SSNL130 – Indeformable plate on a carpet of springs

Summary:

The objective is to test and validate the possibilities of the order AFFE_CARA_ELEM, option RIGI_PARASOL in $2D$ and in $3D$ and affected of behavior DIS_CHOC.

This case test models a plate, considered as indeformable, posed on a carpet of springs.

- The springs are modelled by DIS_T (K_T_D_L), that makes it possible to impose boundary conditions at the ends of the springs which are not related to the solid.
- The behavior DIS_CHOC a unilateral behavior of the springs allows, which leaves a possibility of separation of the plate with respect to the carpet of spring.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2017 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

1 Problem of reference

1.1 Geometry

A rectangular plate of width a and length b, pressed on a carpet of springs.

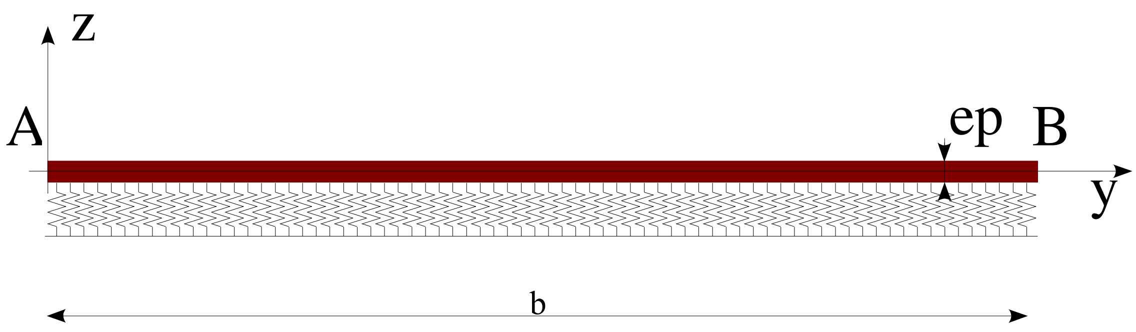

Figure 1.1-a : Diagram of the plate and the springs in the plan (y, z).

Dimensions:

$a=1.00\,m$
$b=2.00\,m$
$ep=0.30\,m$

1.2 Properties of material

Young modulus: $2.0\text{E}+11\,Pa$
Poisson's ratio: 0.3
Total stiffness of the carpet of springs: $K=10000.0\,N/m$

1.3 Boundary conditions and loadings

The loading is a loading of pressure of the form $P=p.(y-b)^2$, with $p=5\text{N}/m^4$

Displacements imposed at the ends of the springs off-line to the plate:

- in the time interval $[0,1]$ displacement is imposed on 0.0 following DX, DY and DZ,
- in the time interval $[1,2]$ displacement is imposed on 0.0 following DX and DY. According to DZ it is imposed by the function $Dz=(t-1.0)*0.5\text{E-02}$.

1.4 Initial conditions

Without object for a static analysis.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2017 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

2 Reference solution

2.1 Method of calculating of the continuous problem

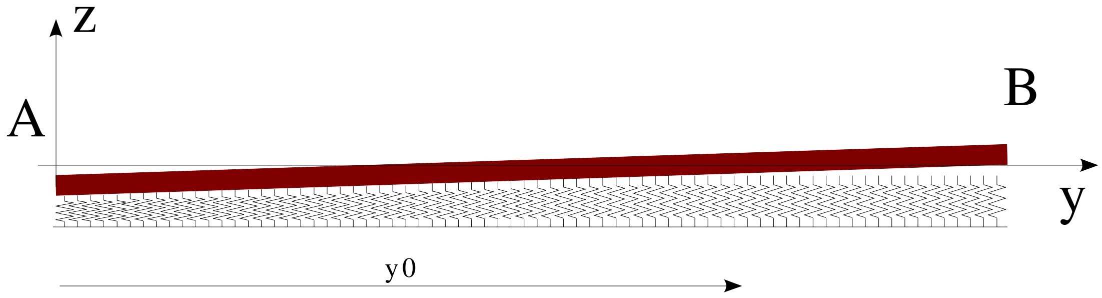

Figure 2.1-a : Diagram of the plate and the springs after loading.

The resolution of the problem consists in calculating vertical displacements of the corners of the plate and the position of the point of separation with respect to the carpet of springs.
The equilibrium equations are the following ones:
Effort resulting due to the loading:

$$Fp=\iint_s P.ds=a.p.\frac{b^3}{3} \quad [2.1\text{-}1]$$

Moment resulting at the point A had with the loading:

$$Mp_A=\iint_s P.y.ds=a.p.\frac{b^4}{12} \quad [2.1\text{-}2]$$

The plate is regarded as rigid, its displacement is form $z(y)=U_a\left(1-\frac{y}{y_0}\right)$. With U_a the vertical displacement of the point A and y_0 the position of separation.
Effort of reaction of the springs:

$$Fr=\iint_s \frac{K}{a.b}U_a\left(1-\frac{y}{y_0}\right)ds=K\,U_a\frac{y_0}{2b} \quad [2.1\text{-}3]$$

Moment of reaction of the springs at the point A :

$$Mr_A=\iint_s \frac{K}{a\,b}U_a\left(1-\frac{y}{y_0}\right)y\,ds=K\,U_a\frac{y_0^2}{6b} \quad [2.1\text{-}4]$$

The resolution of the equations 2.1-1, 2.1-2, 2.1-3, 2.1-4 (balance of the efforts and the moments) gives the following result:

$$y_0=\frac{3b}{4} \quad U_a=-\frac{8\,p\,a\,b^3}{9K} \text{ one from of deduced } U_b=-\frac{U_a}{3}$$

2.2 Method of calculating of the discretized problem

In this analysis the carpet of springs is not regarded any more as continuous. The springs are regularly distributed. As previously vertical displacements of the corners of the plate and the position of the line of separation with respect to the carpet of springs will be calculated.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2017 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

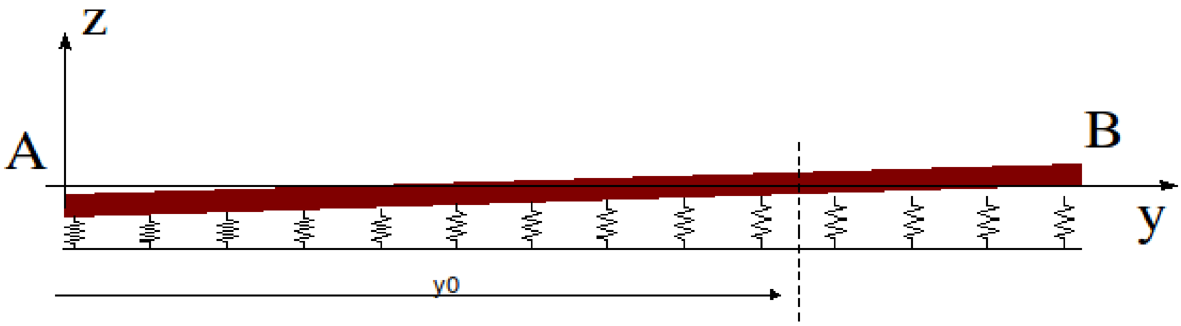

Figure 2.2-a : Diagram of the plate and the springs after loading.

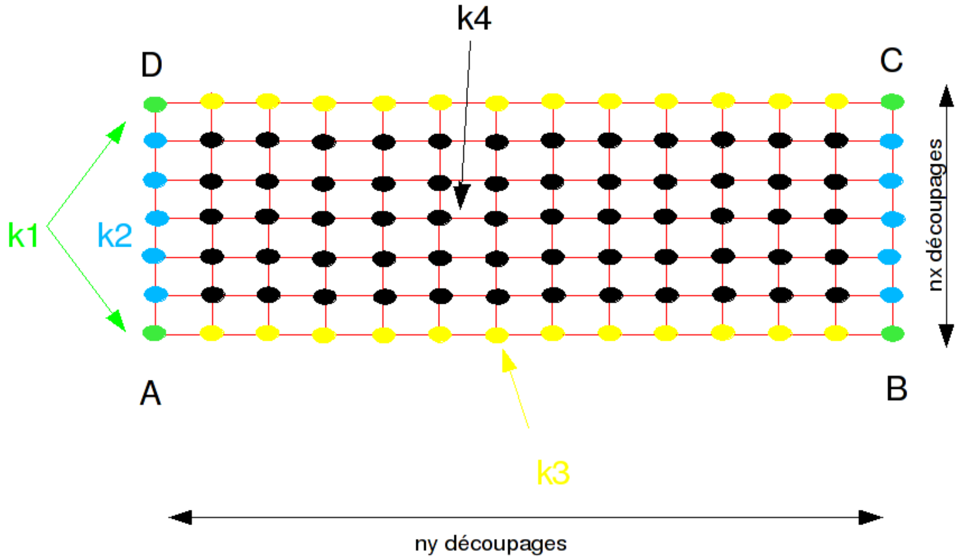

Figure 2.2-b : Discretization of the plate in the plan (x,y).

The figure above locates the springs according to their stiffness. This stiffness is calculated by the option RIGI_PARA_SOL order AFFE_CARA_ELEM. The assignment of the values is made according to the surface of the zone that they affect. If K is the total stiffness of the carpet of spring, one thus has:

$$\mathrm{k4}=\frac{\mathrm{K}}{nx\,ny} \qquad \mathrm{k2}=\mathrm{k3}=\frac{\mathrm{k4}}{2}=\frac{\mathrm{K}}{2\,nx\,ny} \qquad \mathrm{k1}=\frac{\mathrm{k4}}{4}=\frac{\mathrm{K}}{4\,nx\,ny} \tag{2.2-1}$$

The equilibrium equations are the following ones:
Effort of reaction springs:

$$Fr_{(j)}=U_a\cdot\left[K'_x+K''_x\cdot\sum_{j=1}^{n}\left(1-j\frac{b}{ny\,y_0}\right)\right] \tag{2.2-2}$$

Moment of reaction of the springs along the line AB :

$$Mr_{(j)}=U_a\cdot K''_x\cdot\sum_{j=1}^{n}\left(1-j\frac{b}{ny\,y_0}\right)\cdot j\frac{b}{ny} \tag{2.2-3}$$

with $K'_x=(2\,\mathrm{k1}+\mathrm{k2}(nx-1))$ $\quad K''_x=(2\,\mathrm{k3}+\mathrm{k4}(nx-1))$

$$n\frac{b}{ny}\leq y_0\leq(n+1)\frac{b}{ny}$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2017 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

The resolution of the equations (balance of the efforts and the moments) gives the solution of balance:

$$U_a=\frac{p\,a\,b^3 ny(3ny-8n-4)}{6K(1+n+n^2)} \qquad y_0=\frac{b\,n(1+n)(3ny-8n-4)}{3ny(ny+2n(ny-2)-4n^2)} \qquad [2.2\text{-}4]$$

where n and y_0 must observe the following conditions:

$$n.\frac{b}{ny}\le y_0\le(n+1)\frac{b}{ny} \qquad 0\le y_0\le b \qquad n \text{ entirety}$$

2.3 Sizes and results of reference

The sizes tested will be vertical displacements with the 4 corners of the plate.

2.4 Uncertainties on the solution

No, the solution is analytical.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2017 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

3 Modeling A

3.1 Characteristics of modeling

The plate is modelled by elements DKT. The springs are modelled by SEG2 affected of a modeling DIS_T whose characteristics are K_T_D_L. They are the discrete ones in translation having a diagonal matrix, to see the documentation of AFFE_CARA_ELEM.

3.2 Characteristics of the grid

The plate is cut out with $ny=16$ and $nx=4$. Dimensions of the plate are $a=1\,m$ and $b=2\,m$

3.3 Sizes tested and results

For the step of time n°1, displacements of the ends of the springs, off-line to the plate, are imposed on zero. Results of *Code_Aster* are compared with the discrete solution, which corresponds to the solution of the modelled problem. This solution is obtained for $n=12$, equation 2.2-4.

Nature of the results	$U_A=U_D$	$U_B=U_C$
Continuous solution	$\frac{-4}{1125}$	$\frac{4}{3375}$
Discrete solution ($n=12$)	$\frac{-208}{58875}$	$\frac{176}{153075}$
Tolerance	4.0E-04	7.0E-03

For the step of time n°2, displacements of the ends of the springs, off-line to the plate, are moved of $+5.0\text{E-}03\,m$. Results of *Code_Aster* are compared with the discrete solution, which corresponds to the solution of the modelled problem.

Nature of the results	$U_A=U_D$	$U_B=U_C$
Continuous solution	$\frac{-4}{1125}+\frac{5}{1000}$	$\frac{4}{3375}+\frac{5}{1000}$
Discrete solution ($n=12$)	$\frac{-208}{58875}+\frac{5}{1000}$	$\frac{176}{153075}+\frac{5}{1000}$
Tolerance	4.0E-04	7.0E-03

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2017 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4 Modeling B

4.1 Characteristics of modeling

The plate is modelled in $2D$ in plane deformations, by elements QUAD4. The springs are modelled by SEG2 affected of a modeling 2D_DIS_T whose characteristics are K_T_D_L. They are the discrete ones in translation having a diagonal matrix, to see the documentation of AFFE_CARA_ELEM.

4.2 Characteristics of the grid

The plate is cut out with $ny=16$. The length of the plate is $b=2m$.

4.3 Sizes tested and results

For the step of time n°1, displacements of the ends of the springs, off-line to the plate, are imposed on zero. Results of *Code_Aster* are compared with the discrete solution, which corresponds to the solution of the modelled problem. The solution of balance is obtained for $n=12$, equation 2.2-4.

Nature of the results	U_A	U_B
Continuous solution	$\frac{-4}{1125}$	$\frac{4}{3375}$
Discrete solution ($n=12$)	$\frac{-208}{58875}$	$\frac{176}{153075}$
Tolerance	2.0E-07	2.0E-07

For the step of time n°2, displacements of the ends of the springs, off-line to the plate, are moved of $+5.0\text{E-}03m$. Results of *Code_Aster* are compared with the discrete solution, which corresponds to the solution of the modelled problem.

Nature of the results	U_A	U_B
Continuous solution	$\frac{-4}{1125}$	$\frac{4}{3375}$
Discrete solution ($n=12$)	$\frac{-208}{58875}+\frac{5}{1000}$	$\frac{176}{153075}+\frac{5}{1000}$
Tolerance	2.0E-07	2.0E-07

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2017 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

5 Summary of the results

The use of the discrete, affected elements on nodes or segments, with a material of the type DIS_CONTACT and used with STAT_NON_LINE (behavior COMP_INCR and relation DIS_CHOC) allows to model a unilateral behavior of the springs.
In $2D$ as in $3D$, the use of the keyword RIGI_PARASOL order AFFE_CARA_ELEM allows to assign to the springs stiffnesses proportional to the length or the surface of the elements to which they are connected.
The behavior being unilateral, it is necessary that *Code_Aster* makes several iterations to find the position of balance. It is also possible to encounter problems of convergence related to a loss of precision, due to a bad conditioning of the matrix of stiffness during iterations. Stiffness of the springs being able to cancel itself of an iteration with the other.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2017 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)