SSNV121 - Rotation and traction very-rubber band of a bar

Summary:

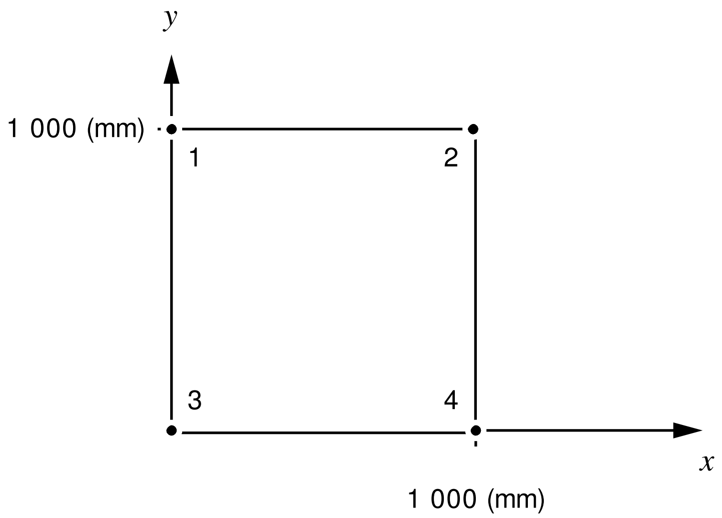
This test of quasi-static mechanics consists in making turn of $90°$ a parallelepipedic bar, to subject it to an important traction for finally letting it return in a discharged state. One thus validates the kinematics of the great deformations very-rubber bands (order STAT_NON_LINE, keyword BEHAVIOR), and thus in particular great rotations, for a relation of elastic behavior linear.

The bar is modelled by a voluminal element (HEXA8, modeling A) or plan (QUAD4, assumption of plane deformations, modeling B).

Results got by *Code_Aster* do not differ from the theoretical solution.

1 Problem of reference

1.1 Geometry



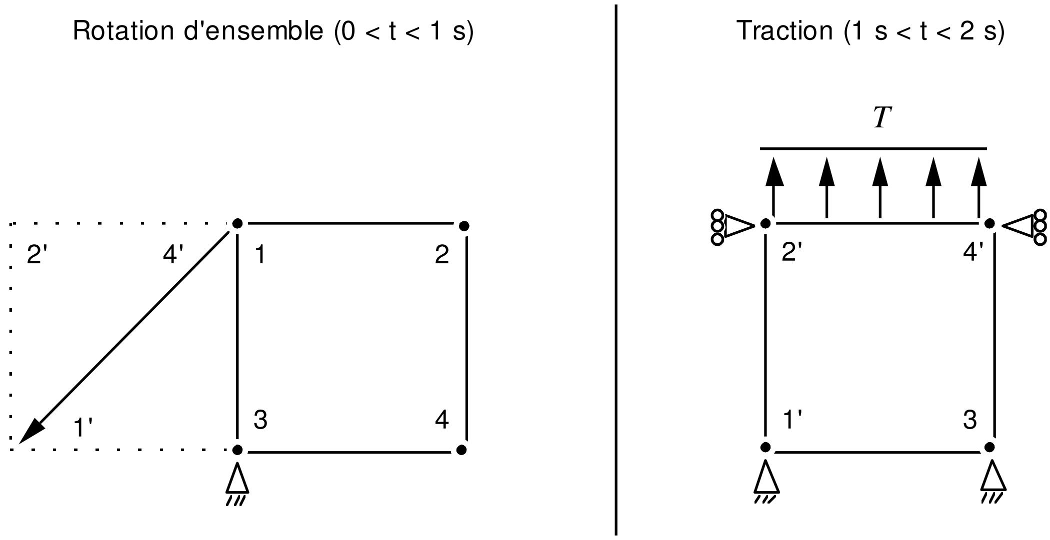
1.2 Material properties

Behavior very-rubber band of Coming St - Kirchhoff:

$$\mathbf{S} = \frac{\nu E}{(1+\nu)(1-2\nu)} \operatorname{tr}(\mathbf{E})\, \mathbf{1} + \frac{E}{1+\nu} \mathbf{E} \qquad E = 200\,000.\text{MPa} \quad \nu = 0.3$$

1.3 Boundary conditions and loadings

The loading is applied in two times: first of all, an overall rotation of the structure, followed by a traction in the new configuration:



2 Reference solution

2.1 Method of calculating used for the reference solution

It is about a problem plan. One can seek the solution in the form of a rigid rotation and a lengthening of a factor λ in the direction y.

$$\mathbf{U}(X,Y,Z) = \begin{pmatrix} -X-Y \\ (1+\lambda)X-Y \\ 0 \end{pmatrix}$$

The gradient of the transformation and the deformation of Green-Lagrange are then:

$$\mathbf{F} = \begin{bmatrix} 0 & -1 & 0 \\ 1+\lambda & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ avec } e = \frac{\lambda(\lambda+2)}{2}$$

The relation of behavior leads then to a tensor of Lagrangian constraints diagonal:

$$\begin{cases} S_{xx} = \dfrac{(1-\nu)\,E}{(1+\nu)(1-2\nu)}\,e \\ S_{yy} = S_{zz} = \dfrac{\nu\,E}{(1+\nu)(1-2\nu)}\,e \end{cases}$$

The boundary condition of the equilibrium equation then enables us to determine the value of lengthening λ :

$$T = (\mathbf{FS})_{yx} = (1+\lambda)S_{xx} \Rightarrow \frac{(1-\nu)\,E}{(1+\nu)(1-2\nu)}\frac{\lambda(\lambda+1)(\lambda+2)}{2} = T$$

The constraint of Cauchy is given by:

$$\boldsymbol{\sigma} = \frac{1}{\text{Det}\,\mathbf{F}}\,\mathbf{F}\,\mathbf{S}\,\mathbf{F}^T \Rightarrow \begin{cases} \sigma_{xx} = \sigma_{zz} = \dfrac{S_{yy}}{1+\lambda} \\ \sigma_{yy} = (1+\lambda)\,S_{xx} \end{cases}$$

Lastly, the force exerted on the faces:

- [2.4]: $\mathsf{F}_y = \sigma_{yy}\,S_{[2,4]} = \sigma_{yy}\,S_{o[2,4]}$
- [4.3]: $\mathsf{F}_x = \sigma_{xx}\,S_{[4,3]} = \sigma_{xx}(1+\lambda)\,S_{o[4,3]}$
- [1,2,3,4]: $\mathsf{F}_z = \sigma_{zz}\,S_{[1,2,3,4]} = \sigma_{zz}(1+\lambda)\,S_{o[1,2,3,4]}$

where $S_{o[\,]}$ initial surfaces of the faces represent.

2.2 Results of reference

One adopts like results of reference displacements, the constraint of Cauchy and the force exerted on the faces [2.4] and [4.3].

At time $t=2\ s$ **:**

One seeks T such as lengthening $\lambda = 0.1$

that is to say $T=31\ 096.154\ MPa$.

The constraint of Cauchy is then:

$$\begin{cases} \sigma_{xx} = \sigma_{zz} = 11\ 013.986 \text{ MPa} \\ \sigma_{yy} = 31\ 096.154 \text{ MPa} \end{cases}$$

The exerted forces are:

$$F_x = 12\ 115.385 \times S_{o[4,3]}\ N$$
$$F_y = 31\ 096.154 \times S_{o[2,4]}\ N$$
$$F_z = 12\ 115.385 \times S_{o[1,2,3,4]}\ N$$

At time $t=3\ s$ **:**

The bar returned in its initial state:

$$\begin{cases} \lambda = 0 \\ \sigma = 0 \\ F = 0 \end{cases}$$

2.3 Uncertainty on the solution

Analytical solution.

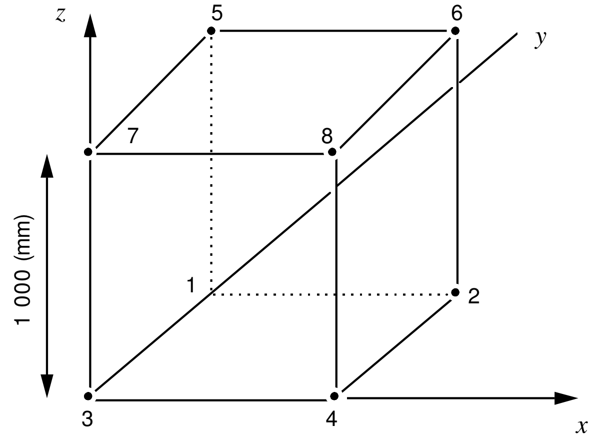
2.4 Bibliographical references

1) Eric LORENTZ "a nonlinear relation of behavior hyperelastic" Notes intern EDF/DER HI - 74/95/011/0

3 Modeling A

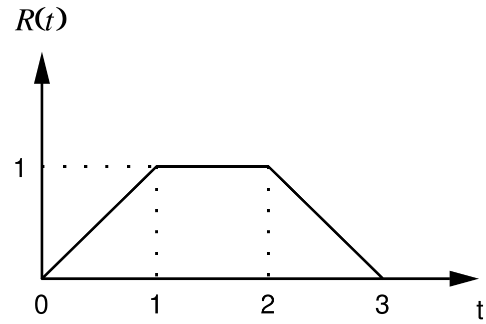
3.1 Characteristics of modeling

Voluminal modeling: 1 mesh `HEXA 8`
1 mesh `QUAD4`



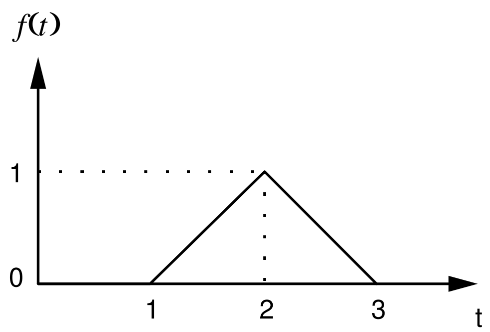
Boundary conditions:

(3,7) : DX = 0 DY = 0
(1,5) : DX = - 1 000 R(t) DY = - 1 000 R(t)
(2,6) : DX = - 2 000 R(t)
(4,8) : DX = - 1 000 R(t)



Loading: Traction on the face $[2,4,8,6]$

mesh $[2,4,8,6]$ (`QUAD4`): $FY=31\ 096.154\ f(t)\ MPa$



3.2 Characteristics of the grid

Many nodes: 8

Many meshs: 2
1 `HEXA8`
1 `QUAD4`

3.3 Sizes tested and results

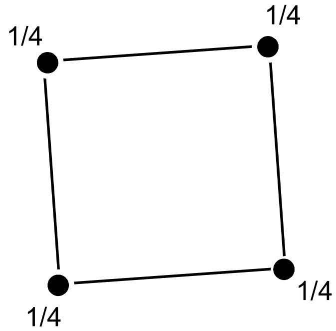
Identification	Reference	*Aster*	% difference
T = 2 Displacement DX (NO2)	100	100	0
T = 2 Displacement DY (NO4)	1100	1100	0
T = 2 Constraints SIGXX (PG1)	11013.986	11013.986	0
T = 2 Constraints SIGYY (PG1)	31096.154	31096.154	0
T = 2 Constraints SIGZZ (PG1)	11013.986	11013.986	0
T = 2 Constraints SIGXY (PG1)	0	□ 10^{-9}	/
T = 2 Constraints SIGXZ (PG1)	0	□ 10^{-10}	/
T = 2 Constraints SIGYZ (PG1)	0	□ 10^{-10}	/
T = 3 Displacement DX10 (NO2)	0	□ 10^{-11}	/
T = 3 Displacement DY (NO4)	0	□ 10^{-12}	/
T = 3 Constraints SIGXX (PG1)	0	□ 10^{-9}	/
T = 3 Constraints SIGYY (PG1)	0	□ 10^{-10}	/
T = 3 Constraints SIGZZ (PG1)	0	□ 10^{-9}	/
T = 3 Constraints SIGXY (PG1)	0	□ 10^{-11}	/
T = 3 Constraints SIGXZ (PG1)	0	□ 10^{-10}	/
T = 3 Constraints SIGYZ (PG1)	0	□ 10^{-11}	/
T = 2 Nodal force DX (NO8)	$3.0289\ 10^{9}$	$3.0288\ 10^{9}$	– 0,002%
T = 2 Nodal force DY (NO8)	$7{,}774\ 10^{9}$	$7{,}774\ 10^{9}$	0
T = 2 Nodal force DZ (NO8)	$3.0289\ 10^{9}$	$3.0288\ 10^{9}$	– 0,002%

3.4 Remarks

Calculation of the nodal force:

The force applied F on a face described by a linear mesh is distributed by:

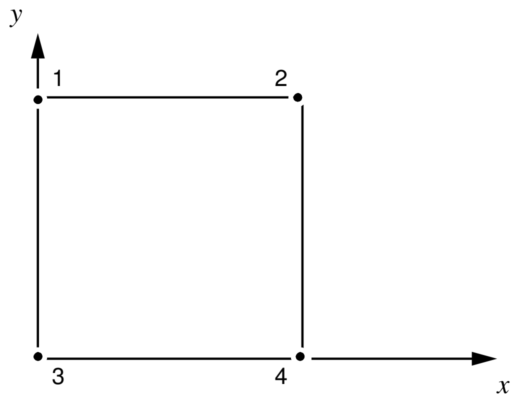
$$F_{noeud} = \frac{1}{4}\mathsf{F}$$



4 Modeling B

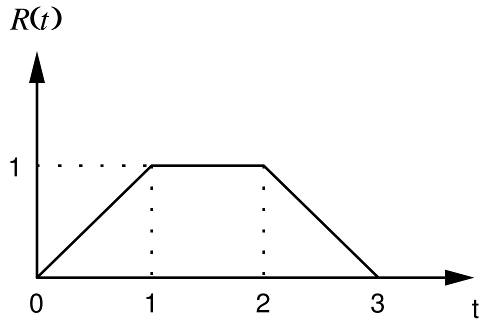
4.1 Characteristics of modeling

Modeling $2D$ plane deformations



Boundary conditions:

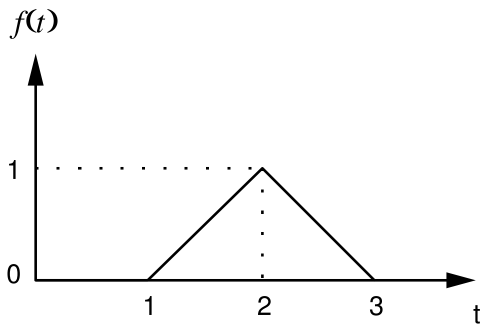
3 :	DX = 0	DY = 0
1 :	DX = - 1 000 R(t)	DY = - 1 000 R(t)
2 :	DX = - 2 000 R(t)	
4 :	DX = - 1 000 R(t)	



Loading:

Traction on the face $[2,4]$

mesh $[2,4]$: $FY=31\ 096.154\ f(t)\ MPa$



4.2 Characteristics of the grid

Many nodes: 4

Many meshs: 2
1 QUAD4
1 SEG2

4.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
T = 2 Displacement DX (NO2)	100	100	0

T = 2 Displacement DY (NO4)	1100	1100	0
T = 2 Constraints SIGXX (PG1)	11013.986	11013.986	0
T = 2 Constraints SIGYY (PG1)	31096.154	31096.154	0
T = 2 Constraints SIGZZ (PG1)	11013.986	11013.986	0
T = 2 Constraints SIGXY (PG1)	0	□ 10^{-10}	/
T = 3 Displacement DX (NO2)	0	□ 10^{-12}	/
T = 3 Displacement DY (NO4)	0	□ 10^{-12}	/
T = 3 Constraints SIGXX (PG1)	0	□ 10^{-10}	/
T = 3 Constraints SIGYY (PG1)	0	□ 10^{-10}	/
T = 3 Constraints SIGZZ (PG1)	0	□ 10^{-10}	/
T = 3 Constraints SIGXY (PG1)	0	□ 10^{-10}	/
T = 2 Nodal force DX (NO4)	$6.0577\ 10^6$	$6.0577\ 10^6$	0
T = 2 Nodal force DY (NO4)	$15.5481\ 10^6$	$15.5481\ 10^6$	0

4.4 Remarks

Calculation of the nodal force:

The force applied F on a face described by a linear mesh is distributed by:

$$F_{noeud} = \frac{1}{2} \mathrm{F}$$

5 Summary of the results

It appears at the conclusion of this test that the digital solution coincides remarkably with the analytical solution. One will notice however that the strong not linearity due to great rotations requires a relatively fine discretization in time, without being penalizing on the precision since, contrary to an incremental law of behavior, the errors do not cumulate a step of time on the other.