SSNL502 - Beam in buckling

Summary:

This test represents a calculation of stability of a beam comforts subjected to a compressive force at an end. It makes it possible to validate modelings finite elements:

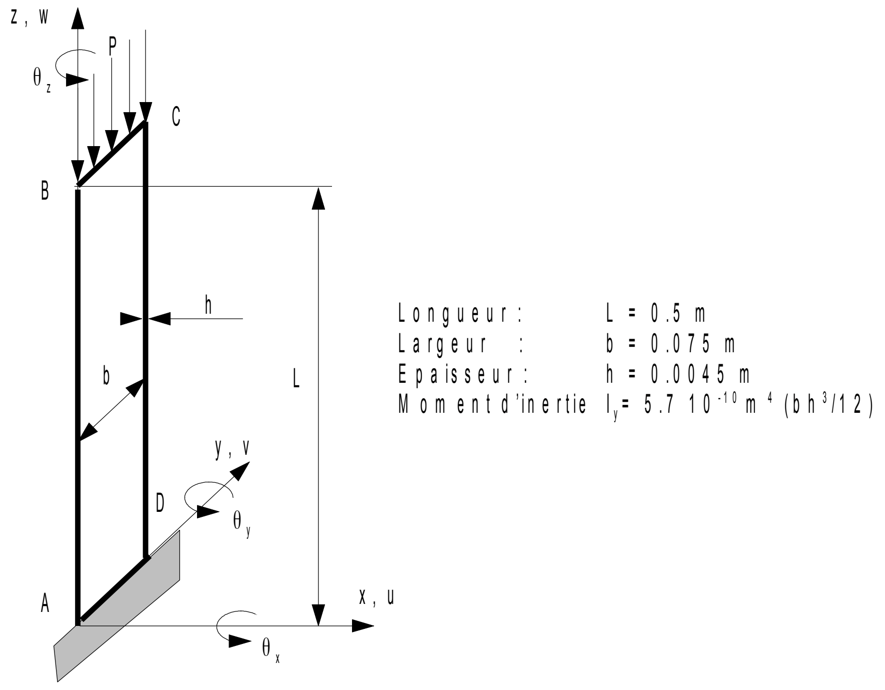
- `COQUE_3D` with the meshs `TRIA7` and `QUAD9`,
- `DKT` with the meshs `TRIA3` and `QUAD4`,
- `DKTG` with the meshs `TRIA3` and `QUAD4`,
- `POU_D_T_GD` and `POU_D_TGM` with the meshs `SEG2`.

in the non-linear quasi-static field in great displacements and great rotations in the presence of instability (Buckling of Euler).

Displacements and the moments obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



1.2 Properties of material

The properties of material constituting the plate are:

$E=2.10^{11}\,Pa$	Young modulus
$\nu=0.3$	Poisson's ratio

1.3 Boundary conditions and loadings

Boundary conditions: Side AD embedded

One seeks the successive states of balance under the loading imposed on the side BC :

$$p(t)=p_{cr}\,t$$

with	t	pseudo_temps
	p_{cr}	critical load of Euler

The load applied corresponds to the critical load of Euler $p_{cr}=\dfrac{\pi E I}{4L^2}=1124{,}21\,N$

1.4 Initial conditions

Without object

2 Reference solution

2.1 Method of calculating used for the reference solution

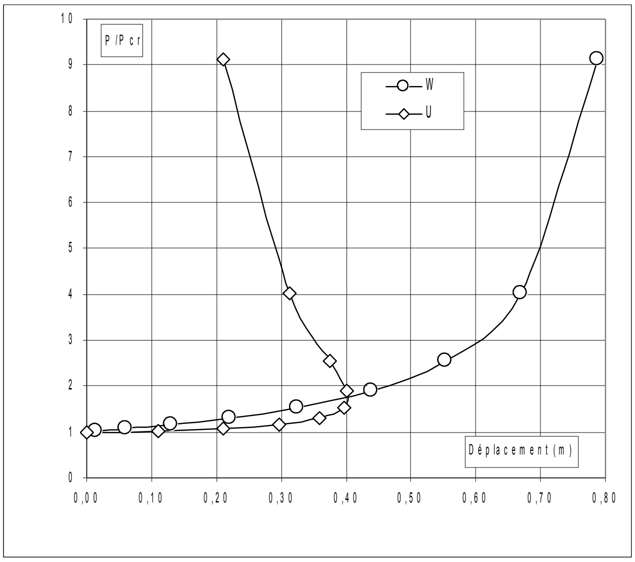
The solution of the problem known as of '' the élastique' 'is presented in [bib1] by making L' assumption of nonextension of the average axis. The analytical solution is obtained by considering elliptic integrals.

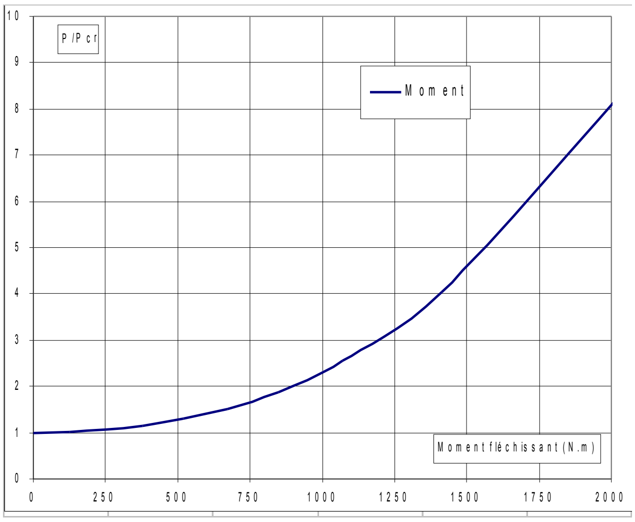
2.2 Results of reference

The results of reference retained for the checks are indicated in characters **fat** in the table below. Displacements are defined in the reference mark of definition of the geometry [§1.1].

P/P_{cr}	u_B/L	w_B/L	$\theta°$	$M_A L/EI$
1.015	**0.220**	**0.030**	**20°**	**0.56**
1.063	0.422	0.119	40°	1.09
1.152	**0.593**	**0.259**	**60°**	**1.67**
1.293	0.719	0.440	80°	2.28
1.518	**0.792**	**0.651**	**100°**	**2.96**
1.884	0.803	0.877	120°	3.73
2.541	**0.750**	**1.107**	**140°**	**4.70**
4.029	0.625	1.340	160°	6.20
9.116	**0.421**	**1.577**	**176°**	**9.44**

Load P (N)	u_B (m)	w_B (m)	M_A ($N.M$)
1141.07	**0.1100**	**0.0150**	**127.58**
1195.03	0.2110	0.0595	248.32
1295.09	**0.2965**	**0.1295**	**380.45**
1453.60	0.3595	0.2200	519.41
1706.55	**0.3960**	**0.3255**	**674.33**
2118.01	0.4015	0.4385	849.74
2856.62	**0.3750**	**0.5535**	**1070.72**
4529.44	0.3125	0.6700	1412.44
10248.29	**0.2105**	**0.7885**	**2150.55**





2.3 Uncertainties on the solution

Analytical solution

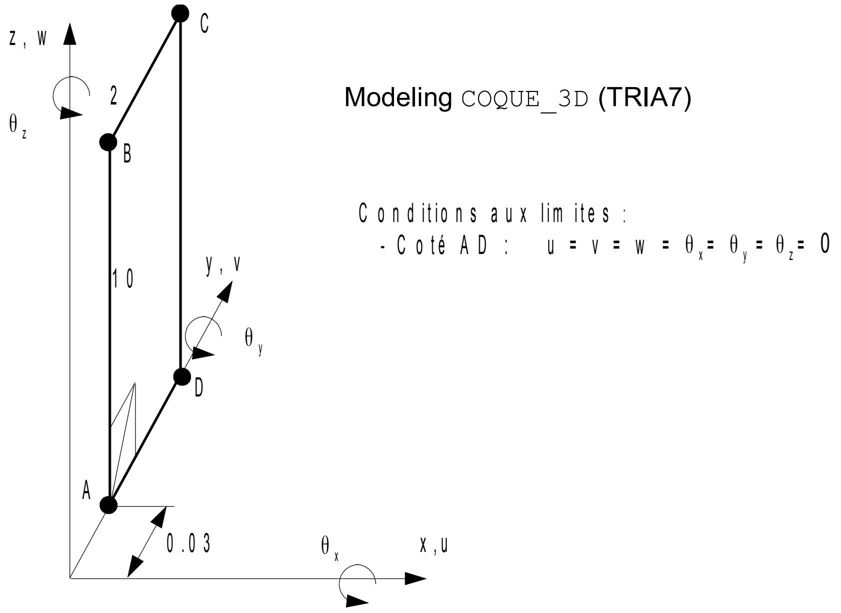
2.4 Bibliographical references

[1] S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, second edition, DUNOD 1966.

[2] J.L. BATOZ: Great displacements and great rotations of elastic thin beams, mechanical Department of engineering University of Technology of Compiegne 1981.

3 Modeling A

3.1 Characteristics of modeling



3.2 Characteristics of the grid

Many nodes: 145
Number of meshs and type: 40 TRIA7

3.3 Sizes tested and results

The strategy of calculation used breaks up into two stages:

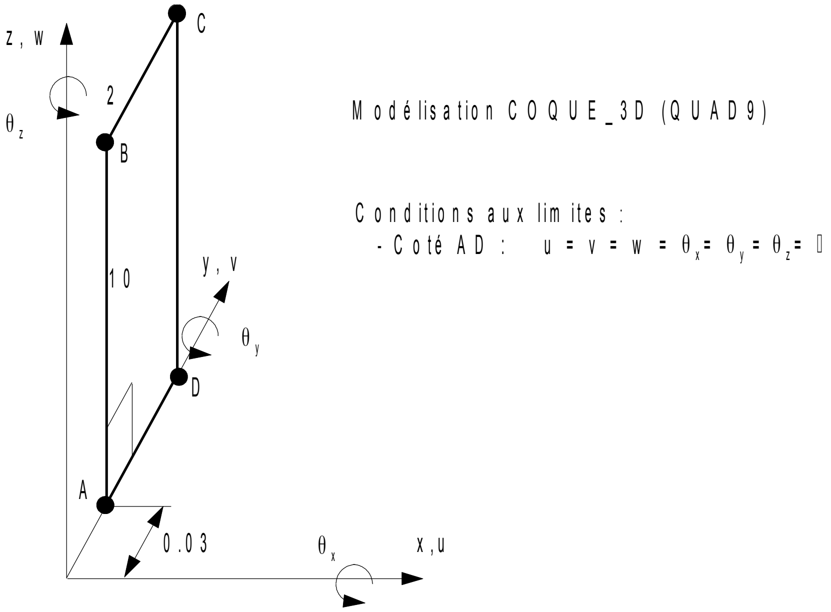
- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load according to X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.98$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond 1.01, the structure became very flexible, one imposes an increase in displacement `DZ` (option `DDL_IMPO` in `STAT_NON_LINE`) to determine the behavior postbuckling.

The use of the technique length of arc makes difficult the definition of the value of reference to be introduced into the order `TEST_RESU`, since these values cannot be imposed. To define the values of reference, we searched the values of `DZ` closest possible to those listed in the table of [§2.2] and we deferred the values of the parameter of piloting and of `DX` that one was to obtain for the values of `DZ` in question.

`DZ`	**Identification**	**Moments**	**Reference**
– 0.0150	`DX`	1.04532	0.1100
	`DZ`	1.04532	– 0.0150
	`ETA_PILOTAGE`	1.04532	1,015
– 0.0595	`DX`	1.09778	0.2110
	`DZ`	1.09778	– 0.0595
	`ETA_PILOTAGE`	1.09778	1,063
– 0.22	`DX`	1.20824	0.3595
	`DZ`	1.20824	– 0.22
	`ETA_PILOTAGE`	1.20824	1,293
– 0.3255	`DX`	1.26646	0,396
	`DZ`	1.26646	– 0.3255
	`ETA_PILOTAGE`	1.26646	1,518
– 0.5535	`DX`	1.38521	0,375
	`DZ`	1.38521	– 0.5535
	`ETA_PILOTAGE`	1.38521	2,541
– 0.67	`DX`	1.46121	0.3125
	`DZ`	1.46121	– 0.67
	`ETA_PILOTAGE`	1.46121	4,029

4 Modeling B

4.1 Characteristics of modeling



4.2 Characteristics of the grid

Many nodes: 105
Number of meshs and type: 20 QUAD9

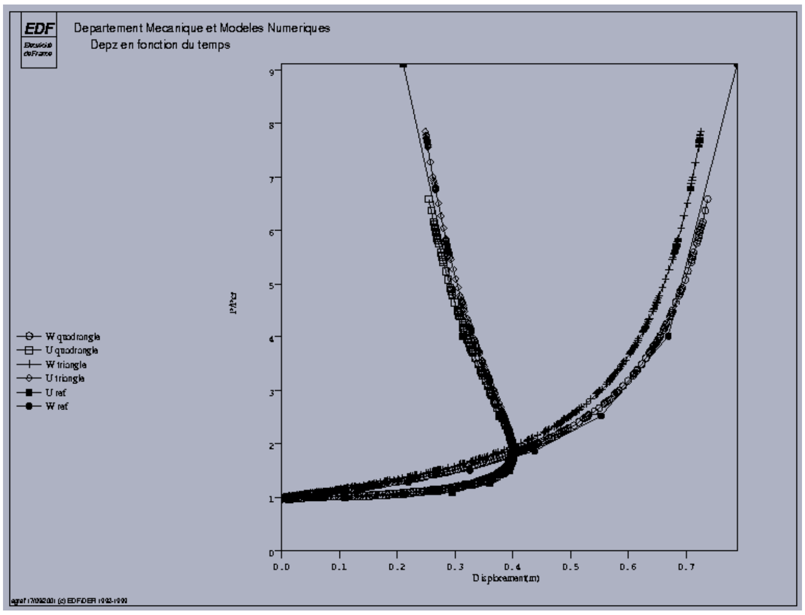
4.3 Sizes tested and results

The strategy of calculation used breaks up into two stages:

- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load according to X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.98$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond 1.01, the structure became very flexible, one imposes an increase in displacement `DZ` (option `DDL_IMPO` in `STAT_NON_LINE`) to determine the behavior postbuckling.

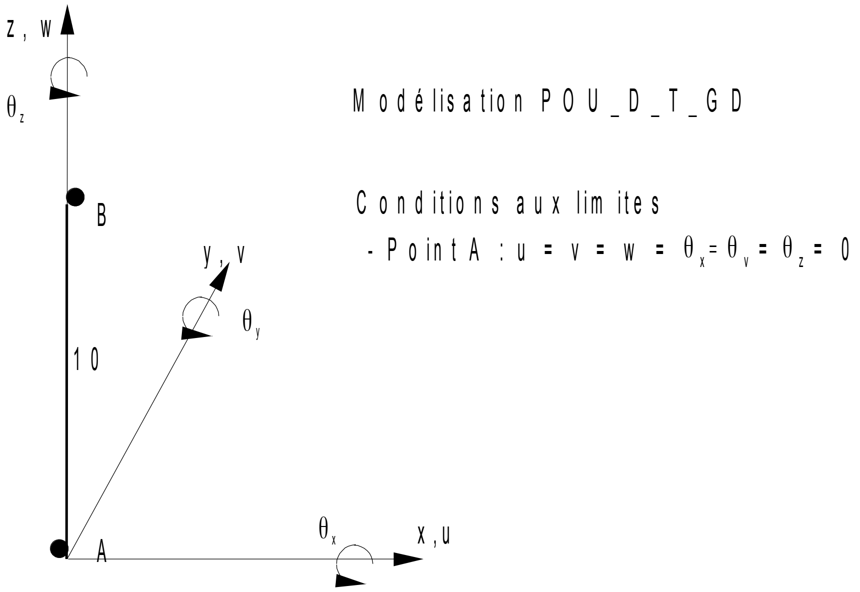
The use of the technique length of arc makes difficult the definition of the value of reference to be introduced into the order `TEST_RESU`, since these values cannot be imposed. To define the values of reference, we searched the values of `DZ` closest possible to those listed in the table of [§2.2] and we deferred the values of the parameter of piloting and of `DX` that one was to obtain for the values of `DZ` in question.

`DZ`	**Identification**	**Moments**	**Reference**
– 0.0150	`DX`	1.03356	0.1100
	`DZ`	1.03356	– 0.0150
	`ETA_PILOTAGE`	1.03356	1,015
– 0.0595	`DX`	1.08921	0.2110
	`DZ`	1.08921	– 0.0595
	`ETA_PILOTAGE`	1.08921	1,063
– 0.22	`DX`	1.20259	0.3595
	`DZ`	1.20259	– 0.22
	`ETA_PILOTAGE`	1.20259	1,293
– 0.3255	`DX`	1.25521	0,396
	`DZ`	1.25521	– 0.3255
	`ETA_PILOTAGE`	1.25521	1,518
– 0.5535	`DX`	1.37521	0,375
	`DZ`	1.37521	– 0.5535
	`ETA_PILOTAGE`	1.37521	2,541
– 0.67	`DX`	1.45321	0.3125
	`DZ`	1.45321	– 0.67
	`ETA_PILOTAGE`	1.45321	4,029



5 Modeling C

5.1 Characteristics of modeling



5.2 Characteristics of the grid

Many nodes: 11
Number of meshs and type: 10 SEG2

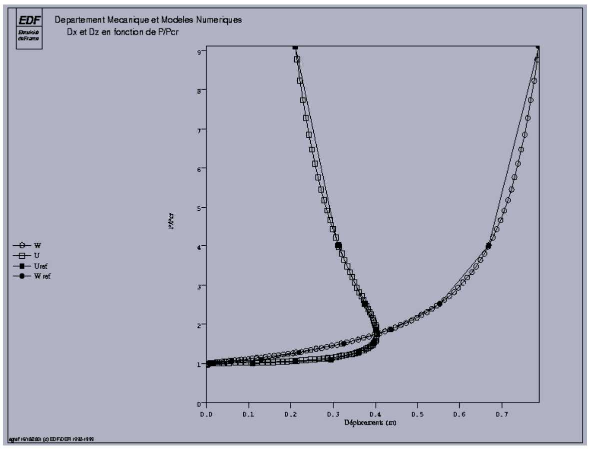
5.3 Sizes tested and results

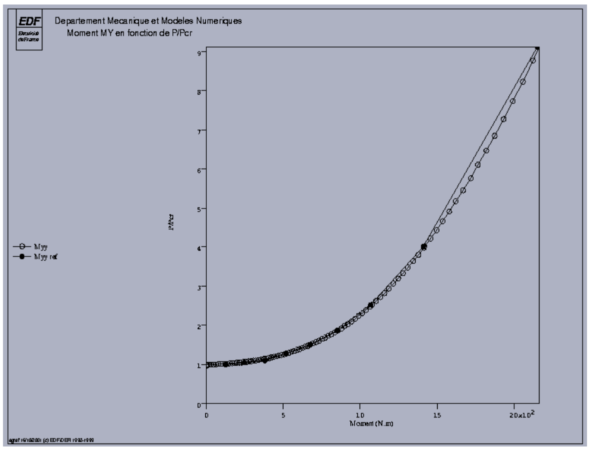
5.3.1 Values tested

The strategy of calculation used breaks up into two stages:

- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load according to X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.98$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond 1.01, the structure became very flexible, one imposes an increase in displacement `DZ` (option `DDL_IMPO` in `STAT_NON_LINE`) to determine the behavior postbuckling.
- The results are in good adequacy with the reference solution from `ETA_PILOTAGE` = 1,293. Before this value, the disturbing load (necessary to obtain buckling) degrades the solution, and the variations with the analytical solution are important (up to 80%). The corresponding values are the object of tests of nonregression. But this variation is only related to the disturbing load, since by increasing the top-load, one finds the good solution.

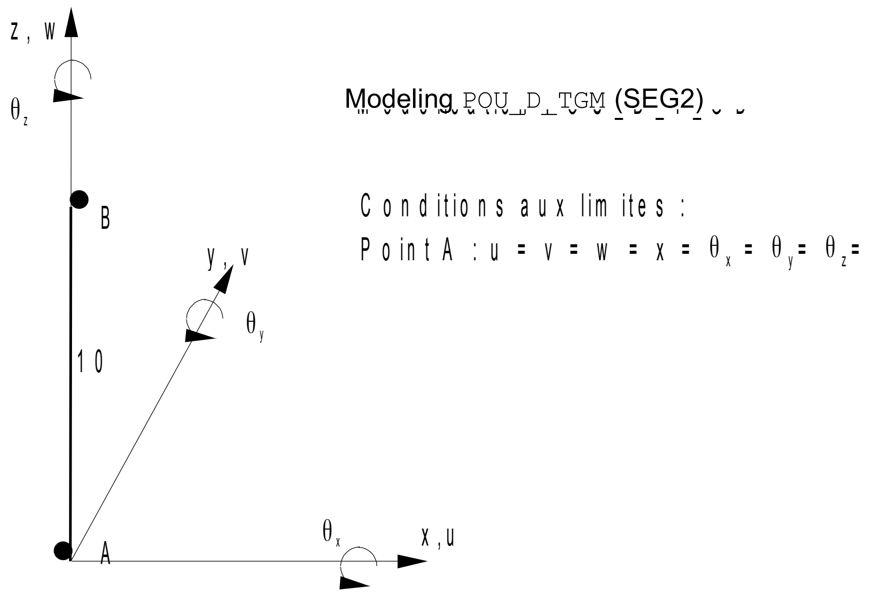
DZ	**Identification**	**Moments**	**Reference**
– 0.22	`DX`	1.18684	0.3595
	`DZ`	1.18684	– 0.22
	`ETA_PILOTAGE`	1.18684	1,293
	`MYY`	1.18684	519.41
– 0.3255	`DX`	1.24521	0,396
	`DZ`	1.24521	– 0.3255
	`ETA_PILOTAGE`	1.24521	1,518
	`MYY`	1.24521	674.3
– 0.4385	`DX`	1.30521	0.4015
	`DZ`	1.30521	– 0.4385
	`ETA_PILOTAGE`	1.30521	1,884
	`MYY`	1.30521	849.74





6 Modeling D

6.1 Characteristics of modeling



6.2 Characteristics of the grid

Many nodes: 11
Number of meshs and type: 10 SEG2 uniformly distributed in the length

6.3 Characteristics of the grid of the transverse section (fibres)

Many fibres: 50 (5 in the width and 10 in the thickness)
Number of meshs and type: 50 QUA4

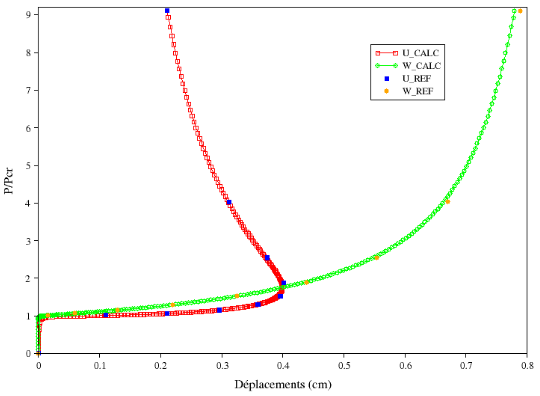
6.4 Sizes tested and results

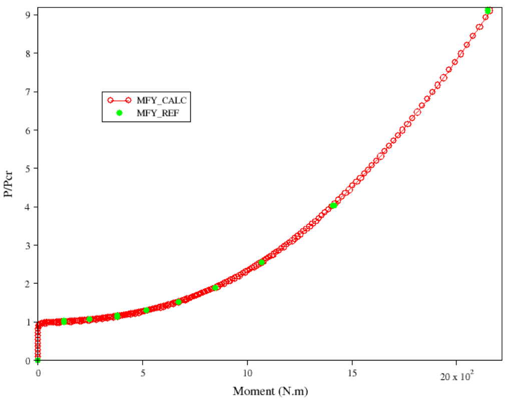
6.4.1 Values tested

The strategy of calculation used breaks up into two stages:

- **Imposed loading** : There is predeformed the structure according to its first mode of buckling and with a very low amplitude in front of the length of the beam (about $5.10^{-4} m$). One applies the loading until $P/P_{cr}=0.95$.
- **Imposed displacement** : beyond 0.95, the structure starts to undergo a strong side displacement for a very weak increase in the loading, one thus controls the structure in length of arc to determine the behavior postbuckling.
- The results are in good adequacy with the reference solution from `ETA_PILOTAGE` = 1,152. Before this value, the predeformation (necessary to obtain buckling) degrades the solution, and the variations with the analytical solutions are important (up to 60%). The corresponding values are the object of tests of nonregression. But this variation is only related to the initial predeformation (arbitrary), since by increasing the top-load, one finds the good solution.

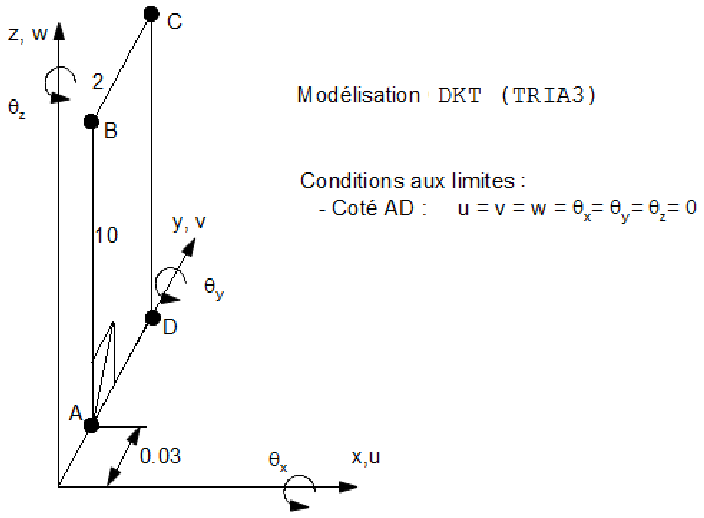
`DZ`	Identification	Moments	Reference
– 0.22	`DX`	3.8	0.3595
	`DZ`	3.8	– 0.22
	`ETA_PILOTAGE`	3.8	1,293
	`MYY`	3.8	519.41
– 0.3255	`DX`	4.56	0,396
	`DZ`	4.56	– 0.3255
	`ETA_PILOTAGE`	4.56	1,518
	`MYY`	4.56	674.3
– 0.4385	`DX`	5,358	0.4015
	`DZ`	5,358	– 0.4385
	`ETA_PILOTAGE`	5,358	1,884
	`MYY`	5,358	849.74


P/Pcr en fonction des déplacements U et W


P/Pcr en fonction du moment MFY

7 Modeling E

7.1 Characteristics of modeling



7.2 Characteristics of the grid

Many nodes: 33
Number of meshs and type: 40 TRIA3, 2 SEG2

7.3 Sizes tested and results

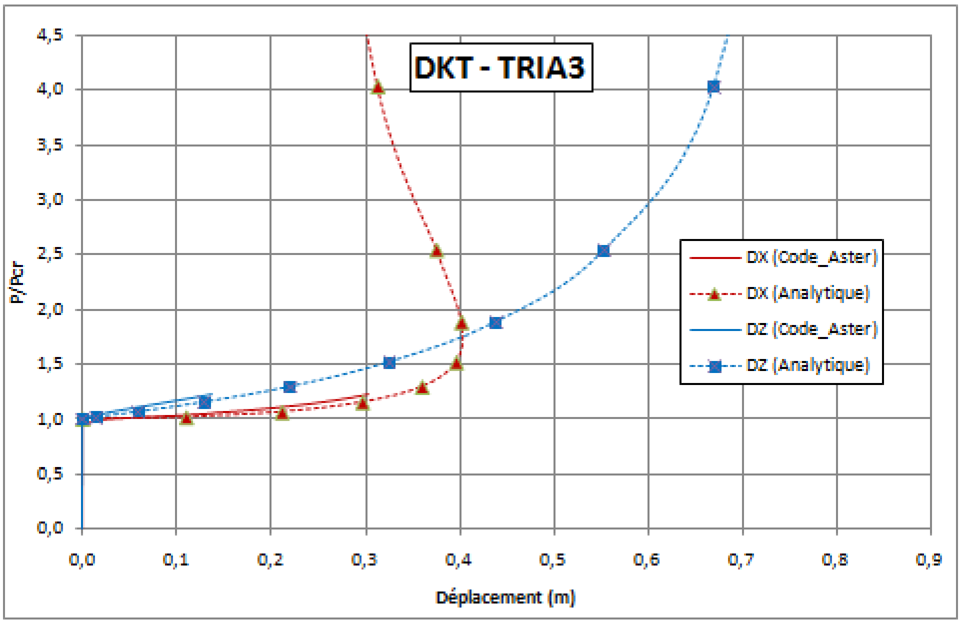
7.3.1 Values tested

The strategy of calculation used breaks up into two stages:

- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load according to X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.95$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond `0,947`, the structure starts to undergo a strong side displacement for a very weak increase in the loading, one thus controls the structure in length of arc to determine the behavior postbuckling.
- Calculations do not converge any more beyond `ETA_PILOTAGE = 1.24936`. The evolution of displacements according to the loading is presented on the figure hereafter. The critical load is well detected. In the postbuckling part, one notes a maximum change of 5% on the critical load with the analytical solution.

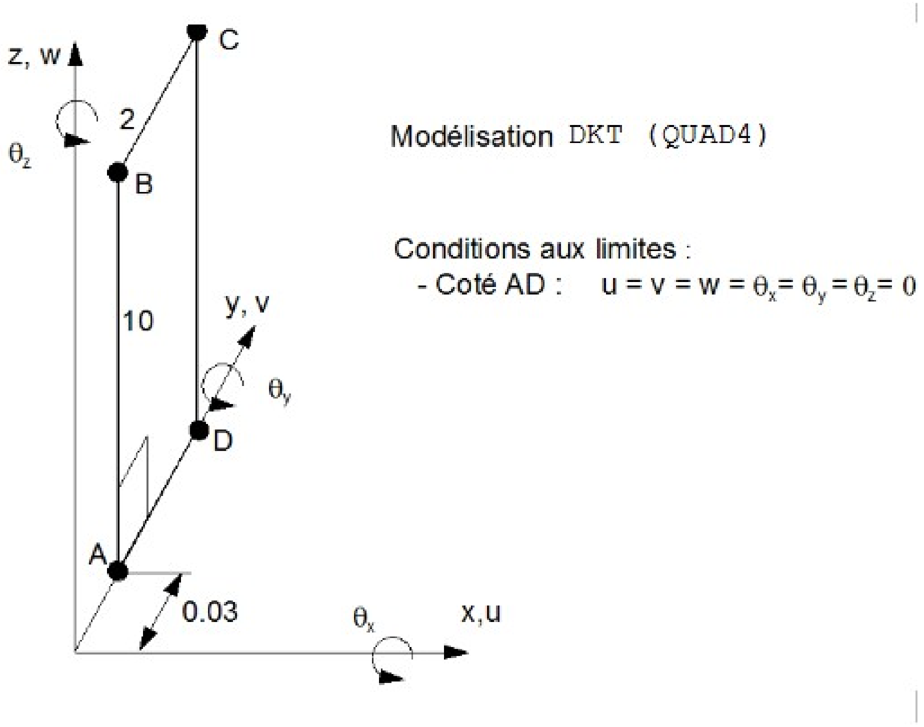
The use of the technique length of arc makes difficult the definition of the value of reference to be introduced into the order `TEST_RESU` , since these values cannot be imposed. To define the values of reference, we searched the values of `DZ` closest possible to those listed in the table of [§2.2] and we deferred the values of the parameter of piloting and of `DX` that one was to obtain for the values of `DZ` in question.

`DZ`	**Identification**	**Moments**	**Type of reference**	**Value of reference**	**Tolerance (%)**
– 0.0150	`DX`	1,001	`'ANALYTICAL'`	0,110	1.0
	`DZ`	1,001	`'ANALYTICAL'`	– 0,015	2.0
	`ETA_PILOTAGE`	1,001	`'ANALYTICAL'`	1,015	3.0
– 0.0595	`DX`	1.05867	`'ANALYTICAL'`	0.2110	2.0
	`DZ`	1.05867	`'ANALYTICAL'`	– 0.0595	3.5
	`ETA_PILOTAGE`	1.05867	`'ANALYTICAL'`	1,063	5.0



8 Modeling F

8.1 Characteristics of modeling



8.2 Characteristics of the grid

Many nodes: 33
Number of meshs and type: 20 QUAD4, 2 SEG2

8.3 Sizes tested and results

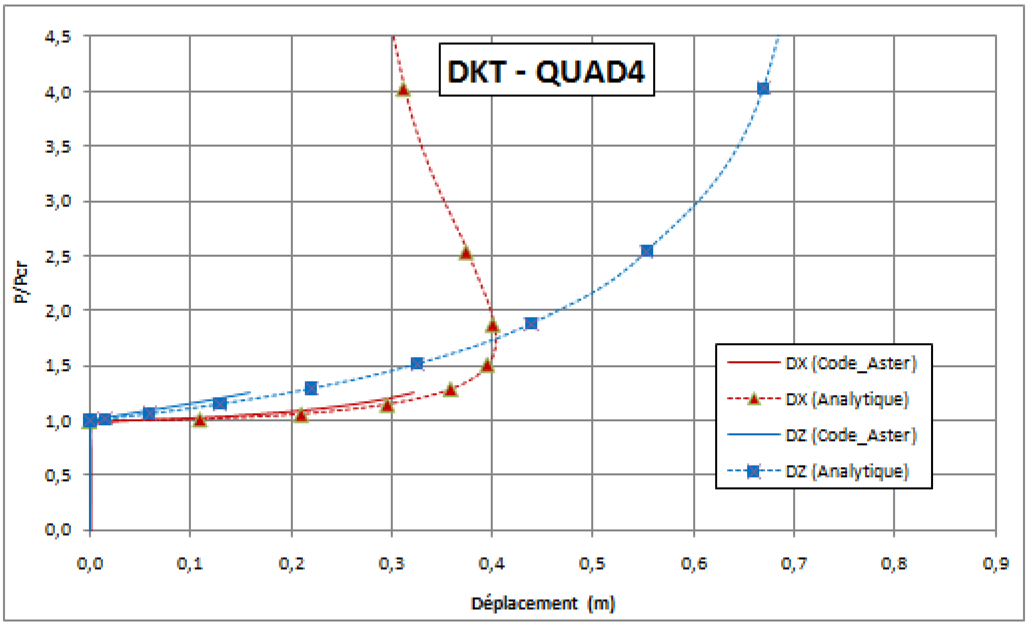
8.3.1 Values tested

The strategy of calculation used breaks up into two stages:

- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load according to X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.95$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond `0.94`, the structure starts to undergo a strong side displacement for a very weak increase in the loading, one thus controls the structure in length of arc to determine the behavior postbuckling.
- Calculations do not converge any more beyond `ETA_PILOTAGE = 1.1277`. The evolution of displacements according to the loading is presented on the figure hereafter. The critical load is well detected. In the postbuckling part, one notes a maximum change of 3.5% on the critical load with the analytical solution.

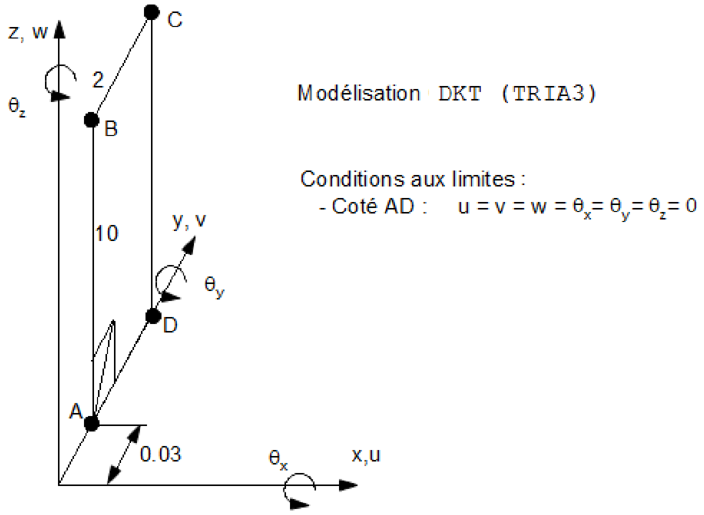
The use of the technique length of arc makes difficult the definition of the value of reference to be introduced into the order `TEST_RESU`, since these values cannot be imposed. To define the values of reference, we searched the values of `DZ` closest possible to those listed in the table of [§2.2] and we deferred the values of the parameter of piloting and of `DX` that one was to obtain for the values of `DZ` in question.

DZ	Identification	Moments	Type of reference	Value of reference	Tolerance (%)
– 0.0150	DX	0,995	'ANALYTICAL'	0,110	2.0
	DZ	0,995	'ANALYTICAL'	– 0,015	2.5
	ETA_PILOTAGE	0,995	'ANALYTICAL'	1,015	1.5
– 0.0595	DX	1.05136	'ANALYTICAL'	0.2110	0.5
	DZ	1.05136	'ANALYTICAL'	– 0.0595	0.7
	ETA_PILOTAGE	1.05136	'ANALYTICAL'	1,063	3.5



9 Modeling G

9.1 Characteristics of modeling



9.2 Characteristics of the grid

Many nodes: 33
Number of meshs and type: 40 TRIA3, 2 SEG2

9.3 Sizes tested and results

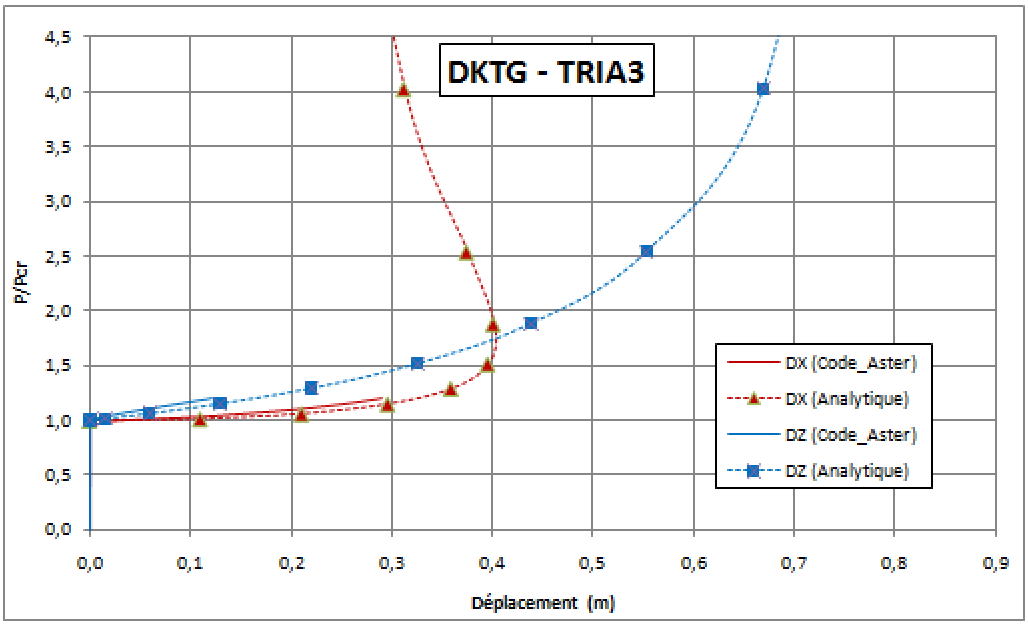
9.3.1 Values tested

The strategy of calculation used breaks up into two stages:

- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load according to X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.95$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond `0,947`, the structure starts to undergo a strong side displacement for a very weak increase in the loading, one thus controls the structure in length of arc to determine the behavior postbuckling.
- Calculations do not converge any more beyond `ETA_PILOTAGE = 1.20914`. The evolution of displacements according to the loading is presented on the figure hereafter. The critical load is well detected. In the postbuckling part, one notes a maximum change of 5% on the critical load with the analytical solution.

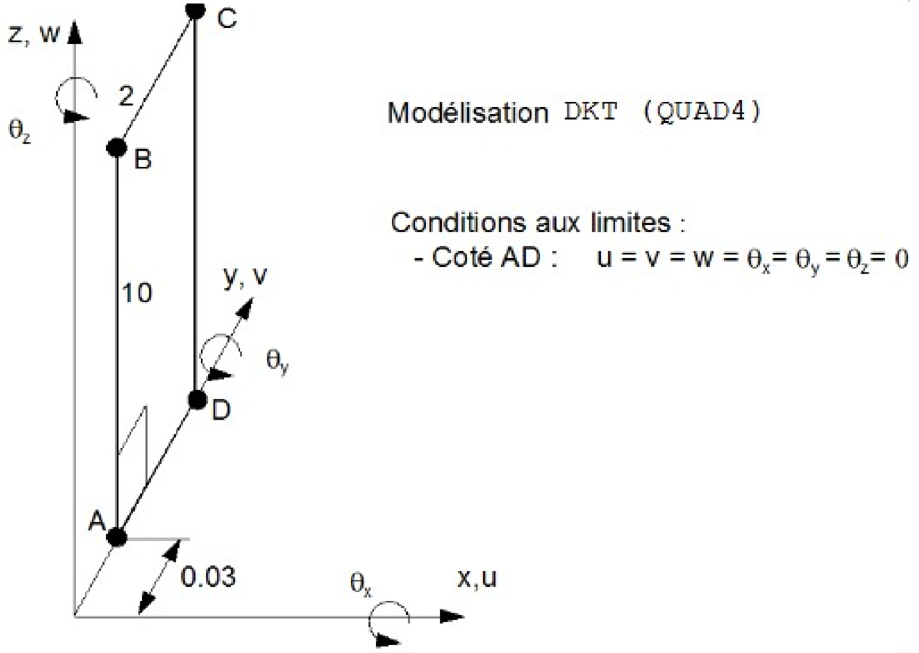
The use of the technique length of arc makes difficult the definition of the value of reference to be introduced into the order `TEST_RESU`, since these values cannot be imposed. To define the values of reference, we searched the values of `DZ` closest possible to those listed in the table of [§2.2] and we deferred the values of the parameter of piloting and of `DX` that one was to obtain for the values of `DZ` in question.

DZ	**Identification**	**Moments**	**Type of reference**	**Value of reference**	**Tolerance (%)**
– 0.0150	DX	1,001	'ANALYTICAL'	0,110	0.1
	DZ	1,001	'ANALYTICAL'	– 0,015	2.0
	ETA_PILOTAGE	1,001	'ANALYTICAL'	1,015	3.0
– 0.0595	DX	1.054217	'ANALYTICAL'	0.2110	2.5
	DZ	1.054217	'ANALYTICAL'	– 0.0595	5.0
	ETA_PILOTAGE	1.054217	'ANALYTICAL'	1,063	5.0



10 Modeling H

10.1 Characteristics of modeling



10.2 Characteristics of the grid

Many nodes: 33
Number of meshs and type: 20 QUAD4, 2 SEG2

10.3 Sizes tested and results

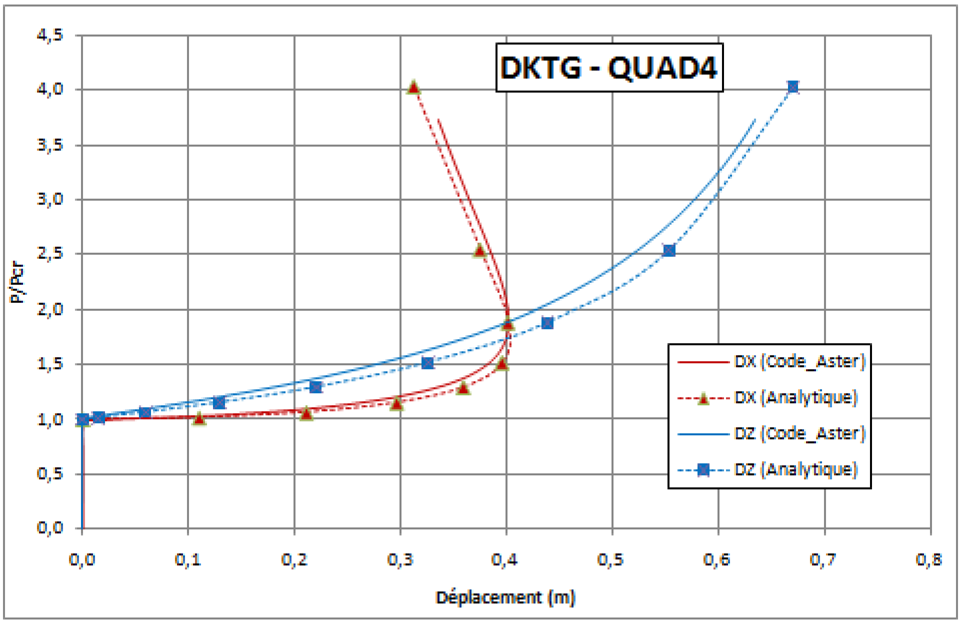
10.3.1 Values tested

The strategy of calculation used breaks up into two stages:

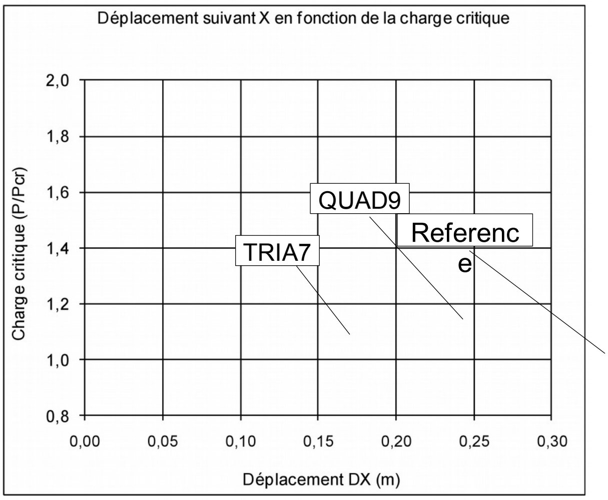
- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load according to X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.95$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond `0,947`, the structure starts to undergo a strong side displacement for a very weak increase in the loading, one thus controls the structure in length of arc to determine the behavior postbuckling.
- Calculations do not converge any more beyond `ETA_PILOTAGE = 3.73969`. The evolution of displacements according to the loading is presented on the figure hereafter. The critical load is well detected. In the postbuckling part, one notes a maximum change of 10% on the critical load with the analytical solution.

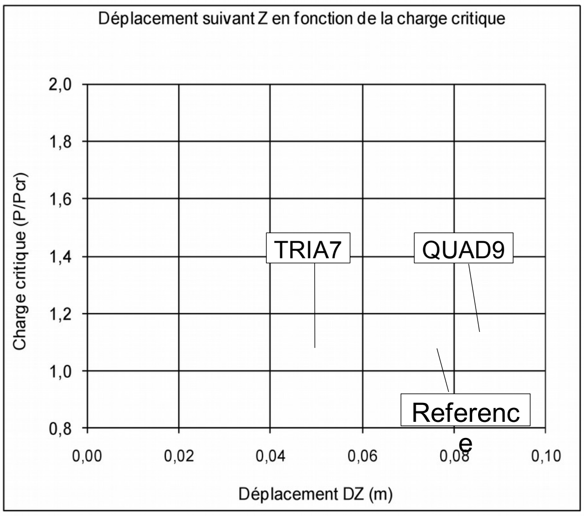
The use of the technique length of arc makes difficult the definition of the value of reference to be introduced into the order `TEST_RESU`, since these values cannot be imposed. To define the values of reference, we searched the values of `DZ` closest possible to those listed in the table of [§2.2] and we deferred the values of the parameter of piloting and of `DX` that one was to obtain for the values of `DZ` in question.

`DZ`	**Identification**	**Moments**	**Type of reference**	**Value of reference**	**Tolerance (%)**
– 0.0150	`DX`	0,995	`'ANALYTICAL'`	0,110	2.0
	`DZ`	0,995	`'ANALYTICAL'`	– 0,015	2.5
	`ETA_PILOTAGE`	0,995	`'ANALYTICAL'`	1,015	1.5
– 0.1295	`DX`	1.10671	`'ANALYTICAL'`	0.2965	0.7
	`DZ`	1.10671	`'ANALYTICAL'`	– 0.1295	1.0
	`ETA_PILOTAGE`	1.10671	`'ANALYTICAL'`	1,152	5.0
– 0.3255	`DX`	1.22068	`'ANALYTICAL'`	0,396	0.5
	`DZ`	1.22068	`'ANALYTICAL'`	– 0.3255	0.7
	`ETA_PILOTAGE`	1.22068	`'ANALYTICAL'`	1,518	8.0
– 0.5535	`DX`	1.34521	`'ANALYTICAL'`	0,375	0.1
	`DZ`	1.34521	`'ANALYTICAL'`	– 0.5535	0.5
	`ETA_PILOTAGE`	1.34521	`'ANALYTICAL'`	2,541	10.0



11 Summary of the results





The critical load is well detected. The first two results corresponding to the loads $P/P_{cr}=1.015$ and 1.063 are correct, the maximum error is of 3.5% for the mesh TRIA7 and of 2.2% for the mesh QUAD9. The mesh QUAD9 give better results.

If one continues calculations with the elements of hulls, the mesh QUAD9 continue to give better results. In the zone where displacements in DZ are most important, the mistake made on the load reaches 9% on the quadrangles and goes up to 30% on the triangles. The errors increase in this area because of the slopes of the curves.

The coefficient of correction of transverse shearing A_CIS was put at 0,833, corresponding to the thick hulls. The value ($9000=10^6\times H/L$) who should have been taken into account does not allow to carry out calculations. It introduces a bad conditioning of the matrices of rigidity by increasing their disparities.

The solution beam of the code, that it is modeling POU_D_T_GD or POU_D_TGM, provides very good performances compared to the analytical solution of reference. However, the computing times are different, modeling POU_D_T_GD who exact is geometrically led to a computing time almost 40% inferior compared to modeling POU_D_TGM. Indeed deformation 'PETIT_REAC' used for the model multifibre is only one approximation of great displacements and especially of great rotations, it requires to make small increments of loading. On the other hand, it should be recalled that one is able to treat behaviors other than elastic with POU_D_TGM.

Some is the mesh TRIA3 or QUAD4, modelings DKT and DKTG, detect the critical load well. Modelings DKT (QUAD4 and TRIA3) and DKTG (TRIA3) do not converge any more beyond $P/P_{cr}=1.2$. On the other hand for modeling DKTG (QUAD4) calculations go beyond that, to reach $P/P_{cr}=3.73969$. The mistake made on displacements is to the maximum of 3% and 5% for $P/P_{cr}=1.063$ and 10% for $P/P_{cr}=2.451$.