SSNV104 - Contact of two phères

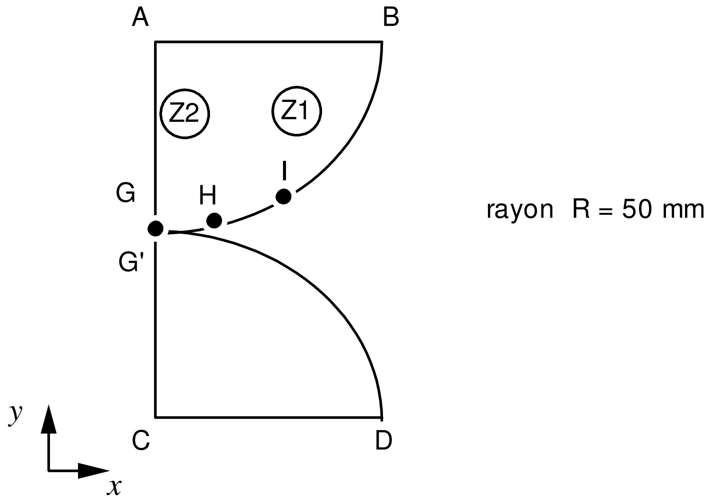
Summary:

Calculation consists in crushing two hemispheres one on the other to test the algorithm of unilateral contact in statics. The solution is compared with the analytical result of Hertz.
Modelings implement calculations into axisymmetric as in $3D$ with linear/quadratic elements and various methods of resolutions of the contact (dualisation, penalization, continuous formulation).
This classical test for which one has an analytical solution is also used to validate several options of the algorithm of pairing.

1 Problem of reference

1.1 Geometry

The geometry consists of two half-spheres of identical ray which one represents a cut in the figure below.



Taking into account the differences in grid, positions of the nodes H and I indicated here in an approximate way are different in 3 modelings.

1.2 Properties of material

$E=20\,000.\,MPa$

$\nu=0.3$

1.3 Boundary conditions and loadings

Crushing imposed on the spheres:

- DY imposed $-2\,mm$ for the nodes of AB
- DY imposed $+2\,mm$ for the nodes of CD

For axisymmetric calculations:

- DX blocked on the axis AC

For calculations $3\mathrm{D}$ (2 modelled quarters of sphere):

- conditions of symmetry on each quarter of sphere

2 Reference solution

2.1 Method of calculating used for the reference solution

In 1881, Hertz established under certain assumptions a solution with the problem which bears its name[1]. Thus by supposing a contact without friction and for which the half width of contact a is very small in front of the ray of the spheres R ($a \ll R$), the contact pressure at the points $C1$ and $C2$ is worth [2] :

$$P_0 = -\frac{E}{\pi(1-\nu^2)}\sqrt{\frac{2h}{R}}$$ éq 2.1-1

where h corresponds to the imposed crushing, which is worth here $4\,mm$. That is to say $P_0 = -2798.3\,Mpa$.

The half-width of contact a express yourself according to imposed crushing and of the ray of the spheres:

$$a = \sqrt{\frac{Rh}{2}}$$ éq 2.1-2

In this test, for a crushing of $4\,mm$, $a = 10\,mm$.
The surface of contact is a disc of ray a, the distribution of pressure in this zone is the following one:

If $x \leqslant a$ then $P(x) = P_0\sqrt{1-\left(\frac{x}{a}\right)^2}$ éq 2.1-3

2.2 Sizes and results of reference

σ_{yy} at the point G (analytical solution).

Displacements in three points of the edge (not-regression except for G analytical).

σ_{yy} in meshs resting on G (not-regression).

Statute and game in several points of the surface of contact (analytical).

2.3 Uncertainties on the solution

Analytical calculation although valid for $a \ll R$ in general give a very good approximation of the solution.

2.4 Bibliographical reference

Bibliography

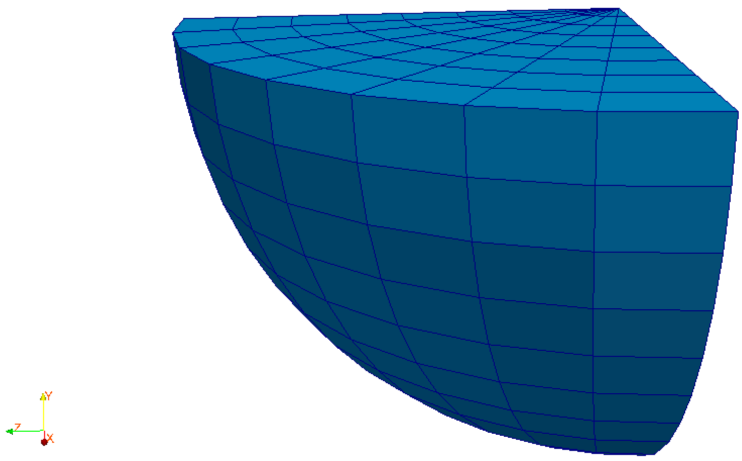
1: HERTZ H, Über die Berührung fester elastischer Körper, 1881
2: DUMONT G, La méthode des contraintes actives appliquée au contact unilatéral, 1993

3 Modeling A

3.1 Characteristics of modeling

A modeling is used 3D. Due to symmetry, only one quarter of hemisphere is represented, a condition of type unilateral connection makes it possible to supplement modeling.
In this modeling, points H and H_p (respectively I and I_p) are the same points (compared to those described in the geometry) but located on the two faces supporting the conditions of symmetry. The results got with this modeling are used as reference (AUTRE_ASTER) for three-dimensional modelings I and J.

3.2 Characteristics of the grid



Nodes: 1849.

Meshs: 96 TETRA4, 2936 PENTA6, 112 PYRAM5.

3.3 Sizes tested and results

The first calculation (formulation 'LIAISON_UNIL')

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M2948$ not G	'ANALYTICAL'	-2798.3 N	14.0%
σ_{yy} mesh $M2948$ not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
σ_{yy} mesh $M2960$ not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
σ_{yy} mesh $M2972$ not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
σ_{yy} mesh $M2984$ not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
σ_{yy} mesh $M2996$ not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
σ_{yy} mesh $M3008$ not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
σ_{yy} mesh $M3020$ not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%
σ_{yy} mesh $M3032$ not G	'NON_REGRESSION'	-3176.3 N	1,0E-4%

DX not G	'ANALYTICAL'	0 mm	1.0E-10
DX not H	'NON_REGRESSION'	-0.11343 mm	1,0E-4%
DZ not H_p	'NON_REGRESSION'	-0.11343 mm	1,0E-4%
DY not H	'NON_REGRESSION'	-0.16291 mm	1,0E-4%
DY not H_p	'NON_REGRESSION'	-0.16291 mm	1,0E-4%
DX not I	'NON_REGRESSION'	-0.17845 mm	1,0E-4%
DZ not I_p	'NON_REGRESSION'	-0.17845 mm	1,0E-4%
DY not I	'NON_REGRESSION'	-0.62966 mm	1,0E-4%
DY not I_p	'NON_REGRESSION'	-0.62966 mm	1,0E-4%

3.4 Remarks

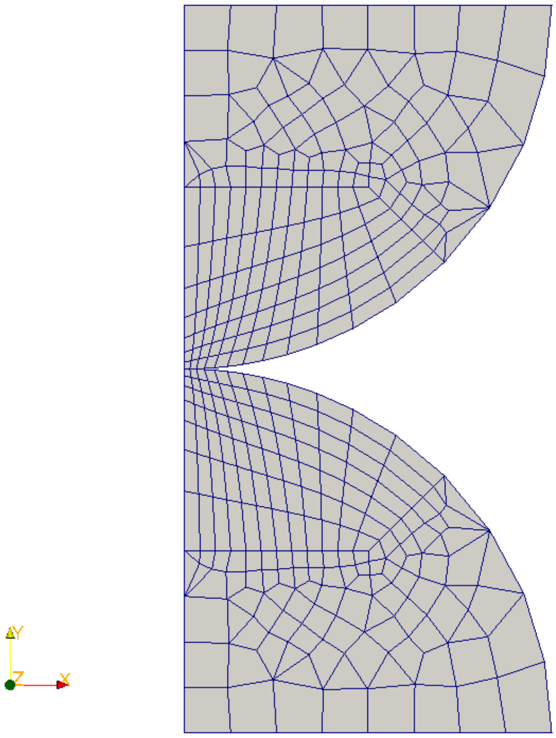
The results got in this three-dimensional modeling are in concord with the analytical solution. One notes moreover than the radiant grid makes it possible to preserve the axisymetry of the problem, this is why the results got in each mesh which contributes to the point G are similar.
This modeling validates the use of a unilateral condition function of the space, i.e. whose “game” is not the same one for all the constrained nodes. The got results are identical to a modeling of the contact.

4 Modeling B

4.1 Characteristics of modeling

A modeling is used AXIS. Three calculations are carried out with options of pairing or different algorithms of contact.

4.2 Characteristics of the grid



Nodes: 376 nodes.

Meshs: 30 TRIA3 and 324 QUAD4.

4.3 Sizes tested and results

The first calculation (nodal pairing, normal slave)

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$	'ANALYTICAL'	-2798.3 N	7,0%
σ_{yy} mesh $M31$ node $N291(G)$	'NON_REGRESSION'	-2971.27 N	0,1%
DX node $N291(G)$	'ANALYTICAL'	0 mm	1.0E-10
DX node $N287(H)$	'NON_REGRESSION'	-0.111283 mm	0,1%
DY node $N287(H)$	'NON_REGRESSION'	-0.161842 mm	0,1%
DX node $N285(I)$	'NON_REGRESSION'	-0.168540 mm	0,1%
DY node $N285(I)$	'NON_REGRESSION'	-0.628587 mm	0.1%

The second calculation (algorithm 'LAGRANGIAN')

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$	'ANALYTICAL'	-2798.3 N	7,0%
σ_{yy} mesh $M31$ node $N291(G)$	'NON_REGRESSION'	-2973.0 N	0,1%
DX node $N291(G)$	'ANALYTICAL'	0 mm	1.0E-10
DX node $N287(H)$	'NON_REGRESSION'	-0.108104 mm	0,1%
DY node $N287(H)$	'NON_REGRESSION'	-0.164375 mm	0,1%
DX node $N285(I)$	'NON_REGRESSION'	-0.160912 mm	0,1%
DY node $N285(I)$	'NON_REGRESSION'	-0.630986 mm	0.1%

The third calculation (algorithm 'FORCED')

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$	'ANALYTICAL'	-2798.3 N	7,0%
σ_{yy} mesh $M31$ node $N291(G)$	'NON_REGRESSION'	-2973.0 N	0,1%
DX node $N291(G)$	'ANALYTICAL'	0 mm	1.0E-10
DX node $N287(H)$	'NON_REGRESSION'	-0.108104 mm	0,1%
DY node $N287(H)$	'NON_REGRESSION'	-0.164375 mm	0,1%
DX node $N285(I)$	'NON_REGRESSION'	-0.160912 mm	0,1%
DY node $N285(I)$	'NON_REGRESSION'	-0.630986 mm	0.1%

4.4 Remarks

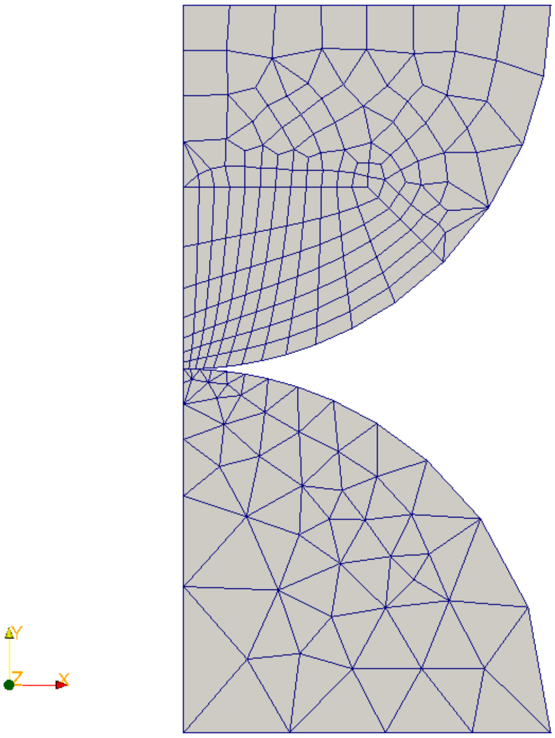
The variation enters the contact pressure at the point G and the analytical solution is explained on the one hand by extrapolation (passage points of Gauss towards nodes) and on the other hand by the weak refinement of the grid.
It is noted that the dualized algorithms of contact 'LAGRANGIAN' and 'FORCED' the same results give exactly and that nodal pairing makes it possible to preserve the symmetry of the problem while giving results very close to pairing node-facet.

5 Modeling D

5.1 Characteristics of modeling

A modeling is used AXIS. Two calculations are carried out with different options of pairing. In this modeling, imposed crushing goes until $10\,mm$.

5.2 Characteristics of the grid



Nodes: 243.

Meshs: 100 TRIA3, 162 QUAD4.

5.3 Sizes tested and results

The first calculation (smoothing)

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$ moment 1.0	'ANALYTICAL'	-1978.7 N	7.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	1.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	15.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 2.0	'ANALYTICAL'	-2798.3 N	5.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	15.0%

σ_{yy} mesh $M31$ node $N291(G)$ moment 3.0	'ANALYTICAL'	-3427.2 N	2.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	14.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 4.0	'ANALYTICAL'	-3957.4 N	3.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	4.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	15.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 5.0	'ANALYTICAL'	-4424.5 N	6.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	4.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	15.0%

The second calculation (nodal pairing and normal slave)

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$ moment 1.0	'ANALYTICAL'	-1978.7 N	2.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	1.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	4.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 2.0	'ANALYTICAL'	-2798.3 N	7.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	7.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	4.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 3.0	'ANALYTICAL'	-3427.2 N	18.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	13.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 4.0	'ANALYTICAL'	-3957.4 N	28.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	20.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	2.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 5.0	'ANALYTICAL'	-4424.5 N	38.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	27.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	1.0%

5.4 Remarks

The grid consists of 2 half-spheres with a grid differently (the higher sphere is in QUAD4, the lower sphere in TRIA3). The contact pressure raised on the upper surface is in concord with the analytical solution (the variation of about 5% is explained by extrapolation).
On lower surface, one notes contrary to the considerable variations because there exist 2 meshs which contribute to the point G' (grid of TRIA3), however if one makes the average of the results got on these 2 meshs, one finds a pressure in concord with the analytical solution.

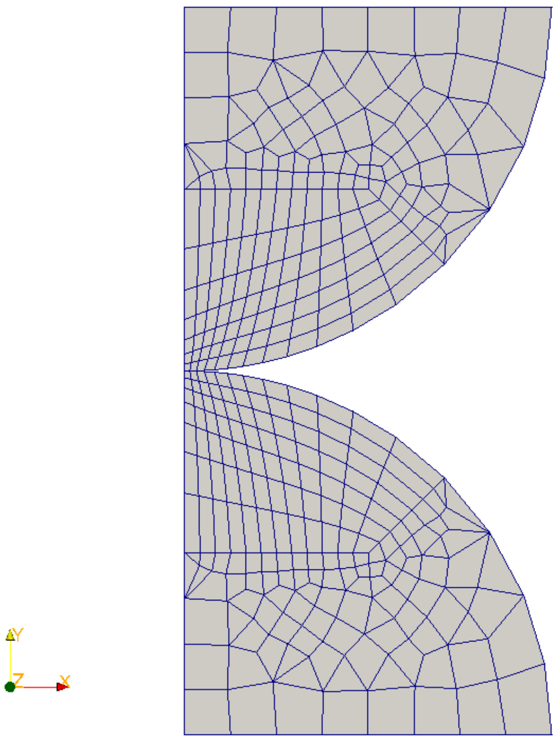
As regards the second calculation the important errors are due to the use of a nodal pairing whereas the grid is not compatible.

6 Modeling E

6.1 Characteristics of modeling

A modeling is used AXIS with the formulation CONTINUOUS contact. This modeling validates the functionality SLIDE by slackening imposed crushing.

6.2 Characteristics of the grid



Nodes: 376 nodes.

Meshs: 30 TRIA3 and 324 QUAD4.

6.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$	'ANALYTICAL'	-2798.3 N	7,0%
σ_{yy} mesh $M31$ node $N291(G)$	'NON_REGRESSION'	-2971.37 N	1.0%
DX node $N291(G)$	'ANALYTICAL'	0 mm	1.0E-10
DX node $N287(H)$	'NON_REGRESSION'	-0.110211 mm	3.0%
DY node $N287(H)$	'NON_REGRESSION'	-0.162911 mm	1.0%
DX node $N285(I)$	'NON_REGRESSION'	-0.165946 mm	4.0%
DY node $N285(I)$	'NON_REGRESSION'	-0.629666 mm	0.5%

Identification	Type of reference	Value of reference	Tolerance

Statute $CONT$ node $N72$ moment 0.2	'ANALYTICAL'	0	0.1
Statute $CONT$ node $N80$ moment 0.2	'ANALYTICAL'	0	0.1
Statute $CONT$ node $N81$ moment 0.2	'ANALYTICAL'	0	0.1
Statute $CONT$ node $N72$ moment 0.5	'ANALYTICAL'	0	0.1
Statute $CONT$ node $N80$ moment 0.5	'ANALYTICAL'	0	0.1
Statute $CONT$ node $N81$ moment 0.5	'ANALYTICAL'	2	0.1
Statute $CONT$ node $N72$ moment 1.0	'ANALYTICAL'	2	0.1
Statute $CONT$ node $N80$ moment 1.0	'ANALYTICAL'	2	0.1
Statute $CONT$ node $N81$ moment 1.0	'ANALYTICAL'	2	0.1
Statute $CONT$ node $N72$ moment 1.5	'ANALYTICAL'	2	0.1
Statute $CONT$ node $N80$ moment 1.5	'ANALYTICAL'	2	0.1
Statute $CONT$ node $N81$ moment 1.5	'ANALYTICAL'	2	0.1
Game JEU node $N72$ moment 1.5	'ANALYTICAL'	0	1.0E-8
Game JEU node $N80$ moment 1.5	'ANALYTICAL'	0	1.0E-8
Game JEU node $N81$ moment 1.5	'ANALYTICAL'	0	1.0E-8
Statute $CONT$ node $N72$ moment 2.0	'ANALYTICAL'	2	0.1
Statute $CONT$ node $N80$ moment 2.0	'ANALYTICAL'	2	0.1
Statute $CONT$ node $N81$ moment 2.0	'ANALYTICAL'	2	0.1
Game JEU node $N72$ moment 2.0	'ANALYTICAL'	0	1.0E-8
Game JEU node $N80$ moment 2.0	'ANALYTICAL'	0	1.0E-8
Game JEU node $N81$ moment 2.0	'ANALYTICAL'	0	1.0E-8

6.4 Remarks

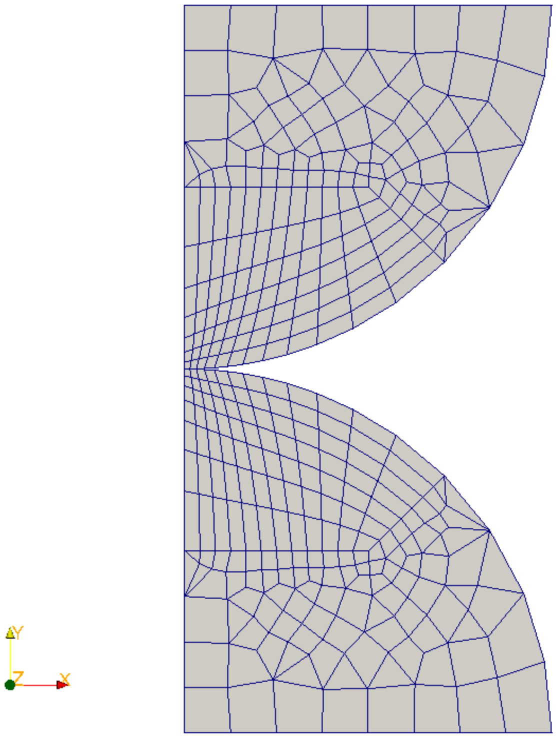
Results got with the formulation CONTINUOUS are very close to those obtained in formulation DISCRETE and in concord with the analytical solution.
The differences noted on displacements are explained by the diagram of integration used in formulation CONTINUOUS : it is about a diagram to the nodes which does not make it possible correctly to treat the nullity of Jacobien on the axis of revolution.

7 Modeling F

7.1 Characteristics of modeling

A modeling is used AXIS. Four calculations are carried out with options of pairing or different algorithms of contact.

7.2 Characteristics of the grid



Nodes: 376 nodes.

Meshs: 30 TRIA3 and 324 QUAD4.

7.3 Sizes tested and results

The first calculation (nodal pairing, master-slave normal and algorithm 'LAGRANGIAN')

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$	'ANALYTICAL'	-2798.3 N	7,0%
σ_{yy} mesh $M31$ node $N291(G)$	'NON_REGRESSION'	-2971.37 N	0,1%
DX node $N291(G)$	'ANALYTICAL'	0 mm	1.0E-10
DX node $N287(H)$	'NON_REGRESSION'	-0.110211 mm	0,1%
DY node $N287(H)$	'NON_REGRESSION'	-0.162911 mm	0,1%
DX node $N285(I)$	'NON_REGRESSION'	-0.165946 mm	0,1%
DY node $N285(I)$	'NON_REGRESSION'	-0.629666 mm	0.1%

The second calculation (master-slave normal, algorithm `'LAGRANGIAN'`)

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$	'ANALYTICAL'	-2798.3 N	7,0%
σ_{yy} mesh $M31$ node $N291(G)$	'NON_REGRESSION'	-2966.0 N	0,1%
DX node $N291(G)$	'ANALYTICAL'	0 mm	1.0E-10
DX node $N287(H)$	'NON_REGRESSION'	-0.110678 mm	0,1%
DY node $N287(H)$	'NON_REGRESSION'	-0.162891 mm	0,1%
DX node $N285(I)$	'NON_REGRESSION'	-0.167194 mm	0,1%
DY node $N285(I)$	'NON_REGRESSION'	-0.628947 mm	0.1%

The third calculation (master-slave normal, algorithm `'FORCED'`)

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$	'ANALYTICAL'	-2798.3 N	7,0%
σ_{yy} mesh $M31$ node $N291(G)$	'NON_REGRESSION'	-2966.0 N	0,1%
DX node $N291(G)$	'ANALYTICAL'	0 mm	1.0E-10
DX node $N287(H)$	'NON_REGRESSION'	-0.110678 mm	0,1%
DY node $N287(H)$	'NON_REGRESSION'	-0.162901 mm	0,1%
DX node $N285(I)$	'NON_REGRESSION'	-0.167194 mm	0,1%
DY node $N285(I)$	'NON_REGRESSION'	-0.628947 mm	0.1%

The fourth calculation (smoothing, master-slave normal and algorithm `'FORCED'`)

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$	'ANALYTICAL'	-2798.3 N	7,0%
σ_{yy} mesh $M31$ node $N291(G)$	'NON_REGRESSION'	-2971.37 N	0,1%
DX node $N291(G)$	'ANALYTICAL'	0 mm	1.0E-10
DX node $N287(H)$	'NON_REGRESSION'	-0.110211 mm	0,1%
DY node $N287(H)$	'NON_REGRESSION'	-0.162911 mm	0,1%
DX node $N285(I)$	'NON_REGRESSION'	-0.165946 mm	0,1%
DY node $N285(I)$	'NON_REGRESSION'	-0.629666 mm	0.1%

7.4 Remarks

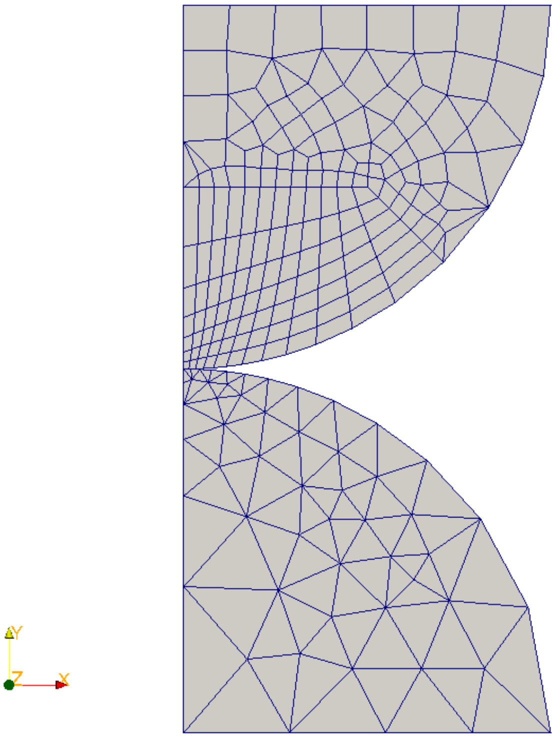
On this modeling, it is noted that smoothing makes it possible to find the symmetry of the problem (nodes perfectly in opposite once the established contact), last calculation indeed which gets the same results as in nodal pairing.
The values obtained are in concord general with the analytical solutions.

8 Modeling G

8.1 Characteristics of modeling

A modeling is used AXIS. Two calculations are carried out with different options of pairing. In this modeling, imposed crushing goes until $10\,mm$.

8.2 Characteristics of the grid



Nodes: 243.

Meshs: 100 TRIA3, 162 QUAD4.

8.3 Sizes tested and results

The first calculation (smoothing, master-slave normal and algorithm 'LAGRANGIAN')

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$ moment 1.0	'ANALYTICAL'	-1978.7 N	8.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	1.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	15.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 2.0	'ANALYTICAL'	-2798.3 N	6.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	4.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	15.0%

σ_{yy} mesh $M31$ node $N291(G)$ moment 3.0	'ANALYTICAL'	-3427.2 N	1.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	14.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 4.0	'ANALYTICAL'	-3957.4 N	4.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	14.5%
σ_{yy} mesh $M31$ node $N291(G)$ moment 5.0	'ANALYTICAL'	-4424.5 N	8.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	15.0%

The second calculation (smoothing, master-slave normal and algorithm 'FORCED')

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$ moment 1.0	'ANALYTICAL'	-1978.7 N	8.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	1.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	15.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 2.0	'ANALYTICAL'	-2798.3 N	6.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	4.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	15.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 3.0	'ANALYTICAL'	-3427.2 N	1.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	14.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 4.0	'ANALYTICAL'	-3957.4 N	4.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	14.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 5.0	'ANALYTICAL'	-4424.5 N	8.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	15.0%

8.4 Remarks

The grid consists of 2 half-spheres with a grid differently (the higher sphere is in QUAD4, the lower sphere in TRIA3). The contact pressure raised on the upper surface is in concord with the analytical solution (the variation of about 5% is explained by extrapolation).
On lower surface, one notes contrary to the considerable variations because there exist 2 meshs which contribute to the point G' (grid of TRIA3), however if one makes the average of the results got on these 2 meshs, one finds a pressure in concord with the analytical solution.

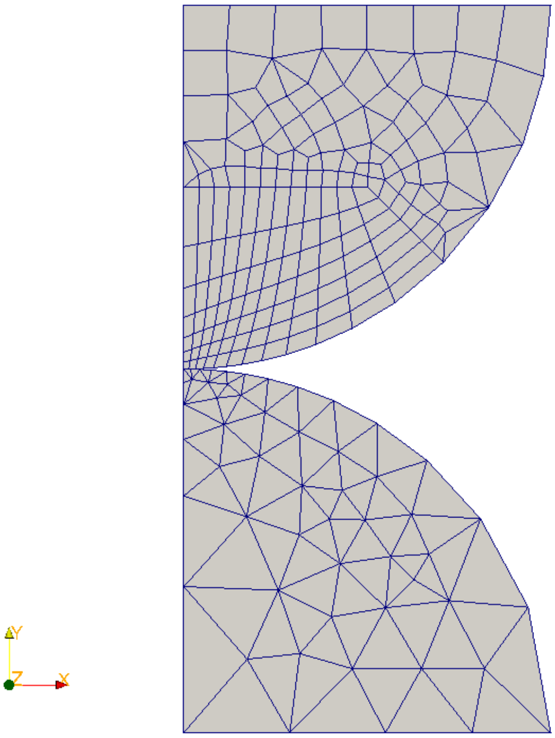
The two algorithms of contact used give the same results appreciably.

9 Modeling H

9.1 Characteristics of modeling

A modeling is used AXIS. Only one calculation with penalization of the contact is carried out. In this modeling, imposed crushing goes until $10\,mm$.

9.2 Characteristics of the grid



Nodes: 243.

Meshs: 100 TRIA3, 162 QUAD4.

9.3 Sizes tested and results

The first calculation (smoothing, algorithm 'PENALIZATION')

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M31$ node $N291(G)$ moment 1.0	'ANALYTICAL'	-1978.7 N	7.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	1.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 1.0	'ANALYTICAL'	-1978.7 N	15.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 2.0	'ANALYTICAL'	-2798.3 N	5.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 2.0	'ANALYTICAL'	-2798.3 N	15.0%

σ_{yy} mesh $M31$ node $N291(G)$ moment 3.0	'ANALYTICAL'	-3427.2 N	2.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	3.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 3.0	'ANALYTICAL'	-3427.2 N	14.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 4.0	'ANALYTICAL'	-3957.4 N	3.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	4.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 4.0	'ANALYTICAL'	-3957.4 N	15.0%
σ_{yy} mesh $M31$ node $N291(G)$ moment 5.0	'ANALYTICAL'	-4424.5 N	6.0%
σ_{yy} mesh $M286$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	4.0%
σ_{yy} mesh $M293$ node $N55(G')$ moment 5.0	'ANALYTICAL'	-4424.5 N	15.0%

9.4 Remarks

The grid consists of 2 half-spheres with a grid differently (the higher sphere is in QUAD4, the lower sphere in TRIA3). The contact pressure raised on the upper surface is in concord with the analytical solution (the variation of about 5% is explained by extrapolation).
On lower surface, one notes contrary to the considerable variations because there exist 2 meshs which contribute to the point G' (grid of TRIA3), however if one makes the average of the results got on these 2 meshs, one finds a pressure in concord with the analytical solution.

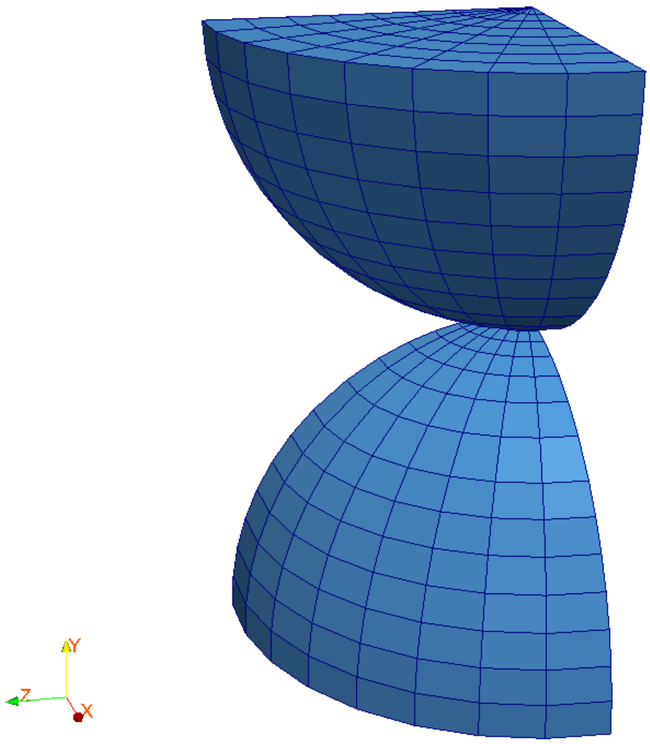
The coefficient of penalization used here (large in front $R_{sphere}*E$) allows to get results comparable to those of the exact algorithms of dualisation (modeling G).

10 Modeling I

10.1 Characteristics of modeling

A modeling is used 3D. Only one calculation with the parameters by default of the contact is carried out.
In this modeling, points H and H_p (resp. I and I_p) are the same points (compared to those described in the geometry) but located on the two faces supporting the conditions of symmetry.

10.2 Characteristics of the grid



Nodes: 3698.

Meshs: 192 TETRA4, 5872 PENTA6, 224 PYRAM5.

10.3 Sizes tested and results

The first calculation (algorithm 'FORCED')

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M5884$ not G	'ANALYTICAL'	-2798.3 N	14.0%
σ_{yy} mesh $M5884$ not G	'AUTRE_ASTER'	-3176.3 N	2.0%
σ_{yy} mesh $M5896$ not G	'AUTRE_ASTER'	-3176.3 N	2.0%
σ_{yy} mesh $M5908$ not G	'AUTRE_ASTER'	-3176.3 N	2.0%

σ_{yy} mesh $M5920$ not G	'AUTRE_ASTER'	-3176.3 N	2.0%
σ_{yy} mesh $M5932$ not G	'AUTRE_ASTER'	-3176.3 N	2.0%
σ_{yy} mesh $M5944$ not G	'AUTRE_ASTER'	-3176.3 N	2.0%
σ_{yy} mesh $M5956$ not G	'AUTRE_ASTER'	-3176.3 N	2.0%
σ_{yy} mesh $M5968$ not G	'AUTRE_ASTER'	-3176.3 N	2.0%
DX not G	'ANALYTICAL'	0 mm	1.0E-10
DX not H	'AUTRE_ASTER'	-0.11343 mm	4.0%
DZ not H_p	'AUTRE_ASTER'	-0.11343 mm	4.0%
DY not H	'AUTRE_ASTER'	-0.16291 mm	2.0%
DY not H_p	'AUTRE_ASTER'	-0.16291 mm	2.0%
DX not I	'AUTRE_ASTER'	-0.17845 mm	3.0%
DZ not I_p	'AUTRE_ASTER'	-0.17845 mm	3.0%
DY not I	'AUTRE_ASTER'	-0.62966 mm	0.1%
DY not I_p	'AUTRE_ASTER'	-0.62966 mm	0.1%

10.4 Remarks

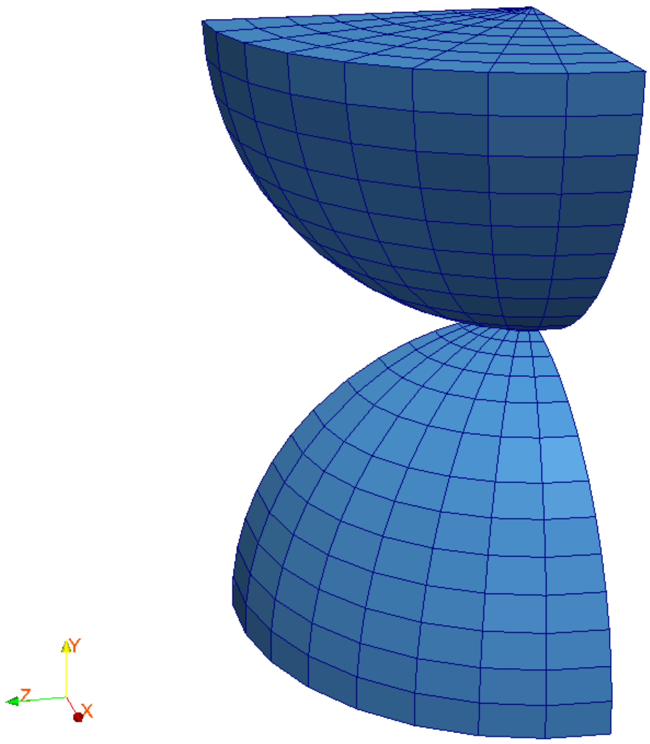
The results got in this modeling differ up to 4 % from the reference (modeling A): it is because the geometrical non-linearity of the contact is voluntarily solved coarsely (criterion fixed at 5 %). One notes that the radiant grid makes it possible to preserve the axisymetry of the problem, this is why the results got in each mesh which contributes to the point G are similar.

11 Modeling J

11.1 Characteristics of modeling

A modeling is used 3D. Only one calculation with an alternative of the algorithm of contact by default is carried out.
In this modeling, points H and H_p (resp. I and I_p) are the same points (compared to those described in the geometry) but located on the two faces supporting the conditions of symmetry.

11.2 Characteristics of the grid



Nodes: 3698.

Meshs: 192 TETRA4, 5872 PENTA6, 224 PYRAM5.

11.3 Sizes tested and results

The first calculation (algorithm 'GCP')

Identification	Type of reference	Value of reference	Tolerance
σ_{yy} mesh $M5884$ not G	'ANALYTICAL'	-2798.3 N	14.0%
σ_{yy} mesh $M5884$ not G	'AUTRE_ASTER'	-3176.3 N	0.1%
σ_{yy} mesh $M5896$ not G	'AUTRE_ASTER'	-3176.3 N	0.1%
σ_{yy} mesh $M5908$ not G	'AUTRE_ASTER'	-3176.3 N	0.1%

σ_{yy} mesh $M5920$ not G	'AUTRE_ASTER'	-3176.3 N	0.1%
σ_{yy} mesh $M5932$ not G	'AUTRE_ASTER'	-3176.3 N	0.1%
σ_{yy} mesh $M5944$ not G	'AUTRE_ASTER'	-3176.3 N	0.1%
σ_{yy} mesh $M5956$ not G	'AUTRE_ASTER'	-3176.3 N	0.1%
σ_{yy} mesh $M5968$ not G	'AUTRE_ASTER'	-3176.3 N	0.1%
DX not G	'ANALYTICAL'	0 mm	1.0E-10
DX not H	'AUTRE_ASTER'	-0.11343 mm	0.1%
DZ not H_p	'AUTRE_ASTER'	-0.11343 mm	0.1%
DY not H	'AUTRE_ASTER'	-0.16291 mm	0.1%
DY not H_p	'AUTRE_ASTER'	-0.16291 mm	0.1%
DX not I	'AUTRE_ASTER'	-0.17845 mm	0.1%
DZ not I_p	'AUTRE_ASTER'	-0.17845 mm	0.1%
DY not I	'AUTRE_ASTER'	-0.62966 mm	0.1%
DY not I_p	'AUTRE_ASTER'	-0.62966 mm	0.1%

11.4 Remarks

The results got in this three-dimensional modeling are in very good agreement with the analytical solution as well as the reference (modeling A). One notes moreover than the radiant grid makes it possible to preserve the axisymetry of the problem, this is why the results got in each mesh which contributes to the point G are similar.
The results differ from those obtained by modeling I not because of the algorithm of resolution of the contact but because the geometrical convergence criteria are not the same one (5 % in modeling I, 1 % in modeling J).

12 Summary of the results

The comparison in each modeling to the analytical reference (limited to the point G) is satisfactory.

The not-regression of the results is ensured by testing displacements in two nodes of the edge, H and I . These nodes occupying of the slightly different positions according to modelings, one should not seek to compare them between modelings.

This test in curved geometry highlights, the utility of certain parameters of pairing such as:

– smoothing: it is about a process of modifications of the normals which are used to write the conditions of contact. Although the grids of sphere are symmetrical one notes always in practice a light shift on the last point of the surface of contact. The smoothing of the normals makes it possible here to find symmetry. This parameter proves also useful to improve geometrical convergence of modelings using of the grids coarse (it is the case of modelings $2\mathrm{D}$ this test).
– Nodal pairing or the average of normals (master-slave): little used in general, these two options of pairing, like smoothing, makes it possible to find the symmetry of the problem.

The modeling A where the contact is replaced by a unilateral connection puts also ahead the very important part played by the geometrical reactualization on the precision of the constraints. Indeed this modeling makes it possible to remove the geometrical non-linearity of this problem, it thus provides **a reference solution**. Thus when one compares the results got on the same problem $3\mathrm{D}$ by modelings A, I and J, one notes that:
– the values of displacements obtained by modeling I present variations going up to 4 % compared to two other modelings
– modelings A and J find a pressure of Hertz identical, while modeling I finds a pressure 10% lower
– in modeling I the geometrical criterion is satisfied to 5% close whereas it is it to 1% in modeling J

It thus appears important to satisfy the geometrical criterion as well as possible to obtain the best possible precision on the values of constraints.