SSNP116 - Coupling creep/cracking - uniaxial Traction

Summary:

This case of validation is intended to check the model of coupling of the laws of creep of Granger with the laws of plasticity/cracking. Coupling, initially restricted with some laws of the nonlinear environment of *Code_Aster*, could be wide thereafter with more laws. The parameters of the models of plasticity/cracking are selected in a particular way to model a nearly perfect elastoplastic behavior, and to be brought back to a problem presenting a relatively simple analytical solution.

The geometry consists of three linear elements (cubic and prisms in $3D$, squares and triangles in $2D$), and three quadratic elements, connected to the precedents by linear relations. Modelings tested here are modelings 3D, C_PLAN, and D_PLAN.

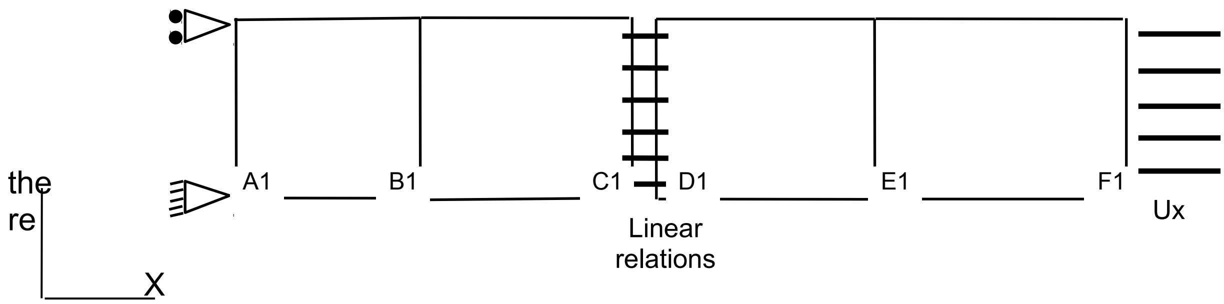
The loading is a uniaxial traction in imposed displacement.

One tests the coupling of the model of creep of Granger with BETON_DOUBLE_DP and VMIS_ISOT_LINE. One has the analytical solution in 3D and C_PLAN, when there is not variation of the water content.

In the cases 3D and D_PLAN, the solutions obtained are also tested when there is variation of the water content and the temperature with activation of the corresponding withdrawals (with opposite sign). They are then tests of not-regression.

1 Problem of reference

1.1 Geometry



1.2 Material properties

The parameters of the laws of behavior are the following:

For the mechanical characteristics in linear elasticity (ELAS) :

Young modulus:	$E = 31\,000$ MPa
Poisson's ratio:	$\nu = 0.2$
Thermal dilation coefficient:	$\alpha = 10^{-5}$
Coefficient of withdrawal of desiccation:	$\kappa = 10^{-5}$

For the nonlinear mechanical characteristics of the model BETON_DOUBLE_DP :

Resistance in uniaxial pressing:	$f'c = 40\ N/mm^2$
Resistance in uniaxial traction:	$f't = 4\ N/mm^2$
Report of resistances in biaxial compression/uniaxial pressing:	$\beta = 1.16$
Energy of rupture in compression:	$Gc = 10\ Nmm/mm^2$
Energy of rupture in traction:	$\mathrm{Gt} = 10000\ Nmm/mm^2$. *to simulate a work hardening quasi no one*
Report of the limit elastic to resistance in uniaxial pressing:	$33.33\,\%$

For the mechanical characteristics of the model with linear work hardening VMIS_ISOT_LINE :

Yield stress:	$Sy = 4\ N/mm^2$
Slope of work hardening:	$\mathrm{D\_sigm\_epsi} = 0.1\ N/mm^2$

For the mechanical characteristics of the model of creep of GRANGER :

Coefficient J_1 *:* $J_1 = 0.2\, MPa^{-1}$
Coefficient τ_1 *:* $\tau_1 = 4\,320\,000\, s$
Coefficient Q/R *:* $\mathrm{QsR\_K} = 0.\mathrm{K}$
The curve of desorption is worth 1 for all values of the hygroscopy, to simplify the analytical solution.

1.3 Boundary conditions and loadings mechanical

For calculations in $3\mathrm{D}$:

- Face in $x=0$ first cube (its): blocked according to ox ,
- Nodes of the faces in $y=0$: blocked according to oy ,
- Nodes of the faces in $z=0$: blocked according to oz ,
- Linear relation (LIAISON_DDL) between the nodes end of the confused faces of the linear and quadratic elements adjacent (nodes $c1$, $c2$, $c3$, $c4$ bound with the nodes $d1$, $d2$, $d3$, $d4$),
- Linear relation (LIAISON_UNIF) on the face sd to bind displacements according to ox quadratic nodes of this vis-a-vis those of the nodes top,
- Face in $x=x_{max}$ last cube (sf) : Traction exerted according to ox .

For calculations in $2\mathrm{D}$:

- Line in $x=0$ first square (it): blocked according to ox ,
- Nodes of the lines in $y=0$: blocked according to oy ,
- Linear relation (LIAISON_DDL) between the nodes end of the confused lines of the linear and quadratic elements adjacent (nodes $c1$, $c2$ bound with the nodes $d1$, $d2$),
- Linear relation (LIAISON_UNIF) on the line ld to bind displacements according to x quadratic nodes of this line to those of the nodes top,
- Line in $x=x_{max}$ last handful (lf) : Traction exerted according to ox .

The field of temperature is either constant (the first calculation), or crescent of $0\,°C$ with $20\,°C$ for all other calculations. If the temperature varies, it is supposed that the field of drying varies from 1 to 0. The characteristics material are constant. Moreover, one applies a coefficient of withdrawal of desiccation not no one, in such way that the withdrawal of desiccation compensates for thermal dilation, to check that these 2 phenomena are well taken into account.

Notice*. The temperature variation impacts the calculation of the withdrawals but not the mechanical law of behavior, which does not depend on the temperature.*

2 Reference solution

2.1 Method of calculating used for the reference solution

To be able to calculate a simple analytical solution, the following choices were carried out, the objective being to validate the coupling and not the laws of plasticity/cracking or creep:

- a law of creep of Granger with only one model of Kelvin in series,
- a law of plasticity/cracking modelling a perfect elastoplastic law,
- a loading of uniaxial traction.

The reference solution is calculated in an analytical way, knowing that in traction, only the criterion of traction is activated. The equations of the model are brought back to scalar equations making it possible to calculate the analytical solution. The only difficulty comes from the determination of the beginning of the plasticity (urgent and deformation of creep) which requires to solve by a digital method a nonlinear equation with an unknown factor.

If the water content is not constant, creep is more complex to solve, the analytical solution was not calculated. They are thus tests of not-regression. However, in the cases `3D` and `D_PLAN` , one can check that one gets the same results with the 2 models.

The imposed deformation (displacement of an end of the structure) is a linear function of time making it possible to bring into play creep and plasticity.

2.2 Calculation of the reference solution

One notes ε , the component xx total deflection ε_e , the component xx elastic strain, ε_{fl} the component xx deformation of creep of Granger, and ε_{pl} the component xx plastic deformation, σ the component xx constraint, and E the Young modulus.
The model of creep selected comprises one model of Kelvin in series and the model of plasticity/cracking is a nearly perfect law elastoplastic (quasi worthless slope of work hardening), which makes it possible to easily calculate the analytical solution of the coupling creep/plasticity, in the case of a uniaxial simple traction. The nearly perfect elastoplastic law can be obtained starting from the laws of *Code_Aster* `BETON_DOUBLE_DP` or `VMIS_ISOT_LINE` , by choosing the set of parameters which is appropriate (work hardening quasi no one). The loading is a uniaxial traction in imposed displacement. One thus imposes a total deflection proportional to passed time, form $\varepsilon_{xx}=\lambda_0 . t$. As there is no effort exerted in the other directions, the stress field is uniaxial. One can thus bring back oneself to a problem $1D$ for the resolution, which makes it possible to calculate in the second time of the deformations in the transverse directions with the loading (yy and zz).

$\sigma=(\sigma_{xx},0,0,0,0,0)$ and $\varepsilon=(\varepsilon_{xx},\varepsilon_{yy},\varepsilon_{zz},0,0,0)$

The equations of the model of creep and the model of plasticity merge with the following scalar equations, by omitting the index xx corresponding to the first component of the tensors:

$\varepsilon=\lambda_0 . t$ (imposed traction)

$\varepsilon=\varepsilon_e+\varepsilon_{fl}+\varepsilon_{pl}$

$\sigma=\mu\dot{\varepsilon}_{fl}+K\varepsilon_{fl}$ with $\mu=\frac{\tau_s}{J_s}$ and $K=\frac{1}{J_s}$

$\sigma=E\varepsilon_e \quad \sigma=E\left[\varepsilon-\varepsilon_{pl}-\varepsilon_{fl}\right]$

Resolution in linear elasticity

Before reaching the threshold of plasticity, the plastic deformation is worthless, which leads to:

$\varepsilon=\lambda_0 . t$ (imposed traction)

$\varepsilon_{pl}=0$

$\varepsilon=\varepsilon_e+\varepsilon_{fl}$

$\sigma=\mu\dot{\varepsilon}_{fl}+K\varepsilon_{fl}$ with $\mu=\frac{\tau_s}{J_s}$ and $K=\frac{1}{J_s}$

$\sigma=E\varepsilon_e \quad \sigma=E\left[\varepsilon-\varepsilon_{fl}\right]$

One obtains the differential equation allowing to calculate the deformation of creep:

$$\sigma=E\left[\varepsilon-\varepsilon_{fl}\right]=\mu\dot{\varepsilon}_{fl}+K\varepsilon_{fl} \text{ with } \varepsilon=\lambda_0 . t$$

The deformation of creep is thus expressed as the sum of a linear function of time and an exponential function, type:

$$\varepsilon_{fl}(t)=a.t+b+\alpha e^{-\beta . t}$$

who gives in the differential equation:

$$0=\mu.\dot{\varepsilon}_{fl}(t)+K.\varepsilon_{fl}(t)+E.\varepsilon_{fl}(t)-E.\lambda_0 . t$$

That is to say:

$$0=\left[(K+E)b+\mu.a\right]+\left[(K+E)a-E.\lambda_0\right]t+\left[(K+E)\alpha-\mu.\beta.\alpha\right]e^{-\beta . t}$$

from where:

$$a=\frac{E.\lambda_0}{K+E} \quad b=-\frac{\mu}{K+E}\frac{E.\lambda_0}{K+E} \quad \beta=\frac{K+E}{\mu}$$

At the initial moment, one starts from a worthless deformation of creep, which leads to:

$$\alpha=\frac{m}{K+E}\frac{E.\lambda_0}{K+E}$$

One obtains finally the expression of the deformation of creep according to time:

$$\varepsilon_{fl}^{xx}(t)=\varepsilon_{fl}(t)=\frac{\lambda_0 . E}{K+E}\left[t-\frac{\mu}{K+E}\left(1-e^{-\frac{K+E}{\mu}t}\right)\right]$$

The component xx elastic strain is worth: $\varepsilon_e=\varepsilon-\varepsilon_{fl}$. That is to say:

$$\varepsilon_e^{xx}(t)=\varepsilon_e(t)=\frac{\lambda_0 . K}{K+E}t+\frac{\lambda_0 . E.\mu}{(K+E)^2}\left(1-e^{-\frac{K+E}{\mu}t}\right)$$

Components yy and zz elastic strain and of creep are obtained by multiplication of the component xx by the Poisson's ratio.

The component xx constraint is worth: $\sigma=E.\varepsilon_e=E\left(\varepsilon-\varepsilon_{fl}\right)$. That is to say:

$$\sigma_{xx}(t)=\sigma(t)=\frac{\lambda_0 . K.E}{K+E}t+\frac{\lambda_0 . E^2 . \mu}{(K+E)^2}\left(1-e^{-\frac{K+E}{\mu}t}\right)$$

Threshold of elasticity

The behavior remains elastic until one reaches limited of elasticity. In the case of a uniaxial traction, the equivalent constraint is equal to the nonworthless component of the constraint. Plasticity thus intervenes when $\sigma_{xx}(t)=\sigma_{eq}=f_t$ (resistance in traction), that is to say:

$$\frac{\lambda_0 . K . E}{K+E} t+\frac{\lambda_0 . E^2 . \mu}{(K+E)^2}\left(1-e^{-\frac{K+E}{\mu} t}\right)=f_t$$

This equation, solved by a digital method, makes it possible to obtain the moment of the beginning of plasticization t_{plas} and deformation of creep at this moment:

$$\varepsilon_{fl plas}=\varepsilon_{fl}(t_{plas})=\frac{\lambda_0 . E}{K+E}\left[t_{plas}-\frac{\mu}{K+E}\left(1-e^{-\frac{K+E}{\mu} t_{plas}}\right)\right]$$

Resolution in plasticity

The model of plasticity was selected in order to obtain a simple analytical resolution. It is about a law of nearly perfect plasticity, obtained by taking a particular game of parameters for the model of behavior leading to a quasi worthless slope of work hardening. Therefore, in plastic phase, the constraint (component xx), equal to the equivalent constraint resistance in traction is worth. The equations of the model are then:

$\varepsilon=\lambda_0 . t$ (imposed traction) and $\varepsilon=\varepsilon_e+\varepsilon_{fl}+\varepsilon_{pl}$

$\sigma=m \dot{\varepsilon}_{fl}+K \varepsilon_{fl}=\mu \dot{\varepsilon}_{fl}+K\left[\varepsilon-\varepsilon_e-\varepsilon_{pl}\right]$ with $\mu=\frac{\tau_s}{J_s}$ and $K=\frac{1}{J_s}$

$\sigma=E \varepsilon_e \quad \sigma=E\left[\varepsilon-\varepsilon_{pl}-\varepsilon_{fl}\right]=f_t$

with like initial conditions:

$t=t_{plas} \quad \varepsilon_{fl}(t_{plas})=\varepsilon_{fl plas}$

what leads to the differential equation making it possible to calculate the deformation of creep:

$\sigma=f_t=\mu \dot{\varepsilon}_{fl}+K \varepsilon_{fl}$

The deformation of creep is thus expressed in the form:

$\varepsilon_{fl}(t)=a+\alpha e^{-\beta . t}$

who gives in the differential equation:

$0=\mu . \dot{\varepsilon}_{fl}(t)+K . \varepsilon_{fl}(t)-f_t=\left[K . a-f_t\right]+\left[K . \alpha-\mu . \beta . \alpha\right] e^{-\beta . t}$ from where:

$a=\frac{f_t}{K} \quad \beta=\frac{K}{\mu}$

At the moment t_{plas} , the deformation of creep is worth $\varepsilon_{fl plas}$, which leads to:

$\alpha=\left(\varepsilon_{fl plas}-\frac{f_t}{K}\right) e^{\beta . t}$

One obtains finally the expression of the deformation of creep according to time:

$\varepsilon_{fl xx}(t)=\frac{f_t}{K}+\left(\varepsilon_{fl plas}-\frac{f_t}{K}\right) e^{-\frac{K}{\mu}(t-t_{plas})}$ with $\varepsilon=\lambda_0 . t$

The component xx elastic strain is worth: $\varepsilon_e=\frac{\sigma}{E}=\frac{f_t}{E}$

The component xx plastic deformation is worth: $\varepsilon_{pl}=\varepsilon-\varepsilon_e-\varepsilon_{fl}=\lambda_0 . t-\varepsilon_e-\varepsilon_{fl}$. That is to say:

$\varepsilon_{plas xx}(t)=\lambda_0 . t-\frac{f_t}{E}-\frac{f_t}{K}-\left(\varepsilon_{fl plas}-\frac{f_t}{K}\right) e^{-\frac{K}{m}(t-t_{plas})}$

Components yy and zz elastic strain and of creep are obtained by multiplication of the component xx by the Poisson's ratio.

The component xx constraint is worth: $\sigma=f_t$

Digital application:

One imposes a deformation of 10^{-3} in 100 seconds, which gives $\lambda_0 = 10^{-5}$

The only difficulty consists in calculating the moment of plasticization t_{plas}, and deformation of creep $\varepsilon_{fl\,plas}$ who corresponds to him, by dichotomy for example. One obtains finally the parameters:

$$t_{plas} = 13.024296$$

$$e_{fl\,plas} = 1.20969985.10^{-6}$$

$$e = 1.2903226.10^{-4}$$

who allow to obtain the values of reference after plasticization of the concrete.

At 10 seconds, the behavior is a coupling creep/elasticity. At 100 seconds, the behavior is a coupling creep/plasticity:

time	10	100
σ	3.0778607	4.0
ε	1.10^{-4}	1.10^{-3}
ε_{fl}	$7.1417140.10^{-7}$	$1.7316168.10^{-5}$
ε_e	$9.9285829.10^{-5}$	$1.2903226.10^{-4}$
ε_{fpl}	0.0	$8.5365157.10^{-4}$

2.3 Uncertainty on the solution

It is negligible, about the precision machine.

2.4 Bibliographical references

The model was defined in the document of specification:

1) CS SI/311- 1/420AL 0/RAP/00.019 Version 1.1, “Development of the coupling creep/cracking in *Code_Aster* - Specifications “

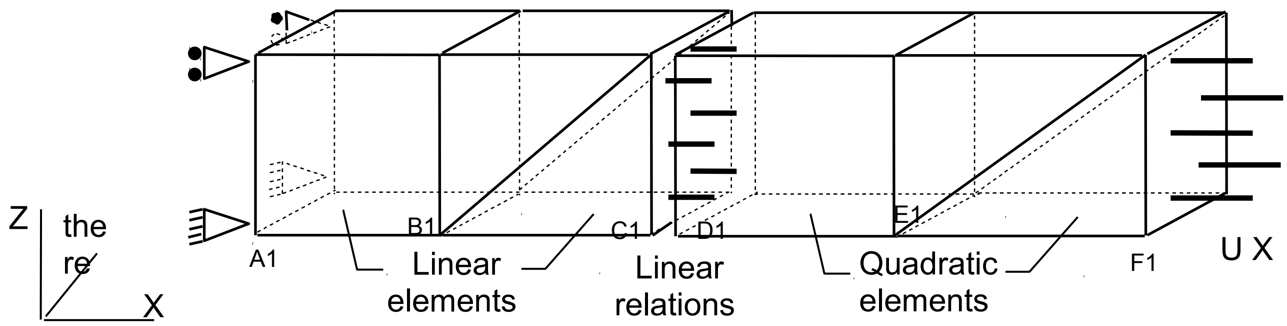
3 Modeling A

3.1 Characteristics of modeling

3D (1 HEXA8, 2 **PENTA6**, 1 **HEXA20**, 2 **PENTA15)**

They are a cube with 8 nodes and two prisms with 6 nodes bound by linear relations to a cube to 20 nodes and two prisms to 15 nodes. The unit is subjected to a uniaxial traction according to the direction x. Dimensions according to y and z are unit. Dimensions according to the direction x are selected so that all the elements have the same characteristic length (this one is worth the cubic root of volume for the quadratic elements, and the cubic root of volume multiplied by $\sqrt{2}$ for the linear elements).

The deformation and stress fields are uniform.
In $3D$, the coupling of the laws is validated BETON_DOUBLE_DP and VMIS_ISOT_LINE with the law GRANGER_FP.



3.2 Characteristics of the grid

Many nodes: 46
Number of meshs and type: 1 HEXA8, 2 PENTA6, 1 HEXA20, 2 PENTA15

3.3 Sizes tested and results

The components are tested xx stress field SIGM_ELNO, field of deformations of creep EPFP_ELNO, and of the field of plastic deformation EPSP_ELNO.
For the coupling with the law BETON_DOUBLE_DP , if drying is constant and the solution analytical known, these values were tested at the point $C1$ located at the interface enters the linear elements and the quadratic elements, and at the point $F1$ located at the end of the structure, where the displacement imposed is applied (in x_{max}).
When drying varies, the analytical solution was not calculated: one thus tests the same components as previously but only at the point $F1$ located at the end of the structure. The solution obtained with BETON_DOUBLE_DP is tested as a not-regression, but the values obtained are used then as reference for model VMIS_ISOT_LINE .

The tests are carried out at moment 10, when plasticity did not start, only creep is present, and at moment 100, after the beginning of the plasticization of the concrete.

3.3.1 Calculation with the law BETON_DOUBLE_DP at a constant temperature (Reference)

Coupling GRANGER_FP/BETON_DOUBLE_DP

- **at the point** $C1$

Identification	Reference	*Aster*	% difference
σ_{xx} for $\varepsilon_{xx}\,10^{-4}$	3.07786	3.07787	1.9.10 -4
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-4}$	7.14171 10 -7	7.140035 10 -7	-0,023
σ_{xx} for $\varepsilon_{xx}\,10^{-3}$	4.0	3.999999	-2.0.10 -5
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-3}$	1.73162 10 -5	1.731596 10 -5	-0,001
ε_{xx}^{p} for $\varepsilon_{xx}\,10^{-3}$	8.53652 10 -4	8.536546 10 -4	3.1 10 -4

- **at the point** $F1$

Identification	Reference	*Aster*	% difference
σ_{xx} for $\varepsilon_{xx}\,10^{-4}$	3.07786	3.07787	1.9.10 -4
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-4}$	7.14171 10 -7	7.140035 10 -7	-0,023
σ_{xx} for $\varepsilon_{xx}\,10^{-3}$	4.0	3.999998	-6.0.10 -5
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-3}$	1.73162 10 -5	1.731596 10 -5	-0,001
ε_{xx}^{p} for $\varepsilon_{xx}\,10^{-3}$	8.53652 10 -4	8.536023 10 -4	-0,006

3.3.2 Calculation with the law BETON_DOUBLE_DP in nonisotherm (Not regression)

They are tests of nonregression, the values are not indicated.

- **at the point** $F1$

Variable tested
σ_{xx} for $\varepsilon_{xx}\,10^{-4}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-4}$
σ_{xx} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{p} for $\varepsilon_{xx}\,10^{-3}$

3.4 Calculation with the law VMIS_ISOT_LINE in nonisotherm

They are tests of nonregression, the values are not indicated.

- **at the point** $F1$

Identification
σ_{xx} for $\varepsilon_{xx}\,10^{-4}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-4}$
σ_{xx} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{p} for $\varepsilon_{xx}\,10^{-3}$

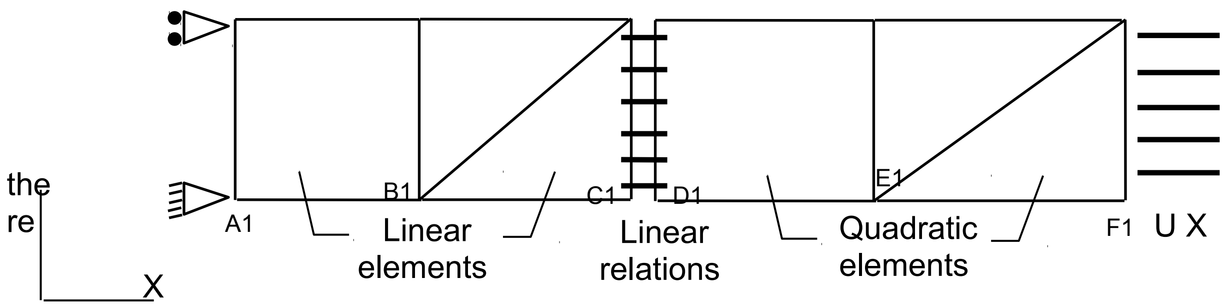
4 Modeling B

4.1 Characteristics of modeling

D_PLAN (1 QUAD4, 2 TRI3, 1 Q UAD8, 2 TRI6)

They are a square with 4 nodes and two triangles with 3 nodes bound by linear relations to a square to 8 nodes and two triangles to 6 nodes. The unit is subjected to a uniaxial traction according to the direction x. Dimensions according to y are unit. Dimensions according to the direction x are selected so that all the elements have the same characteristic length (root of surface for the quadratic elements, and root of the surface multiplied by $\sqrt{2}$ for the linear elements).

The deformation and stress fields are uniform.
In $2D$ plane deformations (D_PLAN), one tests the coupling between law BETON_DOUBLE_DP with the law GRANGER_FP. One tests also the coupling of the law VMIS_ISOT_LINE with the law GRANGER_FP. The analytical solution was not calculated in D_PLAN.



4.2 Characteristics of the grid

Many nodes: 20
Number of meshs and type: 1 QUAD4, 2 TRI3, 1 QUAD8, 2 TRI6

4.3 Features tested

The components are tested xx stress field SIGM_ELNO and of the field of deformations of creep EPFP_ELNO, and of the field of plastic deformation EPSP_ELNO at the point $F1$ located at the end of the structure, where the displacement imposed is applied (in x_{max}).

The analytical solution was not calculated in plane deformation. One thus carries out only same calculation with the 2 models of cracking to variable drying. The tests are of standard not-regression.

The tests are carried out at moment 10, when plasticity did not start, only creep is present, and at moment 100, after the beginning of the plasticization of the concrete.

4.3.1 Calculation with the law BETON_DOUBLE_DP in nonisotherm (Not regression)

They are tests of nonregression, the values are not indicated.
At the point $C1$

Variable tested
σ_{xx} for $\varepsilon_{xx}\,10^{-4}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-4}$
σ_{xx} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{p} for $\varepsilon_{xx}\,10^{-3}$

At the point $F1$

Variable tested
σ_{xx} for $\varepsilon_{xx}\,10^{-4}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-4}$
σ_{xx} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{p} for $\varepsilon_{xx}\,10^{-3}$

4.3.2 Calculation with the law `VMIS_ISOT_LINE` in nonisotherm (Not regression)

They are tests of nonregression, the values are not indicated.

At the point $F1$

Variable tested
σ_{xx} for $\varepsilon_{xx}\,10^{-4}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-4}$
σ_{xx} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{fl} for $\varepsilon_{xx}\,10^{-3}$
ε_{xx}^{p} for $\varepsilon_{xx}\,10^{-3}$

5 Summary of the results

If one knows the analytical solution (not-variation of drying), this case test offers very satisfactory results with a lower deviation than 0.02% for all the calculation cases. The iteration count for the plastic phase is generally about ten; This is explained by the choice of the law of nearly perfect plasticity, obtained with the model VMIS_ISOT_LINE, with particular games of parameters. In fact, this same model used **without** the coupling creep/cracking, under the same conditions of loading and with the same parameters, presents the same difficulties of convergence.

Lastly, in 3D and in C_PLAN, it is checked that the 2 models which were degenerated give many quasi-similar results. On the other hand, in D_PLAN, the model BETON_DOUBLE_DP is not equivalent to the model VMIS_ISOT_LINE because of writing of the criterion which depends on the trace of the tensor of the deformations and is thus not equivalent to the perfectly plastic model.