SSNP108 - Prestressed concrete element in compression

Summary:

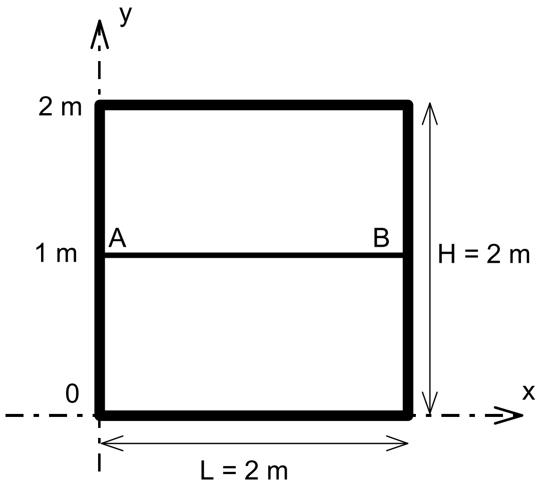
One considers an elementary structure made up of a square concrete plate crossed by a cable of prestressed whose neutral fibre is confused with the horizontal axis of symmetry of the plate. The left vertical edge of the plate is fixed. The cable is put in traction at its two ends in order to prestress the toggle plate. The losses of tension along the cable are neglected.

The goal of this CAS-test is to validate, on a simple configuration, the method of calculating of the state of balance of a prestressed structure of concrete. The results are validated by comparison with an analytical solution.

1 Problem of reference

1.1 Geometry

The concrete plate is square; the sides have even length $L=H=2m$.
The thickness of the plate is worth $e=0,6m$.
The cable crosses the plate horizontally, with middle height, without eccentricity in the thickness. The surface of the cross-section of the cable is worth $S_a=1,5.10^{-4}m^2$.



1.2 Properties of materials

Material concrete constituting the plate:	Young modulus $E_b=3.10^{10}Pa$
Material steel constituting the cable:	Young modulus $E_a=2,1.10^{11}Pa$

The Poisson's ratio is taken equal to 0 for two materials; one thus privileges the direction of normal load application (direction x).

The losses of tension being neglected, the various parameters being used for their estimate are fixed at 0.

1.3 Boundary conditions and loadings

The lower top of the left edge of the plate, i.e. the node origin $(0;0)$, is embedded: all the degrees of freedom of translation and rotation are blocked.
The higher top of this same left edge, i.e. the node $(0;2)$, is supported bilaterally: the degrees of freedom of translation blocked are DX and DZ .

One applies at the two ends of the cable (which are fixed on the concrete in A and B) a normal effort of traction: $(-F_0;0)$ with the node A $(0;1)$ and $(F_0;0)$ with the node B $(2;1)$, with $F_0=2.10^5N$.

2 Reference solution

2.1 Formal solution

Displacements in the cable and the concrete are continuous and homogeneous. Thus horizontal displacement on the interval $[0;L]$ is worth $u(x)=\frac{u_L}{L}x$

The normal constraints in the concrete plate and the steel wire rope are written respectively (assumption of elasticity):

$$\begin{cases} \sigma_b=E_b\frac{\partial u}{\partial x}=E_b\frac{u_L}{L} \\ \sigma_a=E_a\frac{\partial u}{\partial x}+\sigma_0=E_a\frac{u_L}{L}+\sigma_0 \end{cases}$$

where $\sigma_0=\frac{F_0}{S_a}$ is initial prestressing in the cable

and u_L is horizontal displacement with the X-coordinate L .

The balance of the unit plates and cables is written: $\sigma_b S_b+\sigma_a S_a=0 \Rightarrow u_L=\frac{-L F_0}{E_b e H+E_a S_a}$

The normal effort in the cable is worth: $N_a=E_a S_a\frac{u_L}{F}+F_0=F_0\frac{E_b e H}{E_b e H+E_a S_a}$

The total effort on the vertical section of the concrete plate is worth:

$$N_b=E_b e H\frac{u_L}{L}=-F_0\frac{E_b e H}{E_b e H+E_a S_a}$$

One from of deduced the linear density of normal effort on the concrete plate

$$N_{xx}=-F_0\frac{E_b e}{E_b e H+E_a S_a}$$

2.2 Digital values of reference

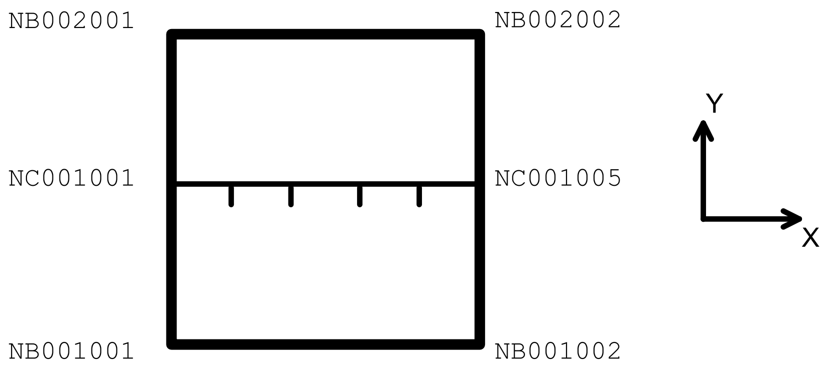
The digital values of reference are:

$$u_L=-1{,}11013974.10^{-5}m$$
$$N_a=1{,}99825153.10^{5}N$$
$$N_b=-1{,}99825153.10^{5}N$$
$$N_{xx}=-9{,}99125765.10^{4}N/m$$

3 Modeling A

3.1 Characteristics of modeling

The figure below gives a simplified representation of the grid.



The concrete plate is represented by an element DKT, supported by a mesh quadrangle with 4 nodes.

A thickness $e=0,6\,m$ he is affected, as well as a material concrete for which the behaviors are defined ELAS (Young modulus $E_b=3.10^{10}\,Pa$) and BPEL_BETON : the parameters characteristic of this relation are fixed at 0 because one neglects the losses of tension along the cable of prestressing.

The node $NB001001$ is embedded: DX, DY, DZ, DRX, DRY MARTINI and DRZ are blocked. The node $NB002001$ is supported bilaterally: DX and DZ are blocked.

The cable is represented by 4 elements MECA_BARRE, supported by 4 meshs segments with 2 nodes.

A surface of cross-section $S_a=1,5.10^{-4}\,m^2$ their is affected, as well as a material steel for which the behaviors are defined ELAS (Young modulus $E_a=2,1.10^{11}\,Pa$) and BPEL_ACIER : the parameters characteristic of this relation are fixed at 0 (neglected losses of tension), except for the elastic ultimate stress for which a zero value is illicit ($f_{prg}=1,77.10^9\,Pa$).

The tension $F_0=2.10^5\,N$ is applied to the nodes $NC001001$ and $NC001005$. This value of tension is coherent with the values of section and yield stress, for a cable of prestressed of standard strand.

The calculation of the state of balance of the unit plates and cable is carried out in only one step, the behavior being elastic.

3.2 Stages of calculation and features tested

The principal stages of calculation correspond to the features which one wishes to validate:

- operator DEFI_MATERIAU : definition of the relations of behavior BPEL_BETON and BPEL_ACIER, in the typical case where the losses of tension along the cable of prestressing are neglected (values by default of the parameters);
- operator DEFI_CABLE_BP : determination of a constant profile of tension along the cable of prestressing, losses being neglected; calculation of the coefficients of the relations kinematics between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes “close” to the concrete plate;
- operator AFFE_CHAR_MECA : definition of a loading of the type RELA_CINE_BP ;
- operator STAT_NON_LINE, option BEHAVIOR : calculation of the state of balance by taking account of the loading of the type RELA_CINE_BP.

4 Results of modeling A

4.1 Values tested

4.1.1 Linear density of normal effort on the vertical section of the concrete plate

One compares the values extracted the field SIEF_ELNO resulting from STAT_NON_LINE with the theoretical values of reference. The extraction is done on the mesh $QD001001$ representing the concrete plate.
The component to which the tests relate is NXX.
The tolerance of relative variation compared to the reference is worth 10^{-6} %.

Node	Value of reference	Tolerance
NB001001	$-9{,}99125765.10^{4}$ N/m	$-4{,}43.10^{-11}$ %
NB001002	$-9{,}99125765.10^{4}$ N/m	$-4{,}43.10^{-11}$ %
NB002001	$-9{,}99125765.10^{4}$ N/m	$-4{,}43.10^{-11}$ %
NB002002	$-9{,}99125765.10^{4}$ N/m	$-4{,}43.10^{-11}$ %

4.1.2 Horizontal displacement of the nodes of the concrete plate

One compares the values extracted the field DEPL resulting from STAT_NON_LINE with the theoretical values of reference.
The component to which the tests relate is DX.
The tolerance of relative variation compared to the reference is worth 10^{-6} %.

Node	Value of reference	Tolerance
NB001002	$-1{,}11013974.10^{-5}$ m	$-1{,}05.10^{-9}$ %
NB002002	$-1{,}11013974.10^{-5}$ m	$-1{,}05.10^{-9}$ %

4.1.3 Normal effort in the cable

One compares the values extracted the field SIEF_ELNO resulting from STAT_NON_LINE with the theoretical values of reference. The extraction is done on the meshs $SG001001$ for the node $NC001001$, $SG001002$ for the node $NC001002$, $SG001003$ for the node $NC001003$, and $SG001004$ for the nodes $NC001004$ and $NC001005$.
The component to which the tests relate is N.
The tolerance of relative variation compared to the reference is worth 10^{-6} %.

Node	Value of reference	Tolerance
$NC001001$	$1{,}99825153.10^{5}$ NR	$-4{,}44.10^{-11}$ %
$NC001002$	$1{,}99825153.10^{5}$ NR	$-4{,}44.10^{-11}$ %
$NC001003$	$1{,}99825153.10^{5}$ NR	$-4{,}44.10^{-11}$ %
$NC001004$	$1{,}99825153.10^{5}$ NR	$-4{,}44.10^{-11}$ %
$NC001005$	$1{,}99825153.10^{5}$ NR	$-4{,}44.10^{-11}$ %

4.1.4 Horizontal displacement of the nodes of the cable

One compares the values extracted the field DEPL resulting from STAT_NON_LINE with the theoretical values of reference.
The component to which the tests relate is DX.
The tolerance of relative variation compared to the reference is worth 10^{-6} %.

Node	Value of reference	Tolerance
NC001002	– 2,77534935.10^{-6} m	– 1,05.10^{-9} %
NC001003	– 5,55069869.10^{-6} m	7,56.10^{-10} %
NC001004	– 8,32604804.10^{-6} m	1,56.10^{-10} %
NC001005	– 1,11013974.10^{-5} m	– 1,05.10^{-9} %

4.2 Remarks

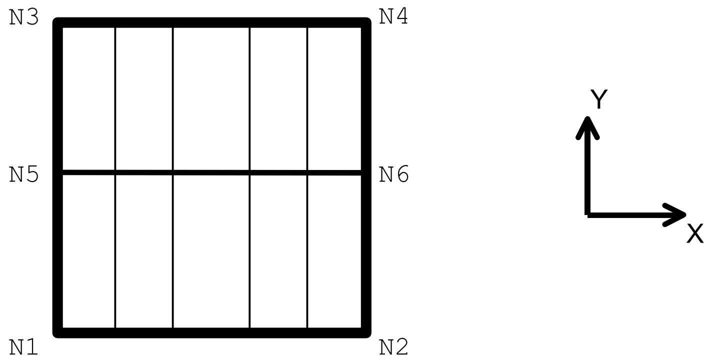
The computed values correspond indeed to those theoretically expected. One obtains well a compactness for the concrete plate.

One observes an infinitesimal difference between horizontal displacement with the node $NC001005$ belonging to the cable and horizontal displacement to the nodes $NB001002$ and $NB002002$ concrete plate. The recorded values should be identical, but the rounding errors appearing in the coefficients of the relations kinematics explain this infinitesimal difference.

5 Modeling B

5.1 Characteristics of modeling

For this modeling, the nodes “cables” and “concrete” are confused.
The figure below gives a simplified representation of the grid.



The concrete plate is represented by 10 elements DKT, supported by 10 meshs QUAD4.

A thickness $e=0,6\,m$ he is affected, as well as a material concrete for which the behaviors are defined ELAS (Young modulus $E_b=3.10^{10}\,Pa$) and BPEL_BETON : the parameters characteristic of this relation are fixed at 0 because one neglects the losses of tension along the cable of prestressing.

Nodes $N1$, $N5$ and $N3$ are embedded: DX , DY , DZ , DRX , DRY and DRZ are blocked.

The cable is represented by 5 elements MECA_BARRE, supported by 5 meshs SEG2.

A surface of cross-section $S_a=1,5.10^{-4}\,m^2$ their is affected, as well as a material steel for which the behaviors are defined ELAS (Young modulus $E_a=2,1.10^{11}\,Pa$) and BPEL_ACIER : the parameters characteristic of this relation are fixed at 0 (neglected losses of tension), except for the elastic ultimate stress for which a zero value is illicit ($f_{prg}=1,77.10^9\,Pa$).

The tension $F_0=2.10^5\,N$ is applied to the nodes $N5$ and $N6$. This value of tension is coherent with the values of section and yield stress, for a cable of prestressed of standard strand.

The calculation of the state of balance of the unit plates and cable is carried out in only one step, the behavior being elastic.

5.2 Stages of calculation and features tested

The principal stages of calculation correspond to the features which one wishes to validate:

- operator DEFI_MATERIAU : definition of the relations of behavior BPEL_BETON and BPEL_ACIER, in the typical case where the losses of tension along the cable of prestressing are neglected (values by default of the parameters);
- operator DEFI_CABLE_BP : determination of a constant profile of tension along the cable of prestressing, losses being neglected; calculation of the coefficients of the relations kinematics between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes “close” to the concrete plate;
- operator AFFE_CHAR_MECA : definition of a loading of the type RELA_CINE_BP ;
- operator STAT_NON_LINE, option BEHAVIOR : calculation of the state of balance by taking account of the loading of the type RELA_CINE_BP.
- operator POST_RELEVE_T, NOM_CMP= ` on all the field SIEF_ELNO.

6 Results of modeling B

6.1 Values tested

6.1.1 Linear density of normal effort on the vertical section of the concrete plate

Node	Value of reference	Tolerance
$N1$	– 9,99125765.10^4 N/m	1.526%
$N3$	– 9,99125765.10^4 N/m	1.526%

6.1.2 Normal effort in the cable

Node	Value of reference	Tolerance
$N5$	1,99825153.10^5 NR	– 0.001%
$N6$	1,99825153.10^5 NR	– 0.193%

6.1.3 Normal effort in the cable via the order POST_RELEVE_T

Node	Value of reference	Computed value
$N5$	1,998224892.10^5 NR	1,9982248921222.10^5 NR
$N6$	1,994393252.10^5 NR	1,9943932520206.10^5 NR

6.2 Remarks

The computed values correspond indeed to those theoretically expected. One obtains well a compactness for the concrete plate.

7 Summary of the results

The got results are validated by comparison with an analytical solution of reference with a very good precision.

The particular features tested are the following ones:

- operator DEFI_MATERIAU : definition of the parameters characteristic of the materials steel and concrete allowing calculation of the tension along the cable of prestressing, following the rules of the BPEL;
- operator DEFI_CABLE_BP : calculation of the tension along the cable and the coefficients of the relations kinematics between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes “close” to the concrete plate;
- operator AFFE_CHAR_MECA : definition of a loading of the type RELA_CINE_BP ;
- operator STAT_NON_LINE, option BEHAVIOR : calculation of the state of balance by taking account of the loading of the type RELA_CINE_BP.