

## SSNP505 - Plate multi-fissured in bitraction-shearing with X-FEM

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### Summary:

The objective of this test is to validate the approach multi-cracking with X-FEM and junction. A plate in plane deformation is requested in bi--traction and shearing. Eight cracks are introduced in the center of the plate, including two with junction. One uses then `RAFF_XFEM` to refine around the funds of crack and `CALC_G` to calculate the factors of intensity of the constraints.

A first test makes it possible to validate the approach without contact by comparison of the factors of intensity of the constraints with an semi-analytical solution. Then one shows in a second test that the addition of the contact, which corrects certain interpenetrations, influences the results.

## 1 Problem of reference

### 1.1 Geometry

One considers a square plate on side  $20\text{ m}$  in plane deformations, centered in the reference mark  $(X, Y)$ . The cracks are defined by the points  $A$  with  $J$  and  $A'$  with  $J'$ , of which the coordinates are given in table 1. The cracks are represented on figure 1.

Points	X	Y	Points	X	Y
A	-3.0851	0.75	A'	3.08512	-0.75
B	0.50000	1.13327	B'	-0.5000	-0.3667
C	0.20309	1.55730	C'	-0.2031	0.05730
D	-2.1454	1.09202	D'	2.14543	-0.4080
E	-1.3794	0.44923	E'	1.37939	-1.0508
F	-3.0851	-0.25	F'	3.08512	-1.75
G	-2.3780	0.45711	G'	2.37802	-1.0429
H	-1.3794	1.09202	H'	1.37939	-0.4080
I	-0.5134	1.59202	I'	0.51336	0.09202
J	-0.4397	0.79125	J'	0.43969	-0.7087

Table 1: coordinates of the points defining the cracks.

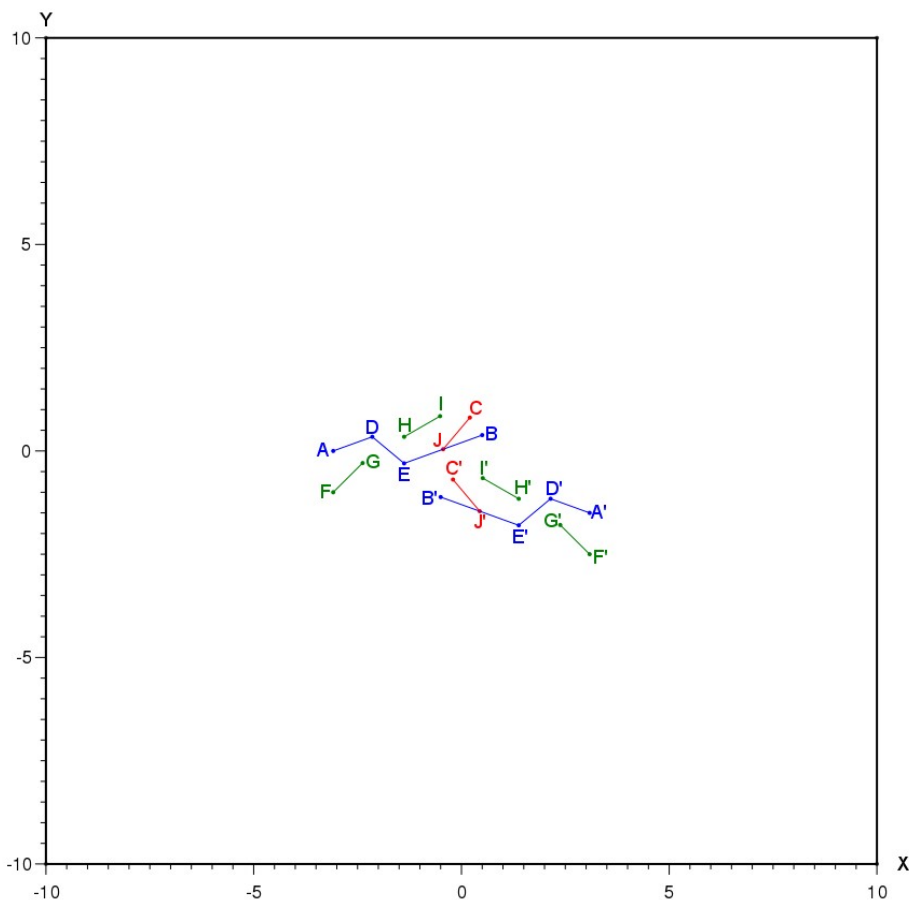


Figure 1: position of the points and the cracks.

## 1.2 Properties of material

The material is elastic isotropic with the following properties:

- $E=0,1 MPa$
- $\nu = 0.3$

## 1.3 Boundary conditions and loadings

The constraint is imposed  $\sigma = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  on all the contour of the structure. That corresponds to a loading in bi-traction and unit shearing. The rigid modes are blocked.

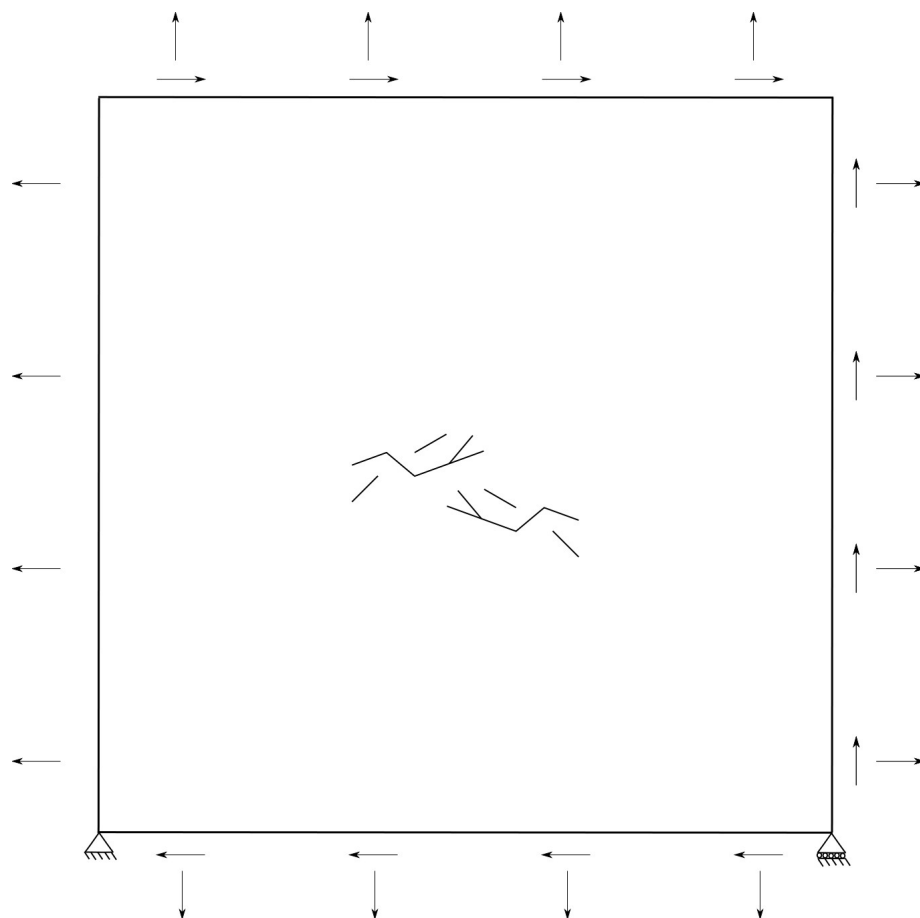


Figure 2: loading of the structure.

## 2 Reference solution

### 2.1 Method of calculating

The calculation of the factors of intensity of the constraints is obtained in [1] by an semi-analytical procedure which consists in superimposing profiles of displacements checking of the conditions of free constraints on the lips of the cracks.

### 2.2 Sizes and results of reference

One compares the values of  $K_1$  and  $K_2$  data in [1] at the bottoms of cracks.

Points	$K_1$	$K_2$
A	1.7943	2,8522
B	1.9932	2,4042
C	-1,6920	-0,1337
F	0,0510	0,2894
G	-0.5317	0,1885
H	-0.0517	-0,1979
I	-0.1933	0,0213

Points	$K_1$	$K_2$
A'	3.7215	2,3379
B'	2,6700	1,0248
C'	5,3966	-0,1143
F'	4,3255	-0,1661
G'	3,6812	0,9279
H'	0,4157	-0,3947
I'	1,0043	0,0648

Table 2: factors of intensity of the constraints at the bottoms of cracks obtained in [1].

### 2.3 Uncertainties on the solution

The number of significant figures given in table 2 reflects the quality of the semi-analytical solution. One can indeed check convergence towards these figures in the case of an increasingly fine grid.

### 2.4 Bibliographical references

- [1] A.K. YAVUZ, S.L. PHOENIX, "Multiple Analysis Ace in Finite Punts", AIAA Newspaper, Flight 44, No 11, November 2006.
- [2] Mr. SIAVELIS, Mr. GUITON, P. MASSIN, NR. MOËS "Broad sliding contact along branched discontinuities xith X-FEM", in second reading, 2012.

## 3 Modeling A

### 3.1 Characteristics of modeling

A modeling is used `D_PLAN`. The cracks are introduced with X-FEM using the operands `GROUP_MA_FISS` and `GROUP_MA_FOND` of the operator `DEFI_FISS_XFEM`. The points given in table 1 indeed make it possible to generate lines with a grid easily representing the cracks of figure 1. The possible contact between the lips of the cracks is not taken into account to respect the assumption of free constraints on the lips of the cracks of the reference solution.

### 3.2 Characteristics of the grid

Initial regulated grid composed of 400 meshes of the type `QUAD4` figure 3 is refined in an adaptive way in 3 iterations using the orders `RAFF_XFEM` and `MACR_ADAP_MAILL`. One obtains the final grid refined around the funds of cracks of figure 4, composed of 17105 meshes of the type `QUAD4` and of 2076 meshes of the type `TRIA3`.

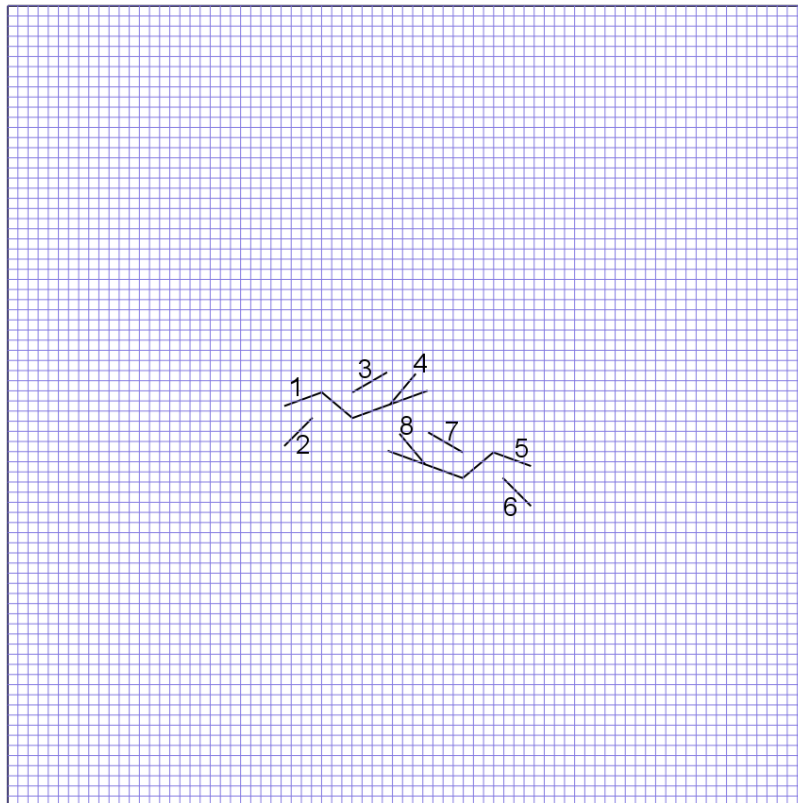


Figure 3: initial grid and classification of the cracks.

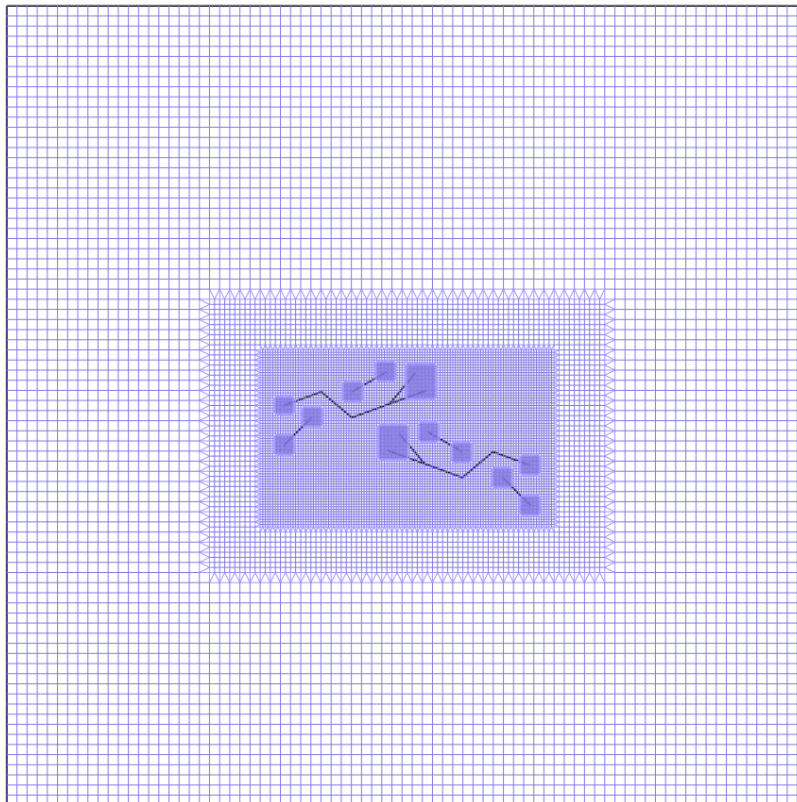


Figure 4: grid refined around the funds of crack used for calculation.

### 3.3 Sizes tested and results

They are tested factors of intensity of the constraints on the points of the funds of crack compared to the values given in table 2. factors of intensity of the constraints are calculated using the option `CALC_K_G` of the operator `CALC_G`.

The values are validated with a tolerance of 1%. It is noted however that it is numerically possible to converge towards the semi-analytical solution by refining the grid.

Identification	Type of reference	Value of reference	Tolerance
Not $A - K_1$	'ANALYTICAL'	0,0000	1.1%
Not $A - K_2$	'ANALYTICAL'	2,8522	1.1%
Not $B - K_1$	'ANALYTICAL'	0,0000	1.1%
Not $B - K_2$	'ANALYTICAL'	2,4042	1.1%
Not $C - K_1$	'ANALYTICAL'	-1,6920	1.1%
Not $C - K_2$	'ANALYTICAL'	-0,1337	1.1%
Not $F - K_1$	'ANALYTICAL'	0,0510	1.1%
Not $F - K_2$	'ANALYTICAL'	0,2894	1.1%
Not $G - K_1$	'ANALYTICAL'	0,0000	1.1%
Not $G - K_2$	'ANALYTICAL'	0,1885	1.1%

Not $H - K_1$	'ANALYTICAL'	0,0000	1.1%
Not $H - K_2$	'ANALYTICAL'	-0,1979	1.1%
Not $I - K_1$	'ANALYTICAL'	0,0000	1.1%
Not $I - K_2$	'ANALYTICAL'	0,0213	1.1%
Not $A' - K_1$	'ANALYTICAL'	0,0000	1.1%
Not $A' - K_2$	'ANALYTICAL'	2,3379	1.1%
Not $B' - K_1$	'ANALYTICAL'	2,6700	1.1%
Not $B' - K_2$	'ANALYTICAL'	1,0248	1.1%
Not $C' - K_1$	'ANALYTICAL'	5,3966	1.1%
Not $C' - K_2$	'ANALYTICAL'	-0,1143	1.1%
Not $F' - K_1$	'ANALYTICAL'	4,3255	1.1%
Not $F' - K_2$	'ANALYTICAL'	-0,1661	1.1%
Not $G' - K_1$	'ANALYTICAL'	3,6812	1.1%
Not $G' - K_2$	'ANALYTICAL'	0,9279	1.1%
Not $H' - K_1$	'ANALYTICAL'	0,4157	1.1%
Not $H' - K_2$	'ANALYTICAL'	-0,3947	1.1%
Not $I' - K_1$	'ANALYTICAL'	1,0043	1.1%
Not $I' - K_2$	'ANALYTICAL'	0,0648	1.1%

## 4 Modeling B

### 4.1 Characteristics of modeling

The characteristics are the same ones as those of modeling A. One however takes into account the possible efforts of contact between the lips of the cracks via the operator `DEFI_CONTACT` in order to correct the interpenetrations. The assumption of free constraints on the lips of the cracks made for the calculation of the reference solution is not respected for the cracks where the contact is active.

### 4.2 Characteristics of the grid

The grid is the same one as for modeling A.

### 4.3 Sizes tested and results

They are tested factors of intensity of the constraints on the points of the funds of crack as for modeling A. the variations related to the taking into account of the contact on certain points do not make it possible to validate the approach with the semi-analytical solution of the section 2.1. A test of nonregression is thus made. However the error obtained compared to an exact solution should be of the same order of magnitude as that obtained between the reference solution and the solution of modeling A. Indeed, the kinematic approximation allowing the calculation of factors of intensity of the constraints is exactly identical for modeling A and B. Only the efforts of contact on certain lips of cracks are added here compared to modeling A.

Identification	Type of reference	Value of reference	Tolerance
Not $A - K_1$	'NON_REGRESSION'	1,7479	1.1%
Not $A - K_2$	'NON_REGRESSION'	2,7678	1.1%
Not $B - K_1$	'NON_REGRESSION'	1,2384	1.1%
Not $B - K_2$	'NON_REGRESSION'	3,1770	1.1%
Not $C - K_1$	'NON_REGRESSION'	-0,5836	1.1%
Not $C - K_2$	'NON_REGRESSION'	0,4408	1.1%
Not $F - K_1$	'NON_REGRESSION'	0,1656	1.1%
Not $F - K_2$	'NON_REGRESSION'	0,2810	1.1%
Not $G - K_1$	'NON_REGRESSION'	-0,3466	1.1%
Not $G - K_2$	'NON_REGRESSION'	0,1555	1.1%
Not $H - K_1$	'NON_REGRESSION'	-0,0002	1.1%
Not $H - K_2$	'NON_REGRESSION'	-0,0410	1.1%
Not $I - K_1$	'NON_REGRESSION'	-0,3366	1.1%
Not $I - K_2$	'NON_REGRESSION'	0,2531	1.1%
Not $A' - K_1$	'NON_REGRESSION'	3,6999	1.1%
Not $A' - K_2$	'NON_REGRESSION'	2,3679	1.1%
Not $B' - K_1$	'NON_REGRESSION'	2,6639	1.1%



Not $B' - K_2$	'NON_REGRESSION'	1,0261	1.1%
Not $C' - K_1$	'NON_REGRESSION'	5,4041	1.1%
Not $C' - K_2$	'NON_REGRESSION'	-0,0961	1.1%
Not $F' - K_1$	'NON_REGRESSION'	4,3020	1.1%
Not $F' - K_2$	'NON_REGRESSION'	-0,1591	1.1%
Not $G' - K_1$	'NON_REGRESSION'	3,6790	1.1%
Not $G' - K_2$	'NON_REGRESSION'	0,9319	1.1%
Not $H' - K_1$	'NON_REGRESSION'	0,4022	1.1%
Not $H' - K_2$	'NON_REGRESSION'	-0,3641	1.1%
Not $I' - K_1$	'NON_REGRESSION'	0,9760	1.1%
Not $I' - K_2$	'NON_REGRESSION'	0,0989	1.1%

## 4.4 Remarks

Figure 5 shows the deformation obtained. One notes a correction of the interpenetrations, which is obvious for crack 4 with the addition of the contact. This correction is at the origin of the variations obtained on factors of intensity of the constraints.

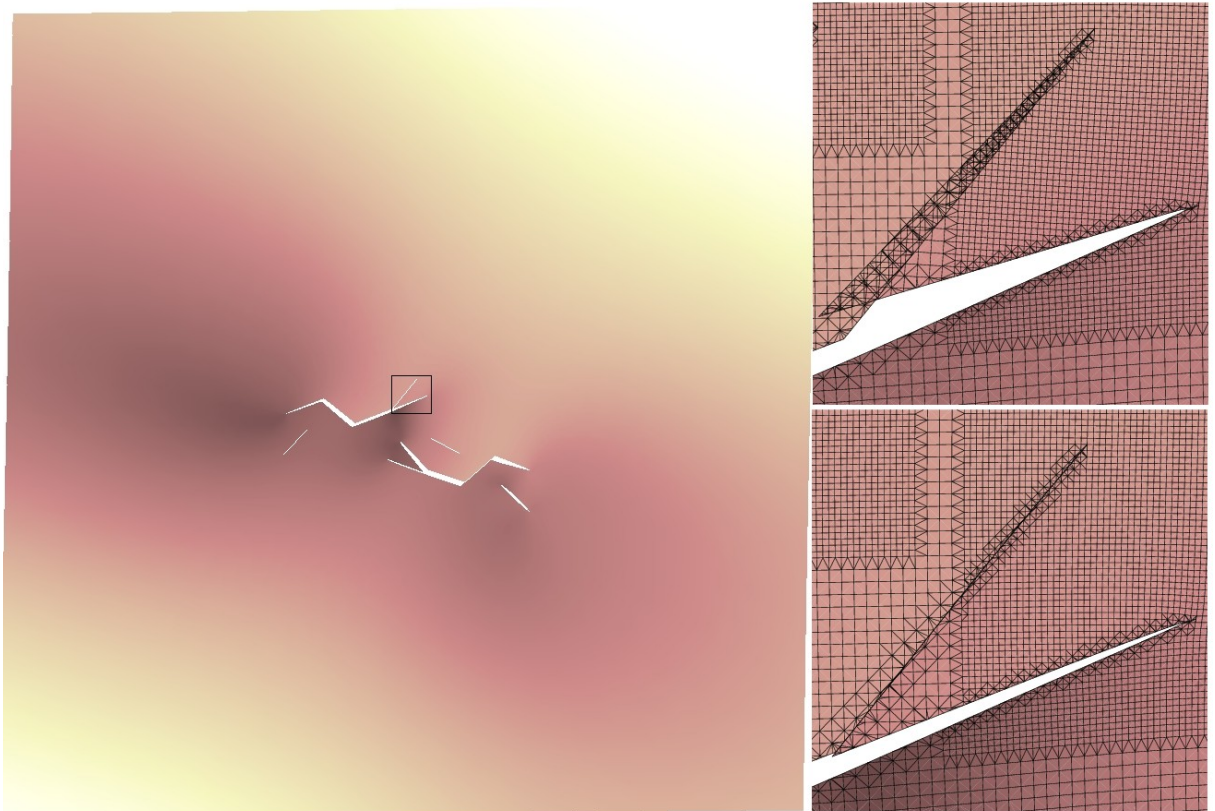


Figure 5: deformation obtained with amplification x1000 (on the left), zoom on crack 4, without and with contact (on the right).

## 5 Summary of the results

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The first calculation without contact makes it possible to validate the approach multi-cracking with junctions in X-FEM by comparison of the factors of intensity of the constraints to an semi-analytical reference solution (which does not take into account the efforts on the lips of the cracks). Interpenetrations however are noted, although the structure is primarily requested in traction. The correction made by the second calculation, with taking into account of the contact, shows that the new values obtained for the factors of intensity of the constraints can lead to light deviations of the directions of propagation [2].