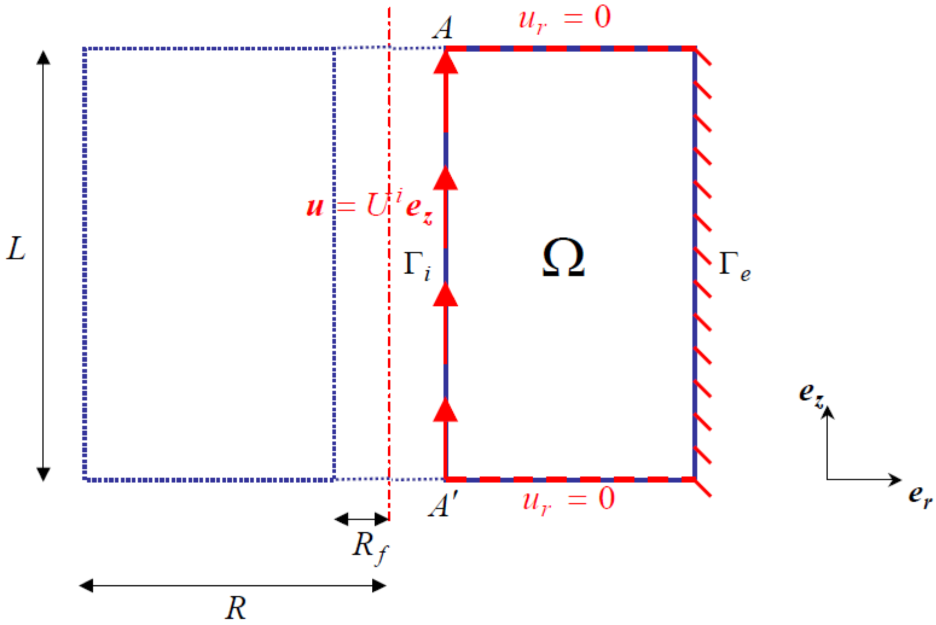

Figure 1.1-a: Diagram of the field and loading

The assumption of an axisymmetric solution is made what enables us to restrict our study has a rectangular 2D field Ω. Dimensions of the field are the following ones: $R_f = 0.5\,\mathrm{mm}$, $R = 5.5\,\mathrm{mm}$, $L = 10\,\mathrm{mm}$. The loading on the rigid reinforcement will be taken into account by applying imposed displacement $U^i \boldsymbol{e}_z$ on all the side Γ_i field $2\mathrm{D}$ as well as a null displacement on the side Γ_e to take into account the embedding of the cylinder. Finally one forces a radial displacement no one on the faces lower and higher of the field in order to avoid a singularity related to a change of boundary condition than the points A and A' (see [Figure 1.1-a]). These boundary conditions will lead to a anti-plane solution (independent of z) what makes it possible to obtain an analytical solution more simply.

2 Reference solution

The reference solution is an analytical solution drawn from [bib3], itself inspired by a unidimensional study suggested in [bib1] and in a more general way being based on the energy approach of the rupture suggested per G.A. Francfort and J.J. Marigo [bib2]. We will not return in the details of the calculation of this solution, we will present just the analytical value of the total answer of the structure. Imposed displacement U according to the corresponding force F is worth:

$$U(F)=\frac{Fl}{2\pi R_f L\mu}+sign(F)\psi'^{-1}\left(\frac{|F|}{2\pi R_f L}\right) \quad \text{éq 2-1}$$

where μ indicate the coefficient of Lamé ($\mu = E/2$ here), ψ density of energy of cracking and where $l=R_f \ln(R/R_f)$ is a length structural feature decisive for the brutal or progressive evolution of decoherence.

The reverse of the derivative of the density of energy surface takes the following values according to whether the cohesive law is adopted CZM_EXP (i.e elements with discontinuity), the law CZM_LIN_REG (i.e elements of joint) or the law CZM_TAC_MIX (i.e elements of interface). (see documentations [R7.02.14] and [R7.02.11]).

CZM_EXP: $\psi'^{-1}(x)=-\frac{G_c}{\sigma_c}\ln\left(\frac{x}{\sigma_c}\right)$

CZM_LIN_REG or CZM_TAC_MIX: $\psi'^{-1}(x)=2\frac{G_c}{\sigma_c}\left(1-\frac{x}{\sigma_c}\right)$

2.1 Bibliographical references

1) CHARLOTTE Mr., FRANKFURT G.A., MARIGO J.J. and TRUSKINOVSKY L.: Revisiting brittle fracture ace year energy minimization problem: comparison of Griffith and Barenblatt surfaces energy models. Proceedings of the Symposium one "Continuous Ramming and Fracture" The dated science library, Elsevier, edited by A. BENALLAL, Paris, pp. 7-18, (2000).

2) FRANKFURT G.A. and MARIGO J.J.: Revisiting brittle fracture ace year energy minimization problem. J. Mech. Phys. Solids, 46 (8), pp. 1319-1342 (1998).

3) LAVERNE J.: Energy formulation of the rupture by models of cohesive forces: digital considerations theoretical and establishments, Doctorate of the University Paris 13, November 2004.

3 Modeling A

3.1 Characteristics of modeling

Simulation is carried out into axisymmetric. The elements with internal discontinuity make it possible to represent the crack along Γ_i. The latter have as a modeling AXIS_ELDI and a cohesive behavior CZM_EXP. The other elements of the grid are QUAD4 with an elastic behavior ELAS in modeling AXIS.

3.2 Parameters material

The values of the Young modulus, of the Poisson's ratio, the critical stress and the tenacity of material are taken in the following way:

$$E = 1.5 \text{ MPa} \text{ , } \nu = 0 \text{ , } \sigma_c = 1.1 \text{ Mpa} \text{ , } G_c = 0.9 \text{ N.mm}^{-1}$$

(NB: they are values "tests" which do not correspond to any material in particular.)

3.3 Characteristics of the grid

One carries out a grid structured in quadrangles of the field Ω with 76 meshs in the height and 28 meshs in the radial direction. One lays out a layer of elements with discontinuity interns along Γ_i using the order CREA_MAILLAGE and of the keyword CREA_FISS (see documentation [U4.23.02]). The orientation of the elements with discontinuity is carried out so that the normal direction is directed according to $-\boldsymbol{e}_r$ (the tangential direction is thus according to $-\boldsymbol{e}_z$). The rest of the field is divided into linear meshs $QUAD4$ (see [Figure 3.2-a]).

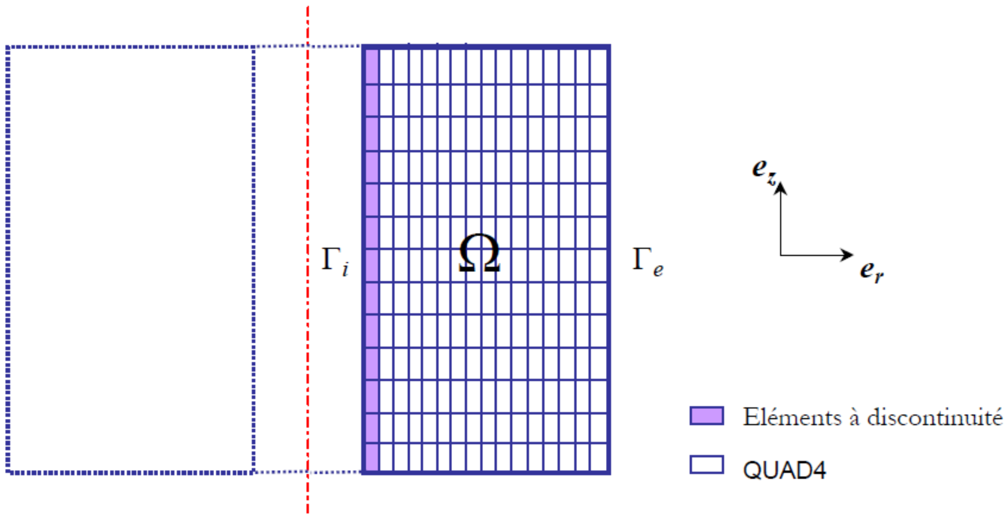

Figure 3.2-a: Grid of the field

3.4 Sizes tested and results

The tangential constraint σ_t along the crack (i.e in the elements with discontinuity) corresponds contrary to the force F divided by the surface of decoherence: $2\pi R_f L$. Moreover while being based on the form of the density of energy of surface ψ defined in [R7.02.12] and according to [éq 2-1] the following relation is deduced:

$$U(\sigma_t) = -\frac{l\sigma_t}{\mu} + sign(\sigma_t)\frac{G_c}{\sigma_c}\ln(\frac{|\sigma_t|}{\sigma_c})$$ éq 3.4-1

The latter will enable us to carry out tests summarized in the table below.

Size tested	Theory	*Code_Aster*	Difference (%)
Tangential constraint: VI7 $PG1$ mesh $MJ38$ Moment: 6.00070E+00	7.69747E-01	7.6974726277784E-01	3.41E-05
Tangential constraint: VI7 $PG1$ mesh $MJ38$ Moment: 1.20004E+01	4.34935E-01	4.3493490987033E-01	-2.07E-05
Tangential constraint: VI7 $PG1$ mesh $MJ38$ Moment: 1.93334E+01	1.28483E-01	1.2848319446210E-01	1.51E-04
Displacement DY Node $N5$ Moment: 1.20004E+01	1.57674E+00	1.5767415306566E+00	9.71E-05

4 Modeling B

4.1 Characteristics of modeling

Simulation is carried out into axisymmetric. The elements of interface make it possible to represent the crack along Γ_i . The latter have as a modeling `AXIS_JOINT` and a cohesive behavior `CZM_LIN_REG`. The other elements of the grid are `QUAD4` with an elastic behavior `ELAS` in modeling `AXIS`.

4.2 Parameters Material

The values of the Young modulus, of the Poisson's ratio, the critical stress and the tenacity of material are taken in the following way:

$$E = 100 \text{ MPa} \text{ , } \nu = 0 \text{ , } \sigma_c = 3\text{Mpa} \text{ , } G_c = 0.9 \text{ N.mm}^{-1}$$

In addition, the parameter of regularization of the cohesive law is taken equal to `PENA_ADHERENCE` = 0.00001. (NB: they are values “tests” which do not correspond to any material in particular.)

4.3 Characteristics of the grid

The grid is identical to the precedent with the difference which the layer of cohesive elements is made up of elements with a low thickness, directed with the order `ORIE_FISSURE`.

4.4 Sizes tested and results

To test the digital solution, the equation is used [éq 2-1]. One notes F^R the resultant of the force along Γ_i multiplied by 2π .

Size tested	Theory	*Code_Aster*	Difference ($\%$)
U^i withmoment: 3	2.298338E-01	2.2983379490657E-01	-2.22E-06
F^R withmoment: 3	1.049985E+01	1.0499850000001E+01	1.05E-11
U^i withmoment: 6	3.884798E-01	3.8847977822122E-01	-5.61E-06
F^R with Instant: 6	5.99982E+00	5.9998200000014E+00	2.27E-11
U^i withmoment: 8	5.2809E-01	5.2809010497189E-01	1.99E-05
F^R withmoment: 8	2.03974E+00	2.0397408000020E+00	3.92E-05

5 Modeling C

5.1 Characteristics of modeling

Simulation is carried out into axisymmetric. The elements of interface make it possible to represent the crack along Γ_i. The latter have as a modeling AXIS_INTERFACE_S and a cohesive behavior CZM_TAC_MIX. The other elements of the grid are QUAD8 with an elastic behavior ELAS in modeling AXIS.

5.2 Parameters Material

The values of the Young modulus, of the Poisson's ratio, the critical stress and the tenacity of material are taken in the following way:

$$E=100\ \text{MPa}\ ,\ \nu=0\ ,\ \sigma_c=3\text{Mpa}\ ,\ G_c=0.9\ \text{N.mm}^{-1}$$

In addition, the parameter of penalization of Lagrangian is taken equal to PENA_LAGR =1000.
(NB: they are values "tests" which do not correspond to any material in particular.)

5.3 Characteristics of the grid

The grid is identical to modeling A with two differences near: all the meshs are quadratic (QUAD8) and lay down it cohesive elements is made up of elements with a low thickness, directed with the order ORIE_FISSURE.

5.4 Sizes tested and results

To test the digital solution, the equation is used [éq 2-1]. One notes F^R the resultant of the force along Γ_i multiplied by 2π.

Size tested	Theory	*Code_Aster*	Difference ($\%$)
U^i withmoment: 3	1.576931E-01	1.5769307873679E-01	-1.35E-05
F^R withmoment: 3	1.25499E+01	1.2549900398014E+01	3.17E-06
U^i with moment: 6	2.656474E-01	2.6564739519277E-01	-1.81E-06
F^R with Instant: 6	9.486833E+00	9.4868329805100E+00	-2.05E-07
U^i withmoment: 8	3.63577E-01	3.6357700583331D-01	1.60E-06
F^R withmoment: 8	6.7082E+00	6.7082039325061D+00	5.86E-05

6 Summary of the results

It is noted that the three types of elements allow a good prediction of decoherence. Indeed the latter develops in an identical way on all the height of the cylinder. Moreover, the digital results are very close to the analytical solution. In addition, the models suggested make it possible to reproduce correctly the evolution brutal (case of modeling A) or progressive (case of modelings B and C) of cracking according to the lengths structural feature and of material. The interested reader will be able to refer to [bib3] for more details.