SSNL134 - Elastoplastic ruin of the gantry of Lee

Summary

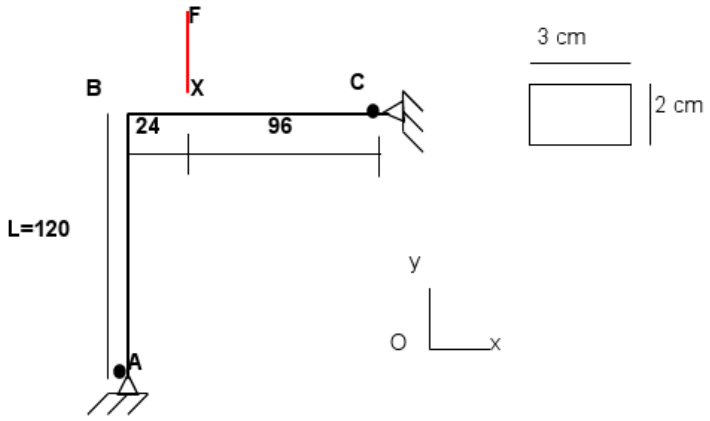
The purpose of this test is to validate the non-linear possibilities simultaneously material and geometrical of the multifibre element of beam POU_D_TGM. The element is implemented on a CAS-test usually treated in the literature with regard to the elastic behavior because it presents an answer complexes with *snap-back* and of *snap-through* : it is the gantry of Lee.

One supposes here an elastoplastic behavior of the gantry, which makes it possible to test the good integration of the law of behavior of the multifibre elements but also the correct treatment of great displacements. The got results are the object of comparisons to results resulting from the literature as well as tests of not-regression.

1 Problem of reference

1.1 Geometry

The gantry of Lee is a gantry with 2 arms, articulated at its ends:



Coordinates of the points (in cm):

	A	B	C	X
x	0	0	120	24
y	0	120	120	120

1.2 Characteristics of the section

The arms are with rectangular section of $3\,cm$ by $2\,cm$. The section is directed so that the weakest axis of the inertia is perpendicular to the plan of the gantry.

$$A = 6\,cm^2$$
$$I_y = 2.0\,cm^4$$
$$I_z = 4.5\,cm^4$$
$$A_y = 1.2$$
$$A_z = 1.2$$

1.3 Properties of material

$$E = 720\,N/cm^2$$
$$\nu = 0.3$$
$$E_t = 72\,N/cm^2$$
$$\sigma_e = 10.44\,N/cm^2$$

1.4 Boundary conditions and loading

1) Boundary conditions are imposed on the points A and C (articulation around Oz) :

$$DX = DY = DZ = DRX = DRY = 0$$

2) One imposes on the point X a top-load F :

$$FY = C \times t$$

The constant C corresponds to the coefficient of given piloting for example by a method length of arc. Indeed, this structure presents a complex answer under nonmonotonous loading.

2 Reference solution

2.1 Method used for calculation of the reference solution

This gantry was studied for the first time in 1968 by Lee [1], who suggested an analytical solution in the elastic case and by neglecting the deformation energy due to the normal effort. Since, several other authors have in their turn studied the structure, in particular in 1984, Cichon [2] was the first to suggest a solution by considering an elastoplastic behavior.

2.2 Results of reference

One is interested in horizontal and vertical displacements in the total reference mark of the point X for various values of the force applied.

One presents below a summary table of the results got by Cichon like by Waszczyszyn and Janus-Michalska [3]. One chooses like reference the average of these two results illustrated in **fat**.

Displacement (cm)	U (horizontal)			V (vertical)		
$F(N)$	Waszczyszyn	Cichon	Average	Waszczyszyn	Cichon	Average
0.968	1.812	1.734	**1.773**	10.414	10.078	**10.246**
1.242	3.482	3.913	**3.6975**	15.61	16.639	**16.1245**
1.4153	6.281	6.711	**6.496**	22.305	23.344	**22.8245**
1.507	12.516	13.836	**13.176**	38.804	36.501	**37.6525**
1.4605	19.025	18.148	**18.5865**	44.18	42.543	**43.3615**
1.4006	23.104	23.073	**23.0885**	49.262	47.374	**48.318**
1.3151	28.998	28.303	**28.6505**	53.012	51.102	**52.057**
1.1062	39.266	37.765	**38.5155**	58.904	56.181	**57.5425**
0.9275	46.871	44.831	**45.851**	61.775	58.442	**60.1085**

2.3 Uncertainty on the solution

Between 1 and 5% (maximum variation relative to the average of the results).

2.4 Bibliographical references

1 Lee S., Manual F.S., and Rossow E.C. “Broad deflections and stability of elastic frames.” *J. Engrg. Mech. Div., ASCE*, 1968; EM2, 521-547.

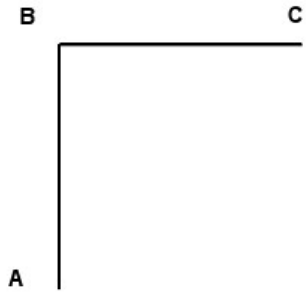
2 Cichon Cz. “Broad displacements in-plane analysis of elasto-plastic frames”. *Comp Struct* 1984; 19,737-45.

3 Waszczyszyn Z., Janus-Michalska Mr. “exact” ‘Numerical approach to the finite element analysis of in-plane finite displacements of framed structures”. *Comput. Struct*. 1998; 69, 525-535.

3 Modeling A

3.1 Characteristics of modeling

Modeling POU_D_TGM



Cutting: 10 elements in the post and the beam

3.2 Characteristics of the grid

Many nodes: 21
Many meshs and types: 20 SEG2

3.3 Characteristics of the grid of the transverse section

Many fibres: 100 (cutting in 10 on each side)
Many meshs and types: 100 QUAD4

3.4 Sizes tested and results

The results are got by an automatic step division of time and with piloting by length of arc of the total answer in displacement of the structure.

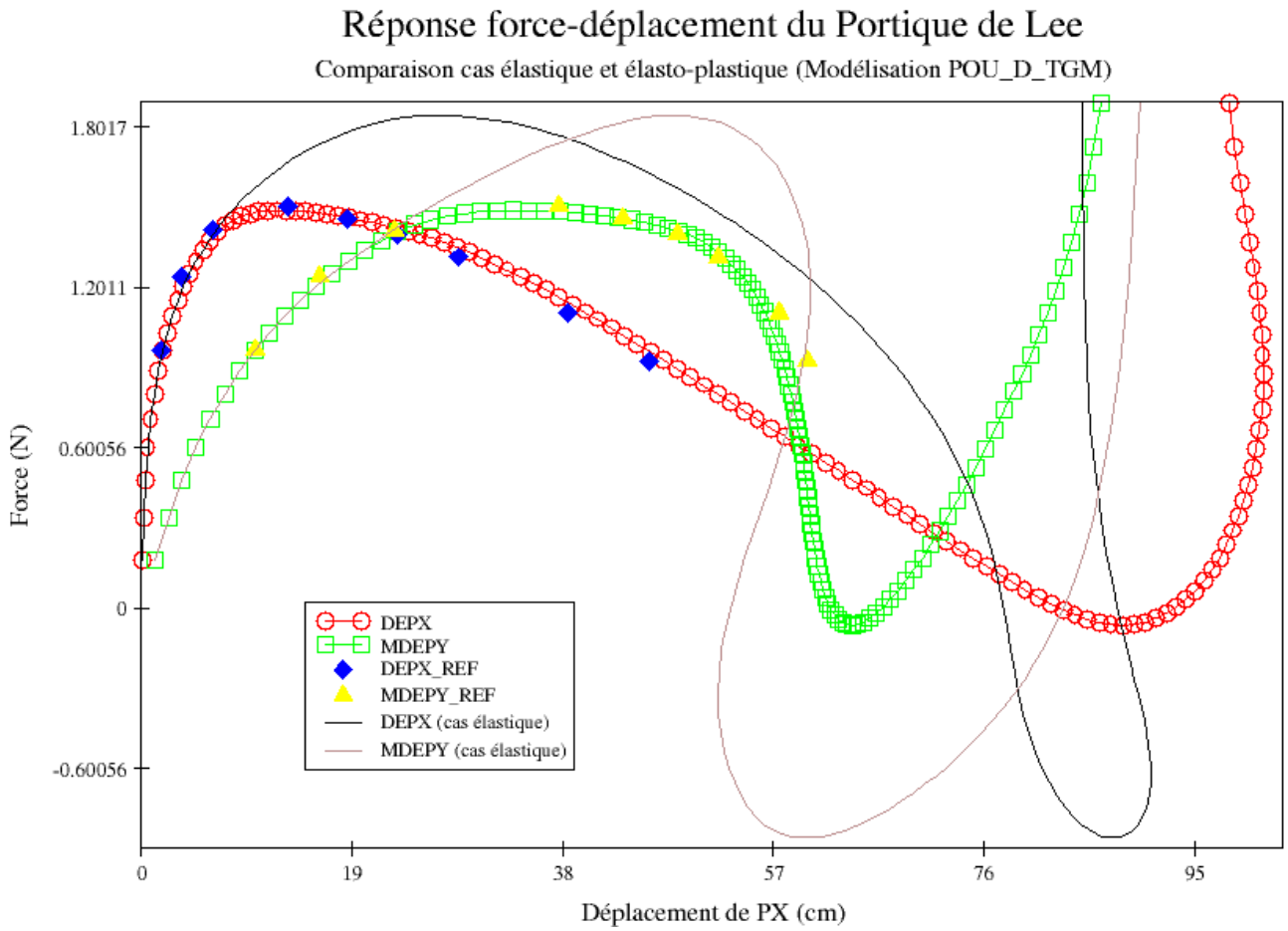
Displacement	Identification	Reference	Type of reference	Tolerance
1.773	DX	0.968	'SOURCE_EXTERNE'	5%
10.246	DY	0.968	'SOURCE_EXTERNE'	5%
3.6975	DX	1.242	'SOURCE_EXTERNE'	5%
16.1245	DY	1.242	'SOURCE_EXTERNE'	5%
6.496	DX	1.4153	'SOURCE_EXTERNE'	5%
22.8245	DY	1.4153	'SOURCE_EXTERNE'	5%
13.176	DX	1.507	'SOURCE_EXTERNE'	5%
37.6525	DY	1.507	'SOURCE_EXTERNE'	5%
18.5865	DX	1.4605	'SOURCE_EXTERNE'	5%
43.3615	DY	1.4605	'SOURCE_EXTERNE'	5%
23.0885	DX	1.4006	'SOURCE_EXTERNE'	5%
48.318	DY	1.4006	'SOURCE_EXTERNE'	5%
28.6505	DX	1.3151	'SOURCE_EXTERNE'	5%
52.057	DY	1.3151	'SOURCE_EXTERNE'	5%
38.5155	DX	1.1062	'SOURCE_EXTERNE'	5%
57.5425	DY	1.1062	'SOURCE_EXTERNE'	15%
45.851	DX	0.9275	'SOURCE_EXTERNE'	5%
60.1085	DY	0.9275	'SOURCE_EXTERNE'	55%

3.5 Remarks

The use of the technique length of arc makes difficult the definition of the value of reference to to introduce into the order TEST_RESU, since these values cannot be imposed. The tests were thus made with back by choosing like parameter the values of displacements and value tested the value of the force (i.e. ETA_PILOTAGE).
It is this manner of testing which explains the high differences noted in DY at the last moments whereas the curves are superimposed perfectly.

4 Summary of the results

4.1 Comparison with the elastic case



One presents above a chart of the answer force-displacement of the gantry of Lee where one made appear the answer in the case of an elastic structure (obtained with same modeling in *Code_Aster*).

It is interesting to notice as the addition of the elastoplastic behavior completely modifies the answer of the structure as of the first plasticization.

4.2 Synthesis

Broadly the mistake made on displacements never exceeds 5% that it is necessary to compare with uncertainty on the solution which is approximately 5%. Moreover as one explained, to carry out tests on values is difficult in the presence of piloting.

If one observes the figure above, one notices a good correlation between the answer of reference and that obtained by *Code_Aster*. One describes in particular well the change of pace due to the first plasticization.