SSNP138 - Crack inclined in 2D with X-FEM

Summary

The purpose of this test is to validate the calculation of the stress intensity factors (K_I and K_{II}) with X-FEM [bib1] in 2D , within the framework of linear elasticity. The purpose of this test is also to test the voluminal loadings of FORCE_INTERNE and of GRAVITY with the method X-FEM in 2D .

This test brings into play a rectangular plate with a right central crack inclined, and subjected to a loading of traction on the edges inferior and superior of the plate (modelings A C , D , E) or subjected to a voluminal force and an embedding on the upper part (modeling B), or subjected to a pressure on the lips (modeling F).

Various modelings are considered:

1. modeling A : X-FEM 2D , surface loading on the edges, meshs QUAD4, geometrical enrichment
2. modeling B : X-FEM 2D , voluminal loading, meshs QUAD4. geometrical enrichment
3. modeling C : X-FEM 2D , surface loading on the edges, meshs TRIA3, geometrical enrichment
4. modeling D : X-FEM 2D , surface loading on the edges, meshs TRIA3, topological enrichment
5. modeling E : X-FEM 2D , surface loading on the edges, meshs TRIA6, topological enrichment
6. modeling F : X-FEM 2D , loading on the lips, meshs TRIA3, topological enrichment

The validation relates to the stress intensity factors (K_I and K_{II}), by comparison with the analytical solution for modelings A , C , D , E, F and a solution of not-regression for modeling B.

1 Problem of reference

1.1 Geometry

The structure $2D$ is a rectangular plate ($LX=0{,}2\,m$, $LY=0{,}5\,m$), comprising a right central crack, tilted of an angle θ variable compared to the horizontal axis [Figure 1.1-1]. The length of the crack is constant ($a=0{,}04\,m$). In this test, the angle θ will take successively the values: $0°$, $15°$, $30°$, $45°$, $60°$ for modeling A and $0°$, $45°$ for modeling B .

One calls "lower line", the line in $y=-LY/2$ and "higher line", the line in $y=LY/2$.

Noted nodes A , B , C and D on Figure 1.1-1 are used to impose the boundary conditions, which is clarified in the paragraph [§1.3].

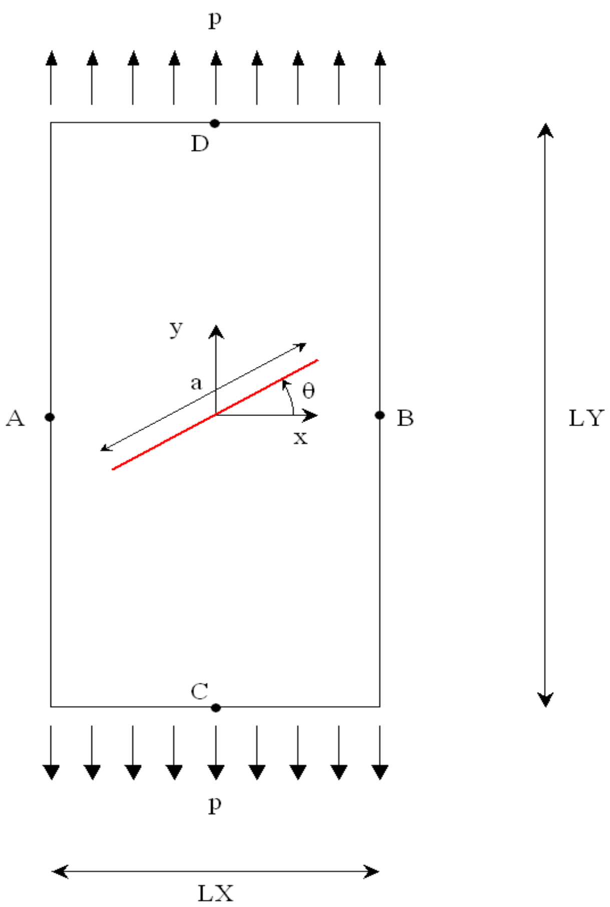

Figure 1.1-1 : geometry of the fissured plate and loading of modeling A .

1.2 Properties of material

Young modulus: $E=210\,10^9\,Pa$
Poisson's ratio: modeling A $\nu=0.3$, modeling B $\nu=0$
Density: modeling b: $\rho=7800\,kg/m^3$

1.3 Boundary conditions and loadings

Modeling A :

The loading consists in applying a force left again to the lines lower and higher $p=10^6 Pa$.

In order to block the rigid modes, displacements of the nodes are blocked A, B, C and D as follows:

- $DY^A=DY^B=0$;
- $DX^C=DX^D=0$.

Modeling B :

The loading consists in applying to the plate an embedding to the higher line and a voluminal force of type `FORCE_INTERNE` or `GRAVITY`. One makes in kind apply the same loading (according to $-Y$) for two successive calculations with these keywords: for the first density of force imposed of $78000\,N/m^3$ and for the second one chooses an acceleration of gravity equalizes with $10\,m.s^{-2}$ and thus density of force of $10.\rho=78000\,N/m^3$.

Modelings C, D, E :

Same assumptions as modeling A

Modeling F :

The loading of traction on the faces higher and lower is replaced by a pressure on the lips of the crack (the 2 loadings are equivalent)

1.4 Reference solution

Modeling A :

Analytical expressions of the stress intensity factors K_I and K_{II} are functions of the force distributed p, length of the crack a, width of the plate LX and of the angle θ :

$$K_I=p\sqrt{\pi\frac{a}{2}}F\left(\frac{a}{Lx}\right)cos^2\theta$$

$$K_{II}=p\sqrt{\pi\frac{a}{2}}F\left(\frac{a}{Lx}\right)\cos\theta\sin\theta$$

where the function F can be given several different manners. We choose that obtained by Brown in 1966 [bib2, p41], whose precision is lower than $0,5\%$ if the relationship between the length of the crack and the width of the plate is lower or equal to $0,7$ (in our case, $a/LX=0,2$):

$$F(x)=1+0,128x-0,288x^2+1,525x^3$$

With the digital values of the test:

Reference		
α (°)	$K_I(Pa.m^{-1/2})$	$K_{II}(Pa.m^{-1/2})$
0	2.5725024656 10 ^{5}	0
15	2.4001774761 10^5	6.4312561642 10^4
30	1.9293768492 10^5	1.1139262432 10^5
45	1.2862512328 10^5	1.2862512328 10^5
60	6.4312561641 10^4	1.1139262432 10^5

Table 1.4-1 : values of reference for K_I and K_{II}

Theoretically, these values are the same ones for the two funds of crack.

Modeling B :
Tests of nonregression are carried out.

Modelings C, D, E, F :
One takes again the values of reference of modeling A

1.5 Bibliographical references

[1] GENIAUT S., MASSIN P.: eXtended Finite Method Element, Handbook of reference of *Code_Aster*, [R7.02.12]

[2] TADA H., PARIS P., IRWIN G.: The stress analysis of aces handbook, 3 ème éd., 2000

2 Modeling A

In this modeling, wide finite element method (X-FEM) is used. One defines a geometrical ray of enrichment with a number of layer of element equal to 3.

2.1 Characteristics of the grid

The structure is modelled by a regular grid composed of 100×100 QUAD4, respectively along the axes x, y . The crack is not with a grid.

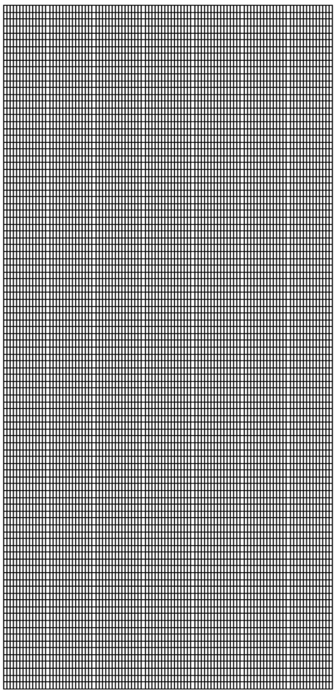

Figure 2.1-1 : grid of the fissured plate

2.2 Sizes tested and results

For each value of the angle θ , one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G (for the two funds of crack) like those given by K1 and K2 of POST_K1_K2_K3 (for the two funds of crack).

For the method $G-thêta$ (order CALC_G), one tests two choices of crowns of field theta:

- $C1$: $R_{inf}=0,1\,a$ and $R_{\sup}=0,3\,a$;
- $C2$: $R_{inf}=h$ and $R_{\sup}=3\text{h}$;

where h is the size characteristic of the meshs:

$$h=\sqrt{\left(\frac{LX}{NX}\right)^2+\left(\frac{LY}{NY}\right)^2}$$

For the method by extrapolation of the jumps of displacements (POST_K1_K2_K3), the maximum curvilinear X-coordinate is equal to $0,3\,a$.

2.2.1 Results for theta= 0°

Identification	Reference	Tolerance
CALC_G		
C1 + fond1: K1	$2.5725\ 10^5$	2,0%
C1 + fond2: K1	$2.5725\ 10^5$	2,0%
C1 + fond1: K2	0	257
C1 + fond2: K2	0	257
C1 + fond1: G	0,29	2,0%
C1 + fond2: G	0,29	2,0%
C2 + fond1: K1	$2.5725\ 10^5$	2,0%
C2 + fond2: K1	$2.5725\ 10^5$	2,0%
C2 + fond1: K2	0	257
C2 + fond2: K2	0	257
C2 + fond1: G	0,29	2,0%
C2 + fond2: G	0,29	2,0%
POST_K1_K2_K3		
fond1: K1	$2.5725\ 10^5$	2,00%
fond2: K1	$2.5725\ 10^5$	2,00%
fond1: K2	0	257
fond2: K2	0	257

Zero values of K_2 are tested in absolute with a tolerance equalizes with $K_1^{ref}/1000$.

2.2.2 Results for thêta= 15°

Identification	Reference
CALC_G	
C1 + fond1: K1	$2.4001\ 10^5$
C1 + fond2: K1	$2.4001\ 10^5$
C1 + fond1: K2	$6.4313\ 10^4$
C1 + fond2: K2	$6.4313\ 10^4$
C2 + fond1: K1	$2.4001\ 10^5$
C2 + fond2: K1	$2.4001\ 10^5$
C2 + fond1: K2	$6.4313\ 10^4$
C2 + fond2: K2	$6.4313\ 10^4$
POST_K1_K2_K3	
fond1: K1	$2.4001\ 10^5$
fond2: K1	$2.4001\ 10^5$
fond1: K2	$6.4313\ 10^4$
fond2: K2	$6.4313\ 10^4$

2.2.3 Results for theta= 30°

Identification	Reference
CALC_G	
C1 + fond1: K1	$1.9294\ 10^5$
C1 + fond2: K1	$1.9294\ 10^5$
C1 + fond1: K2	$1.1139\ 10^5$
C1 + fond2: K2	$1.1139\ 10^5$
C2 + fond1: K1	$1.9294\ 10^5$
C2 + fond2: K1	$1.9294\ 10^5$
C2 + fond1: K2	$1.1139\ 10^5$
C2 + fond2: K2	$1.1139\ 10^5$
POST_K1_K2_K3	
fond1: K1	$1.9294\ 10^5$
fond2: K1	$1.9294\ 10^5$
fond1: K2	$1.1139\ 10^5$
fond2: K2	$1.1139\ 10^5$

2.2.4 Results for thêta= 45°

Identification	Reference
CALC_G	
C1 + fond1: K1	$1.2863\ 10^5$
C1 + fond2: K1	$1.2863\ 10^5$
C1 + fond1: K2	$1.2863\ 10^5$
C1 + fond2: K2	$1.2863\ 10^5$
C2 + fond1: K1	$1.2863\ 10^5$
C2 + fond2: K1	$1.2863\ 10^5$
C2 + fond1: K2	$1.2863\ 10^5$
C2 + fond2: K2	$1.2863\ 10^5$
POST_K1_K2_K3	
fond1: K1	$1.2863\ 10^5$
fond2: K1	$1.2863\ 10^5$
fond1: K2	$1.2863\ 10^5$
fond2: K2	$1.2863\ 10^5$

2.2.5 Results for theta= 60°

Identification	Reference
CALC_G	
C1 + fond1: K1	6.4313 10^4
C1 + fond2: K1	6.4313 10^4
C1 + fond1: K2	1.1140 10^5
C1 + fond2: K2	1.1140 10^5
C2 + fond1: K1	6.4313 10^4
C2 + fond2: K1	6.4313 10^4
C2 + fond1: K2	1.1140 10^5
C2 + fond2: K2	1.1140 10^5
POST_K1_K2_K3	
fond1: K1	6.4313 10^4
fond2: K1	6.4313 10^4
fond1: K2	1.1140 10^5
fond2: K2	1.1140 10^5

2.3 Complementary results

Other values of angles θ were tested in addition, without being part of this test. They are deferred on Figure 2.3-1.

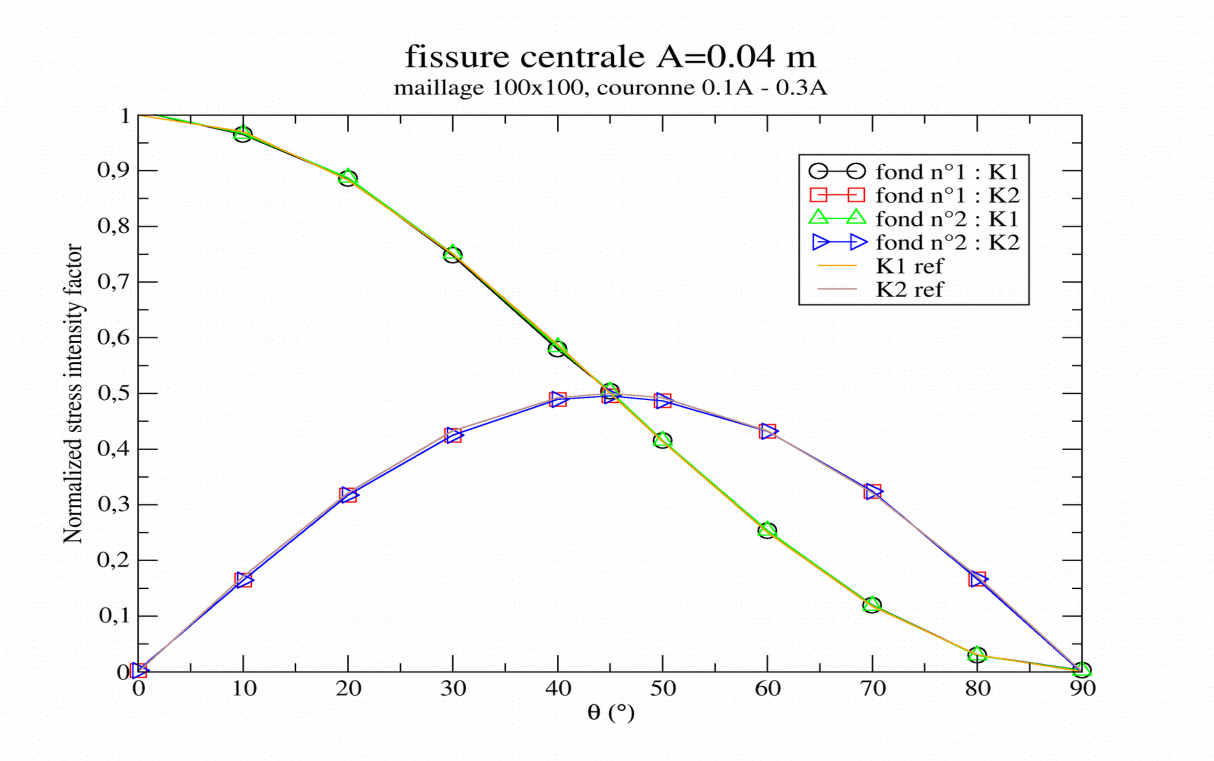

Figure 2.3-1 : factors of intensity of the constraints standardized by K_I (thêta= 0°) obtained by `CALC_G` for the crown $C1$

As illustration, Figure 2.3-2 present a view of the deformation of the plate for an angle $\theta=45°$.

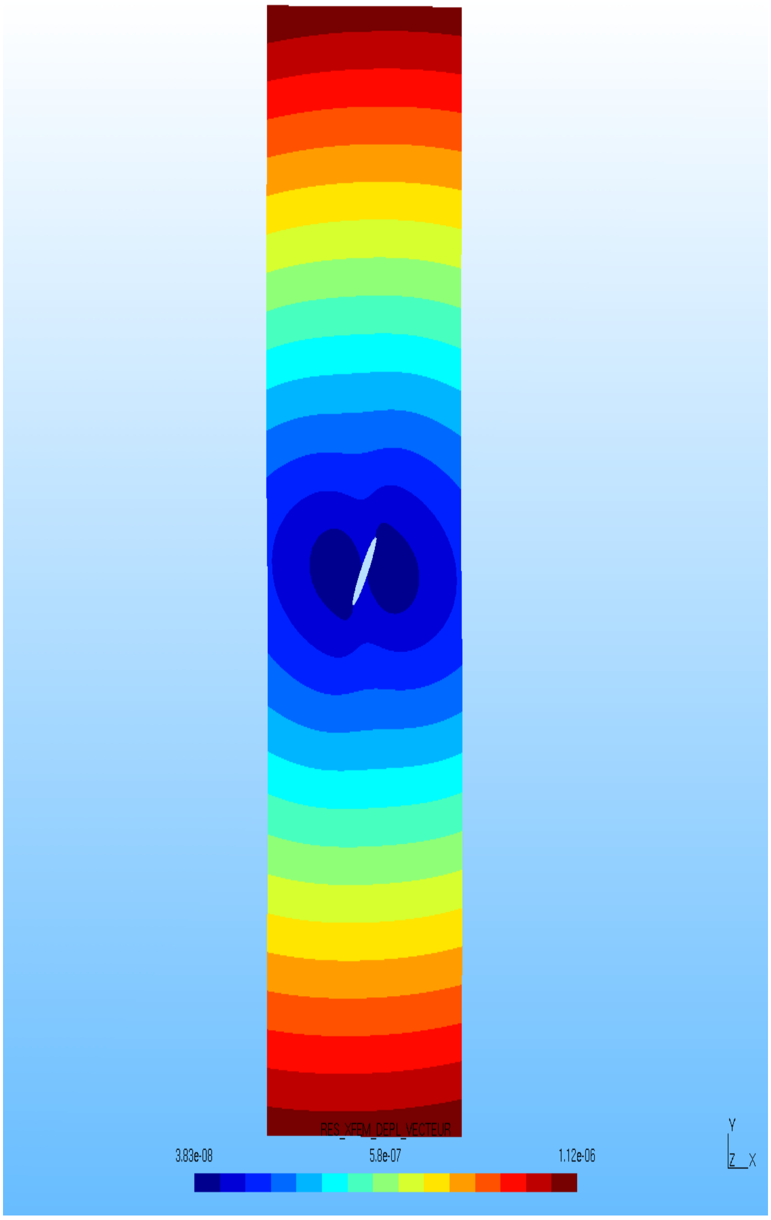

Figure 2.3-2 : deformation for thêta= 45°

3 Modeling B

In this modeling, wide finite element method (X-FEM) is used. One defines a geometrical ray of enrichment with a number of layer of element equal to 3.

3.1 Characteristics of the grid

The structure is modelled by a regular grid (identical to modeling A) composed of 100×100 QUAD4, respectively along the axes x, y . The crack is not with a grid.

3.2 Sizes tested and results

For each value of the angle θ ($0°$ and $45°$), and for each loading: force interns (FI) and gravity ($PESA$), one tests the value of the stress intensity factor KI data by CALC_G (for the two funds of crack) like that given by K1 of POST_K1_K2_K3 (for the 2nd bottom of crack). One also tests the value of G data by CALC_G, option CALC_G, that one compared to that obtained by CALC_G, option CALC_K_G.

For the method $G-thêta$ (order CALC_G), the following crowns of field theta are chosen: $R_{inf}=0{,}1\,a$ and $R_{\sup}=0{,}3\,a$;
For method by extrapolation of the jumps of displacements (POST_K1_K2_K3), the maximum curvilinear X-coordinate is equal to $0{,}3\,a$.

3.2.1 Results for $\theta=0°$

Identification	*Code_Aster*	Tolerance
CALC_G		
FI fond1: K1	5013.598	0.1%
FI fond2: K1	5013.586	0.1%
PES fond1: K1	5013.598	0.1%
PES fond2: K1	5013.586	0.1%
FI melts 1: G	1.19751E-04	0.1%
PES melts 1: G	1.19751E-04	0.1%
POST_K1_K2_K3		
FI fond2: K1	5069,12	0.1%
PES fond2: K1	5069,12	0.1%

3.2.2 Results for $\theta=45°$

Identification	*Code_Aster*	Tolerance
CALC_G		
FI fond1: K1	2454.40	0.1%
FI fond2: K1	2592.14	0.1%
PES fond1: K1	2454.40	0.1%
PES fond2: K1	2592.14	0.1%
FI melts 1: G	5.644E-05	0.1%
PES melts 1: G	5.644E-05	0.1%

POST_K1_K2_K3		
FI fond2: K1	2589,86	0.1%
PES fond2: K1	2589,86	0.1%

4 Modeling C

In this modeling, wide finite element method (X-FEM) is used. One defines a geometrical ray of enrichment with a number of layer of element equal to 3.

4.1 Characteristics of the grid

The field is with a grid with linear triangles (meshs TRIA3). One preserves the refinement of preceding modelings at knowing 100 quadrangles (divided into 2 triangles) along the axis X and 100 quadrangles (divided into 2 triangles) along the axis Y . The crack is not with a grid.

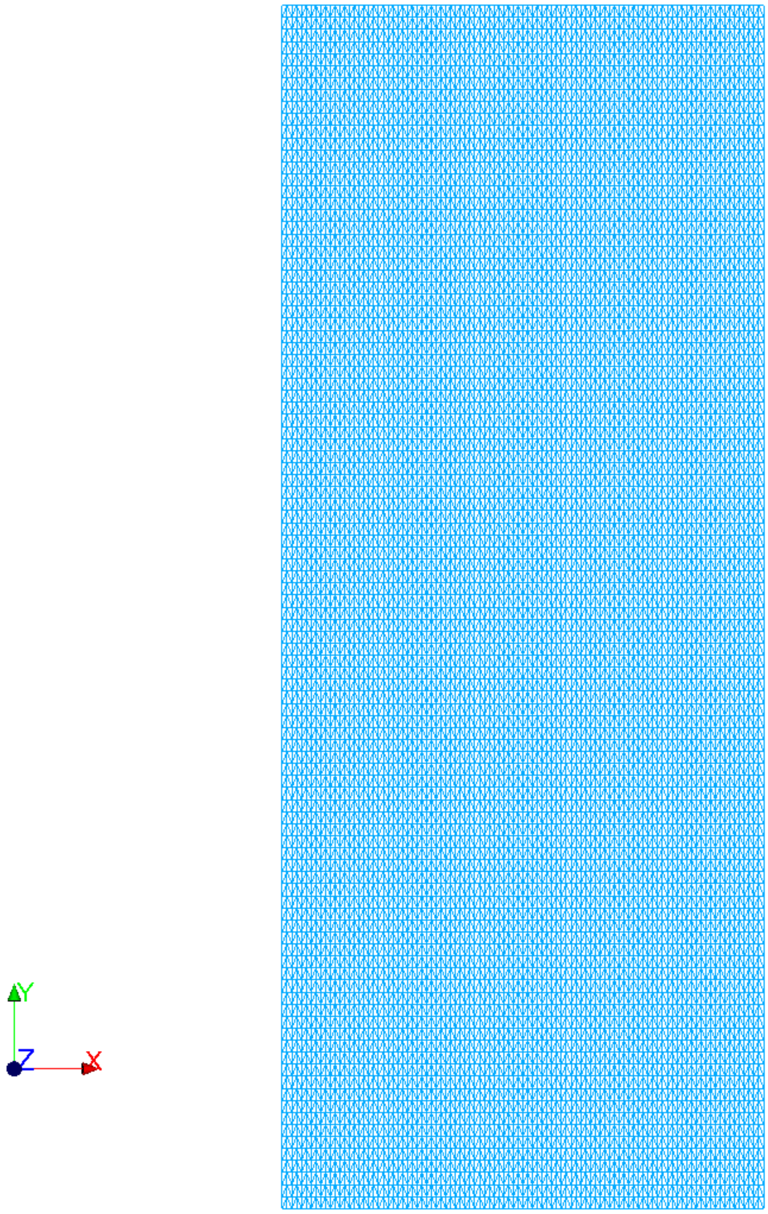

Figure 4.1-1 : grid with triangles

4.2 Sizes tested and results

The crack is tilted according to 3 angular values: $\theta=0°, 30°, 60°$

For each angle of inclination, one tests the stress intensity factors as in modelings A and B, by the method $G-thêta$ and by the method of extrapolation of the jumps of displacements.

For the method $G-thêta$ (order CALC_G), the following crowns of field theta are chosen: $R_{inf}=0,1a$ and $R_{sup}=0,3a$.

4.2.1 Results for $\theta=0°$

Identification	Reference	Tolerance
CALC_G		
C1 + fond1: K1	$2.5725\ 10^5$	2,0%
C1 + fond2: K1	$2.5725\ 10^5$	2,0%
C1 + fond1: K2	0	257
C1 + fond2: K2	0	257
C1 + fond1: G	0,29	2,0%
C1 + fond2: G	0,29	2,0%
C2 + fond1: K1	$2.5725\ 10^5$	2,0%
C2 + fond2: K1	$2.5725\ 10^5$	2,0%
C2 + fond1: K2	0	257
C2 + fond2: K2	0	257
C2 + fond1: G	0,29	2,0%
C2 + fond2: G	0,29	2,0%
POST_K1_K2_K3		
C1 + fond1: K1	$2.5725\ 10^5$	2,0%
C1 + fond2: K1	$2.5725\ 10^5$	2,0%
C1 + fond1: K2	0	514,5
C1 + fond2: K2	0	514,5
C2 + fond1: K1	$2.5725\ 10^5$	2,0%
C2 + fond2: K1	$2.5725\ 10^5$	2,0%
C2 + fond1: K2	0	514,5
C2 + fond2: K2	0	514,5

Zero values of K_2 are tested in absolute with a tolerance equalizes with $K_1^{ref}/1000$ for CALC_G and a tolerance equalizes with $K_1^{ref}/500$ for POST_K1_K2_K3.

4.2.2 Results for $\theta=30°$

Identification	Reference	Tolerance
CALC_G		
C1 + fond1: K1	$1.9294\ 10^5$	2,0%
C1 + fond2: K1	$1.9294\ 10^5$	2,0%
C1 + fond1: K2	$1.1139\ 10^5$	3.0%
C1 + fond2: K2	$1.1139\ 10^5$	3.0%
C1 + fond1: G	0.215	2.0%

C1 + fond2: G	0.215	2,0%
C2 + fond1: K1	1.9294 10^5	2,0%
C2 + fond2: K1	1.9294 10^5	2,0%
C2 + fond1: K2	1.1139 10^5	3.0%
C2 + fond2: K2	1.1139 10^5	3.0%
C2 + fond1: G	0.215	2,0%
C2 + fond2: G	0.215	2,0%
POST_K1_K2_K3		
C1 + fond1: K1	1.9294 10^5	2,0%
C1 + fond2: K1	1.9294 10^5	2,0%
C1 + fond1: K2	1.1139 10^5	3.0%
C1 + fond2: K2	1.1139 10^5	3.0%
C2 + fond1: K1	1.9294 10^5	2,0%
C2 + fond2: K1	1.9294 10^5	2,0%
C2 + fond1: K2	1.1139 10^5	3.0%
C2 + fond2: K2	1.1139 10^5	3.0%

4.2.3 Results for $\theta = 60°$

Identification	Reference	Tolerance
CALC_G		
C1 + fond1: K1	6.4312$10^5$	2,0%
C1 + fond2: K1	6.4312$10^5$	2,0%
C1 + fond1: K2	1.1139 10^5	3.0%
C1 + fond2: K2	1.1139 10^5	3.0%
C1 + fond1: G	7.1692 10^{-2}	2.0%
C1 + fond2: G	7.1692 10^{-2}	2,0%
C2 + fond1: K1	6.4312$10^5$	2,0%
C2 + fond2: K1	6.4312$10^5$	2,0%
C2 + fond1: K2	1.1139 10^5	3.0%
C2 + fond2: K2	1.1139 10^5	3.0%
C2 + fond1: G	7.1692 10^{-2}	2,0%
C2 + fond2: G	7.1692 10^{-2}	2,0%
POST_K1_K2_K3		
C1 + fond1: K1	6.4312$10^5$	2,0%
C1 + fond2: K1	6.4312$10^5$	2,0%
C1 + fond1: K2	1.1139 10^5	3.0%
C1 + fond2: K2	1.1139 10^5	3.0%
C2 + fond1: K1	6.4312$10^5$	2,0%
C2 + fond2: K1	6.4312$10^5$	2,0%
C2 + fond1: K2	1.1139 10^5	3.0%
C2 + fond2: K2	1.1139 10^5	3.0%

5 Modeling D

This modeling is identical to modeling C.
The only difference is that one chooses a topological enrichment here (only one of layer of elements nouveau riches in bottom of crack).

One models only the inclined crack with 30°.

5.1 Sizes tested and results

Identification	Reference	Tolerance
CALC_G		
C1 + fond1: K1	1.9294 10^5	0,5%
C1 + fond2: K1	1.9294 10^5	0,5%
C1 + fond1: K2	1.1139 10^5	3.0%
C1 + fond2: K2	1.1139 10^5	3.0%
C1 + fond1: G	0.215	4,0%
C1 + fond2: G	0.215	4,0%
C2 + fond1: K1	1.9294 10^5	0,5%
C2 + fond2: K1	1.9294 10^5	0,5%
C2 + fond1: K2	1.1139 10^5	3.0%
C2 + fond2: K2	1.1139 10^5	3.0%
C2 + fond1: G	0.215	4,0%
C2 + fond2: G	0.215	4,0%
POST_K1_K2_K3		
C1 + fond1: K1	1.9294 10^5	6,0%
C1 + fond2: K1	1.9294 10^5	6,0%
C1 + fond1: K2	1.1139 10^5	6,0%
C1 + fond2: K2	1.1139 10^5	6,0%
C2 + fond1: K1	1.9294 10^5	6,0%
C2 + fond2: K1	1.9294 10^5	6,0%
C2 + fond1: K2	1.1139 10^5	6,0%
C2 + fond2: K2	1.1139 10^5	6,0%

6 Modeling E

This modeling is identical to modeling E.
The only difference is that one chooses quadratic meshs here (TRIA6).

Identification	Reference	Tolerance
CALC_G		
C1 + fond1: K1	1.9294 10^5	0,5%
C1 + fond2: K1	1.9294 10^5	0,5%
C1 + fond1: K2	1.1139 10^5	2,0%
C1 + fond2: K2	1.1139 10^5	2,0%
C1 + fond1: G	0.215	0,6%
C1 + fond2: G	0.215	0,6%
C2 + fond1: K1	1.9294 10^5	0,5%
C2 + fond2: K1	1.9294 10^5	0,5%
C2 + fond1: K2	1.1139 10^5	2,0%
C2 + fond2: K2	1.1139 10^5	3.0%
C2 + fond1: G	0.215	0,6%
C2 + fond2: G	0.215	0,6%
POST_K1_K2_K3		
C1 + fond1: K1	1.9294 10^5	0,2%
C1 + fond2: K1	1.9294 10^5	0,2%
C1 + fond1: K2	1.1139 10^5	1,5%
C1 + fond2: K2	1.1139 10^5	1,5%
C2 + fond1: K1	1.9294 10^5	0,2%
C2 + fond2: K1	1.9294 10^5	0,2%
C2 + fond1: K2	1.1139 10^5	1,5%
C2 + fond2: K2	1.1139 10^5	1,5%

7 Modeling F

This modeling is similar to modeling D (meshs TRIA3, topological enrichment), but the loading is applied via a pressure to the lips of the crack.
Only the horizontal crack is tested.

7.1 Sizes tested and results

Identification	Reference	Tolerance
CALC_G		
C1 + fond1: K1	$2.5725\ 10^5$	2,0%
C1 + fond2: K1	$2.5725\ 10^5$	2,0%
C1 + fond1: K2	0	257
C1 + fond2: K2	0	257
C1 + fond1: G	0,29	9,0%
C1 + fond2: G	0,29	9,0%
C2 + fond1: K1	$2.5725\ 10^5$	2,0%
C2 + fond2: K1	$2.5725\ 10^5$	2,0%
C2 + fond1: K2	0	257
C2 + fond2: K2	0	257
C2 + fond1: G	0,29	9,0%
C2 + fond2: G	0,29	9,0%
POST_K1_K2_K3		
C2 + fond1: K1	$2.5725\ 10^5$	10,0%
C2 + fond2: K1	$2.5725\ 10^5$	10,0%
C2 + fond1: K2	0	257
C2 + fond2: K2	0	257

The results are less precise than those of modeling D.
Indeed, when one imposes a pressure on the lips of the crack, it is necessary to use small crowns.

8 Summaries of the results

The goals of this test are achieved:

- To validate on a simple case the calculation of the stress intensity factors in mixed mode for the elements X-FEM linear and quadratic
- To test to it not regression of the voluminal forces imposed on a crack X-FEM

Assessment on the linear elements
With the order CALC_G, one obtains a good precision on K_I and K_{II} (2 to 3%) with linear elements (triangles or quadrangles), some is the type of enrichment in bottom of crack (topological or geometrical).
On the other hand, with the order POST_K1_K2_K3, the activation of geometrical enrichment improves significantly the solution compared to topological enrichment by default (5 to 6% → 2 to 3%).

It is thus to recommend to use enrichment by default (topological) and a postprocessing with CALC_G. So for any reason, one wishes post-to treat with POST_K1_K2_K3, then it is preferable to activate geometrical enrichment.

Assessment on the quadratic elements
The quadratic elements (with topological enrichment) make it possible to find results as precise as the linear elements with geometrical enrichment, but for a size of the system to be solved much larger.

Comparison of the errors relative for the crack inclined to 30°:

	TRIA3 + topological (Modeling D)	TRIA3 + geometrical (Modeling C)	TRIA6 + topological (Modeling E)
Size of the system	20788 ddls	21396 ddls	82032 ddls
CALC_G : K_I	0,5%	0,2%	0,3%
CALC_G : K_{II}	2,0%	2,0%	2,0%
CALC_G : G	4,0%	1,0%	0,6%
POST_K1_K2_K3 : K_I	6,0%	0,2%	0,2%
POST_K1_K2_K3 : K_{II}	6,0%	3,0%	1,5%