

SSNV126 - Test-tube in traction-relieving anisotherme with the model VENDOCHAB

Summary:

The model `VENDOCHAB`, a formulation suggested by Chaboche begins again. It is about a coupled formulation which cover a élasto-viscoplastic law with multiplicative isotropic work hardening and isotropic kinetics of damage. This law was initially developed to predict the lifetime and the cracking of the paddles of the turbojets and more generally to envisage the time of ruin of the requested structures at high temperatures.

This test of nonlinear quasi-static mechanics makes it possible to validate the model `VENDOCHAB` in 3D in the case of a test-tube subjected to a uniaxial tensile test anisotherme. The stress and strain states are homogeneous in the test-tube. This test also validates the explicit and implicit integration of this model. The equations of this coupled formulation are described in the booklet of reference [R5.03.15].

Two modelings of the test-tube are tested one with a single element 3D with 8 nodes (`HEXA8`), the other with a quadrangle with 8 nodes (`QUAD8`) in AXIS.

1 Problem of reference

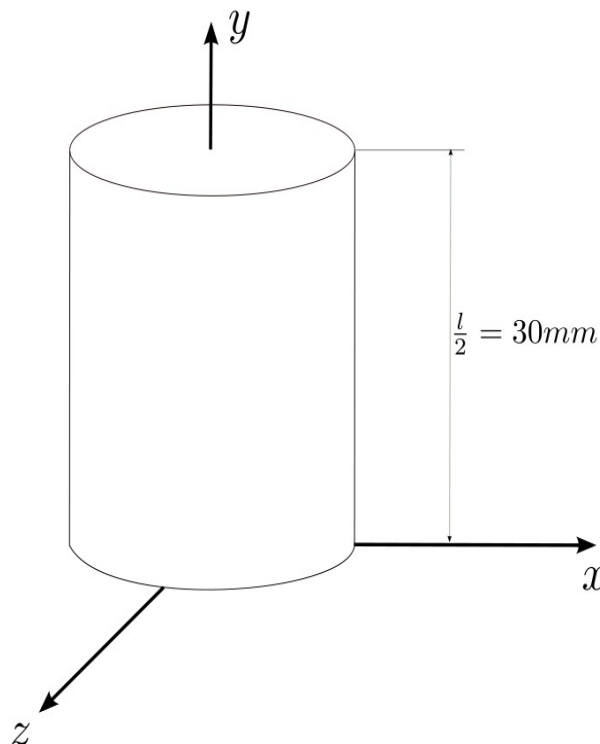
The mechanical problem milked of the loading with displacement imposed (case of relieving) of an axisymmetric test-tube under condition anisotherme.

1.1 Geometry

The geometry of the cylindrical test-tube is defined by:

- ray: $R = 3 \text{ mm}$;
- length: $L = 60 \text{ mm}$.

The plan (X, Z) being a symmetry plane, one models only half of the test-tube.
The test-tube is subjected to a uniform field of temperature which can vary in time.



1.2 Properties of material

The characteristics are the following ones:

Keyword ELAS :

- $YOUNG = 150000.0 \text{ MPa}$
- $NU = 0.30$

Keyword VENDOCHAB :

- $S_{VP} = 0$,
- $\alpha = 0 (SEDVP1)$,
- $\beta = 0 (SEDVP2)$,
- $N_{VP} = f(T)$,
- $M_{VP} = f(T)$
- $K_{VP} = f(T)$

- $A_D = f(T)$,

T (°C)	N_{vp}	M_{vp}	K_{vp}	R_D	A_D
900	12.2	10.5	2110	6.3	3191.62
1000	10.8	9.8	1450	5.2	2511.35
1025	10.45	9.625	1285	4.925	2341.30

If the coefficient K_D is only depend on the temperature (modeling C and d).

T (°C)	900	1000	1025
K_D	15	15	15

If the coefficient K_D is selected depend on the temperature and on σ_0 (modeling A and B).

T (°C)	0 MPa	100 MPa	200 MPa
900	14.355	14.855	15.355
1000	14.5	15	15.5
1025	14.5363	15.0363	15.5363
1050	14.5725	15.0725	15.5725

Table 1.2-1: K_D depend on the temperature and the constraint

1.3 Boundary conditions and loadings

1.3.1 Thermics

The field of temperature is homogeneous but nonstationary. Its variation is the following one: T is constant and is worth 1000°C t=0s with t=200000s (55,55h); then T increases linearly to reach 1025°C with t=2000000s (555.55 H); T is then constant with 1025°C.

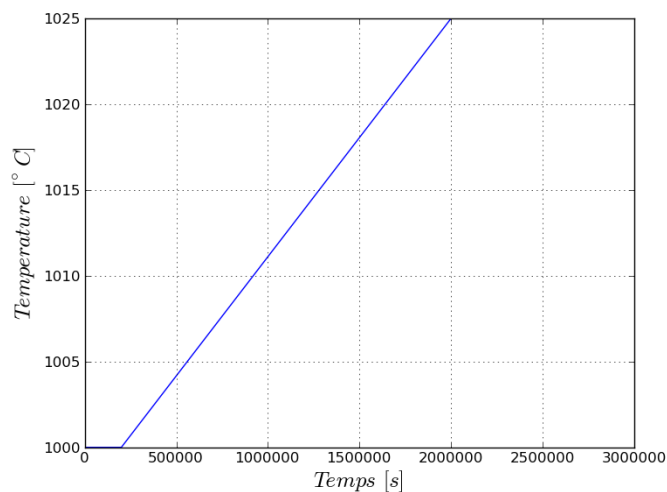


Image 1.3-1: Thermal loading

1.3.2 Mechanics

The symmetry plane (Z, X) imposes a blocking of UY in $y=0\text{mm}$. The mechanical loading is $UY=0,1\text{ mm}$ in $z=30\text{mm}$. Loading is not instantaneous because of problems of convergence. One chose to impose the constraint or displacement linearly in time so as to reach the final loading in 0.1 seconds.

1.4 Initial conditions

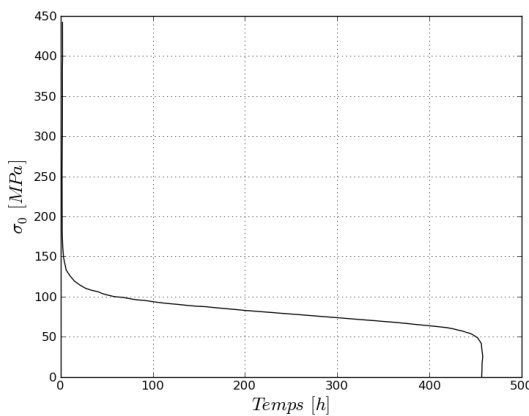
Worthless constraints and deformations.

2 Reference solution

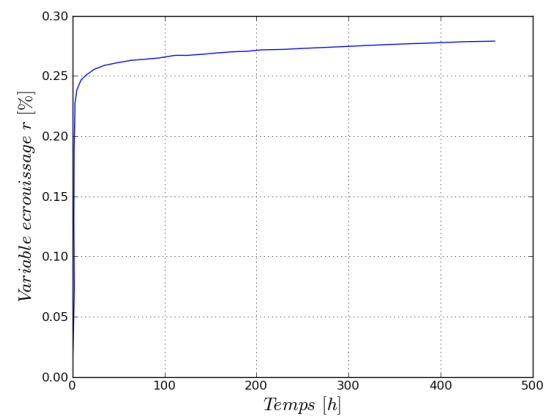
2.1 Method of calculating

The reference solution is obtained by integration of the equations of the model under Mathematica for an axisymmetric modeling. For that, it is enough to write the formal equations of the system, to observe the rule of transformation to them characterizing the law of Hooke and to solve the non-linear differential connection. The user wishing to obtain more information will be able to refer to note HT-2C/97/016/A

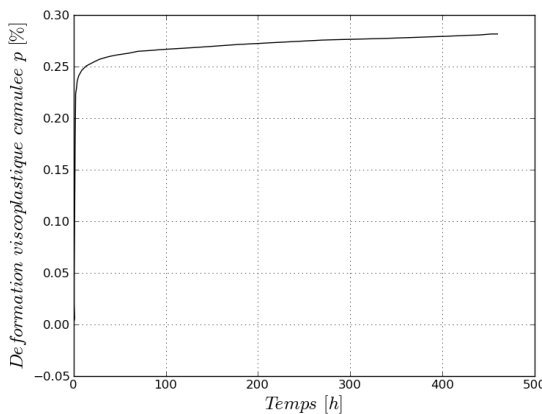
Here results of integration of the behavior, according to the case where the coefficient K_D is selected depend on the temperature and on σ_0 (modeling has and b):



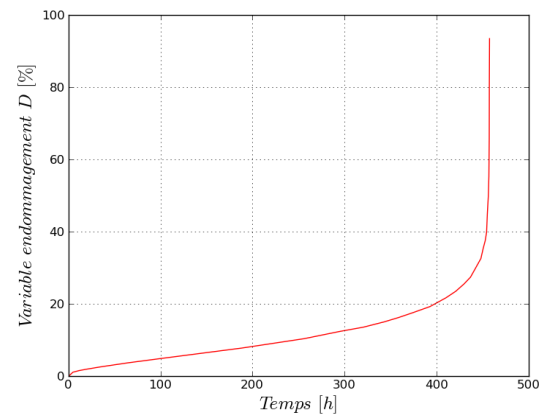
SIGM for K_D (T, SIG)



R for K_D (T, SIG)



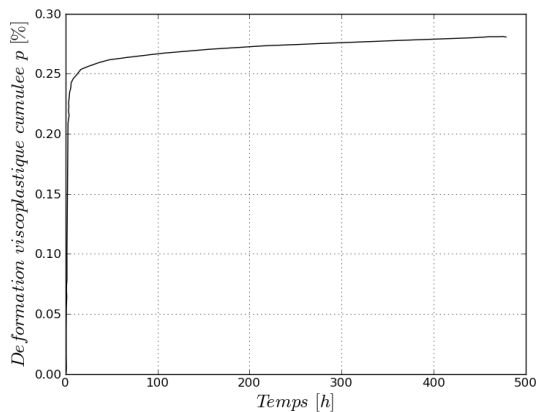
p for K_D (T, SIG)



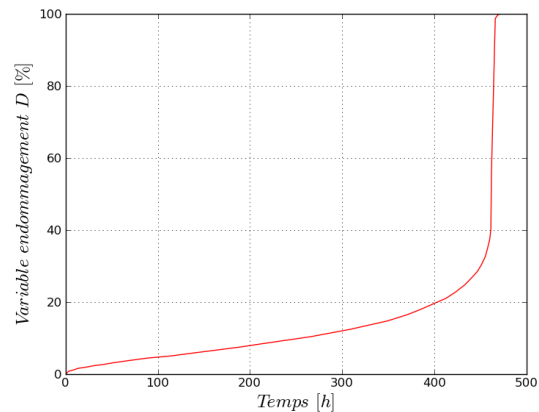
D for K_D (T, SIG)

Here results of integration of the behavior, according to the case where the coefficient K_D is selected depending only on the temperature (modeling C and d):

SIGM for K_D (T)



R for K_D (T)



p for K_D (T)

D for K_D (T)

In the graphs above, D is the variable of damage corresponding to the internal variable $V9$, r is the variable of multiplicative viscoplastic work hardening corresponding to the internal variable $V8$ and p is the cumulated viscoplastic deformation corresponding to the internal variable $V7$.

There is also the correspondence following, by reports with the parameters of the keyword VENDOCHAB :

$$\begin{aligned} N &= N_{VP} \\ M &= M_{VP} \\ K &= K_{VP} \\ A &= A_D \\ R &= R_D \\ k &= K_D \end{aligned}$$

2.2 Results of reference

2.2.1 The coefficient K_D is selected depend on the temperature and on σ_0 .

Evolution of the containte, σ_0 , according to time. One tests this value at various moments:

Moment	Reference
20	252.76091
2000	164,261
200000	101,596
1000000	75.97849999999997
1600000	55.54209999999998

Table 2.2.1-1: Results of reference for K_D (T, SIG)

Evolution of the variable of damage, D according to time. One tests this value at various moments according to modeling:

Moment	Reference
20	2.3168400000000001E-4
2000	2.77144E-3
200000	0.032255100000000002
1000000	0.110134
1600000	0.28131600000000001

Table 2.2.1-2: Results of reference for K_D (T, SIG)

Evolution of the variable of isotropic work hardening viscoplastic, r , according to time. One tests this value at various moments:

Moment	Reference
20	1.64451000000000001E-3
2000	2.23126000000000001E-3
200000	2.62515000000000001E-3
1000000	2.7477999999999999E-3
1600000	2.79276E-3

Table 2.2.1-3: Results of reference for K_D (T, SIG)

Evolution of the variable of isotropic work hardening viscoplastic, p , according to time. One tests this value at various moments:

Moment	Reference
20	1.6445699999999999E-3
2000	2.2318799999999999E-3
200000	2.6301200000000001E-3
1000000	2.7607899999999999E-3
1600000	2.8147799999999998E-3

Table 2.2.1-4: Results of reference for K_D (T, SIG)

2.2.2 The coefficient K_D is selected only depend on the temperature.

Evolution of the containte, σ_0 , according to time. One tests this value at various moments:

Moment	Reference
20	253.02000000000001
2000	164.36000000000001
200000	102.16
1000000	79.92000000000002
1600000	70.90000000000006

Table 2.2.2-1: Results of reference for K_D (T)

Evolution of the variable of damage, D according to time. One tests this value at various moments according to modeling:

Moment	Reference
20	2.32E-4
2000	2.7399E-3
200000	0.027560999999999999
1000000	0.066266500000000006
1600000	0.090278800000000006

Table 2.2.2-2: Results of reference for K_D (T)

Evolution of the variable of isotropic work hardening viscoplastic, r , according to time. One tests this value at various moments:

Moment	Reference
20	1.6459999999999999E-3
2000	2.2339E-3
200000	2.6282800000000002E-3
1000000	2.7522900000000001E-3
1600000	2.7992999999999998E-3

Table 2.2.2-3: Results of reference for K_D (T)

Evolution of the variable of isotropic work hardening viscoplastic, p , according to time. One tests this value at various moments:

Moment	Reference
20	1.6461E-3
2000	2.2344999999999999E-3
200000	2.6329000000000001E-3
1000000	2.762699E-3
1600000	2.8137000000000001E-3

Table 2.2.2-4: Results of reference for K_D (T)

2.3 Uncertainties on the solution

Precision of the codes

2.4 Bibliography

- [1] HT-2C/97/016/A, Deceived P., Description of the viscoplastic law of behavior coupled with the isotropic law of behavior of Chaboche introduced into Code_Aster, 1997.
- [2] Handbook of reference R5.03.15 Aster, Viscoplastic behavior with damage of Chaboche.

3 Modeling A

3.1 Characteristics of modeling

A modeling is used 3D.

The coefficient K_D is selected depend on the temperature and on σ_0 .

T (°C)	0 MPa	100 MPa	200 MPa
900	14.355	14.855	15.355
1000	14.5	15	15.5
1025	14.5363	15.0363	15.5363
1050	14.5725	15.0725	15.5725

Table 3.1-1: K_D of modeling A

The discretization in time is rather fine:

```
( JUSQU_A = 2,          NUMBER = 10 ),  
( JUSQU_A = 2. ,      NUMBER = 10 ),  
( JUSQU_A = 20. ,     NUMBER = 10 ),  
( JUSQU_A = 200. ,   NUMBER = 10 ),  
( JUSQU_A = 2000. ,  NUMBER = 10 ),  
( JUSQU_A = 20000. , NUMBER = 10 ),  
( JUSQU_A = 200000. , NUMBER = 10 ),  
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( JUSQU_A = 1600000. , NUMBER = 30 ),  
( JUSQU_A = 1700000. , NUMBER = 40 ),  
( JUSQU_A = 1800000. , NUMBER = 40 ),  
( JUSQU_A = 1900000. , NUMBER = 40 ),  
( JUSQU_A = 2000000. , NUMBER = 40 ),  
( JUSQU_A = 2100000. , NUMBER = 40 ),  
( JUSQU_A = 2200000. , NUMBER = 40 ),  
( JUSQU_A = 2300000. , NUMBER = 40 ),  
( JUSQU_A = 2400000. , NUMBER = 40 ),  
( JUSQU_A = 2500000. , NUMBER = 40 ),
```

3.2 Characteristics of the grid

One chooses to represent the cylindrical test-tube by a paving stone in order to be able to carry out a calculation on only one element. Modeling has 3 symmetry planes

Many nodes: 8

Many meshes: 1 (HEXA8)

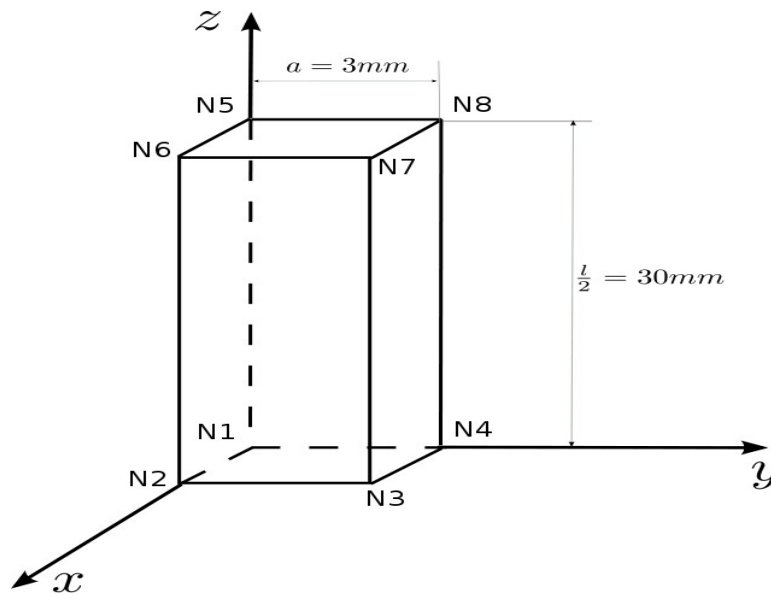


Image 3.2-1: Grid of modeling A

3.3 Sizes tested and results

Evolution of the constraint, σ_0 , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
20	'ANALYTICAL'	252.76091	0.5
2000	'ANALYTICAL'	164,261	0.5
200000	'ANALYTICAL'	101,596	0.5
1000000	'ANALYTICAL'	75.9784999999999997	0.5
1600000	'ANALYTICAL'	55.5420999999999998	10.0

Table 3.3-1: Results of modeling A

Evolution of the variable of damage, D according to time. One tests this value at various moments according to modeling:

Moment	Type of reference	Reference	Tolerance (%)
20	'ANALYTICAL'	2.3168400000000001E-4	0.5
2000	'ANALYTICAL'	2.77144E-3	0.5
200000	'ANALYTICAL'	0.032255100000000002	0.5
1000000	'ANALYTICAL'	0.110134	1.0
1600000	'ANALYTICAL'	0.28131600000000001	10.0

Table 3.3-2: Results of modeling A

Evolution of the variable of isotropic work hardening viscoplastic, r , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
20	'ANALYTICAL'	1.64451000000000001E-3	0.5
2000	'ANALYTICAL'	2.23126000000000001E-3	0.5
200000	'ANALYTICAL'	2.62515000000000001E-3	0.5
1000000	'ANALYTICAL'	2.74779999999999999E-3	0.5
1600000	'ANALYTICAL'	2.79276E-3	0.5

Table 3.3-3: Results of modeling A

Evolution of the variable of isotropic work hardening viscoplastic, p , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
20	'ANALYTICAL'	1.6445699999999999E-3	0.5
2000	'ANALYTICAL'	2.2318799999999999E-3	0.5
200000	'ANALYTICAL'	2.6301200000000001E-3	0.5
1000000	'ANALYTICAL'	2.7607899999999999E-3	0.5
1600000	'ANALYTICAL'	2.8147799999999998E-3	0.5

Table 3.3-4: Results of modeling A

4 Modeling B

4.1 Characteristics of modeling

A modeling is used `AXIS`.

In this case, the coefficient K_D is selected depend on the temperature and on σ_0 .

T (°C)	0 MPa	100 MPa	200 MPa
900	14.355	14.855	15.355
1000	14.5	15	15.5
1025	14.5363	15.0363	15.5363
1050	14.5725	15.0725	15.5725

Table 4.1-1: K_D of modeling B

The discretization in time is rather fine:

```
( JUSQU_A = 2,          NUMBER = 10 ),  
( JUSQU_A = 2. ,      NUMBER = 10 ),  
( JUSQU_A = 20. ,    NUMBER = 10 ),  
( JUSQU_A = 200. ,   NUMBER = 10 ),  
( JUSQU_A = 2000. ,  NUMBER = 10 ),  
( JUSQU_A = 20000. , NUMBER = 10 ),  
( JUSQU_A = 200000. , NUMBER = 10 ),  
( JUSQU_A = 1000000. , NUMBER = 30 ),  
( JUSQU_A = 1600000. , NUMBER = 30 ),  
( JUSQU_A = 1700000. , NUMBER = 40 ),  
( JUSQU_A = 1800000. , NUMBER = 40 ),  
( JUSQU_A = 1900000. , NUMBER = 40 ),  
( JUSQU_A = 2000000. , NUMBER = 40 ),  
( JUSQU_A = 2100000. , NUMBER = 40 ),  
( JUSQU_A = 2200000. , NUMBER = 40 ),  
( JUSQU_A = 2300000. , NUMBER = 40 ),  
( JUSQU_A = 2400000. , NUMBER = 40 ),  
( JUSQU_A = 2500000. , NUMBER = 40 ),
```

4.2 Characteristics of the grid

One chooses to represent the cylindrical test-tube by a quadrangle with 8 nodes in axisymetry.

Many nodes: 8

Many meshes: 1 (`QUAD8`)

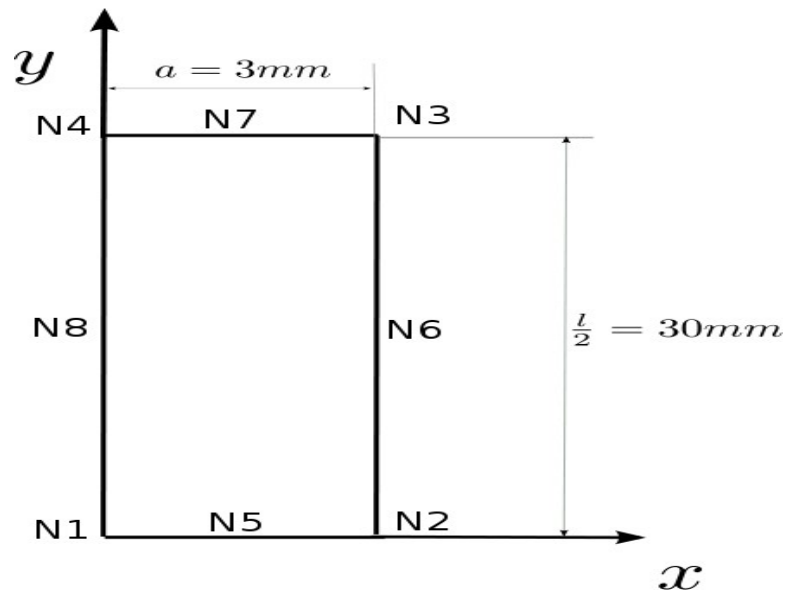


Image 4.2-1: Grid of modeling B

4.3 Sizes tested and results

Evolution of the constraint, σ_0 , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
1000000	'ANALYTICAL'	75.978499999999997	0.5
1600000	'ANALYTICAL'	55.542099999999998	10.0

Table 4.3-1: Results of modeling B

Evolution of the variable of damage, D according to time. One tests this value at various moments according to modeling:

Moment	Type of reference	Reference	Tolerance (%)
1000000	'ANALYTICAL'	0.110134	1.0
1600000	'ANALYTICAL'	0.28131600000000001	10.0

Table 4.3-2: Results of modeling B

Evolution of the variable of isotropic work hardening viscoplastic, r , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
1000000	'ANALYTICAL'	2.7477999999999999E-3	0.5
1600000	'ANALYTICAL'	2.79276E-3	0.5

Table 4.3-3: Results of modeling B

Evolution of the variable of isotropic work hardening viscoplastic, p , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
1000000	'ANALYTICAL'	2.7607899999999999E-3	0.5
1600000	'ANALYTICAL'	2.8147799999999998E-3	0.5

Table 4.3-4: Results of modeling B

5 Modeling C

5.1 Characteristics of modeling

A modeling is used 3D.

One carries out two calculations which differ only by the algorithm from integration: 'NEWTON' and 'RUNGE_KUTTA'.

In this case, the coefficient K_D is selected only depend on the temperature.

T (°C)	900	1000	1025
K_D	15	15	15

Table 5.1-1: K_D of modeling C

The discretization in time is rather fine:

```
( JUSQU_A = 2,          NUMBER = 10 ),  
( JUSQU_A = 2. ,      NUMBER = 10 ),  
( JUSQU_A = 20. ,    NUMBER = 10 ),  
( JUSQU_A = 200. ,   NUMBER = 10 ),  
( JUSQU_A = 2000. ,  NUMBER = 10 ),  
( JUSQU_A = 20000. , NUMBER = 10 ),  
( JUSQU_A = 200000. , NUMBER = 10 ),  
( JUSQU_A = 1000000. , NUMBER = 30 ),  
( JUSQU_A = 1600000. , NUMBER = 30 ),  
( JUSQU_A = 1700000. , NUMBER = 40 ),  
( JUSQU_A = 1800000. , NUMBER = 40 ),  
( JUSQU_A = 1900000. , NUMBER = 40 ),  
( JUSQU_A = 2000000. , NUMBER = 40 ),  
( JUSQU_A = 2100000. , NUMBER = 40 ),  
( JUSQU_A = 2200000. , NUMBER = 40 ),  
( JUSQU_A = 2300000. , NUMBER = 40 ),  
( JUSQU_A = 2400000. , NUMBER = 40 ),  
( JUSQU_A = 2500000. , NUMBER = 40 ),
```

5.2 Characteristics of the grid

One chooses to represent the cylindrical test-tube by a paving stone in order to be able to carry out a calculation on only one element. Modeling has 3 symmetry planes.

Many nodes: 8

Many meshes: 1 (HEXA8)

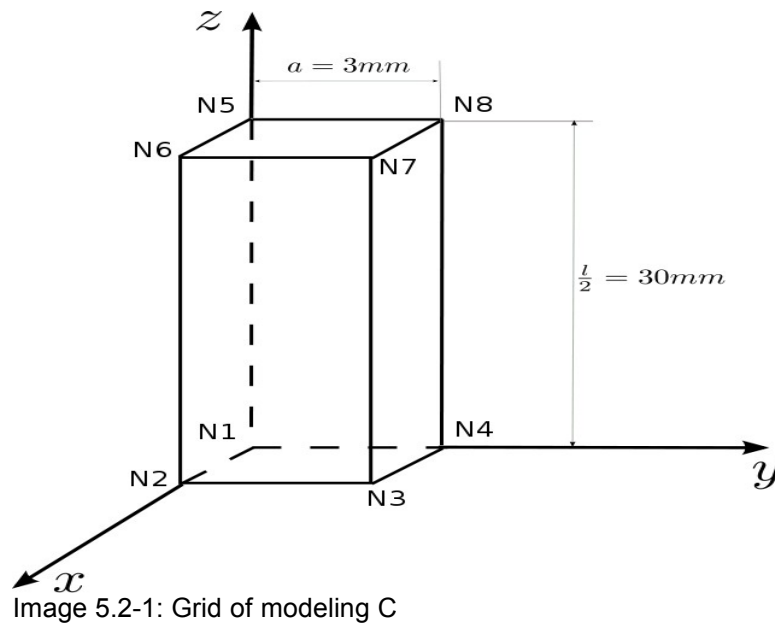


Image 5.2-1: Grid of modeling C

5.3 Sizes tested and results

5.3.1 Case ELASTIC RUNGE_KUTTA + Matrix

Evolution of the constraint, σ_0 , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
20	'ANALYTICAL'	253.02000000000001	0.5
2000	'ANALYTICAL'	164.36000000000001	0.5
200000	'ANALYTICAL'	102.16	0.5
1000000	'ANALYTICAL'	79.920000000000002	0.5
1600000	'ANALYTICAL'	70.900000000000006	1.0

Table 5.3.1-1: Results of modeling C

Evolution of the variable of damage, D according to time. One tests this value at various moments according to modeling:

Moment	Type of reference	Reference	Tolerance (%)
20	'ANALYTICAL'	2.32E-4	0.5
2000	'ANALYTICAL'	2.7399E-3	0.5
200000	'ANALYTICAL'	0.027560999999999999	0.5
1000000	'ANALYTICAL'	0.066266500000000006	1.0
1600000	'ANALYTICAL'	0.090278800000000006	1.0

Table 5.3.1-2: Results of modeling C

Evolution of the variable of isotropic work hardening viscoplastic, r , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
20	'ANALYTICAL'	1.6459999999999999E-3	0.5
2000	'ANALYTICAL'	2.2339E-3	0.5
200000	'ANALYTICAL'	2.6282800000000002E-3	0.5
1000000	'ANALYTICAL'	2.7522900000000001E-3	0.5
1600000	'ANALYTICAL'	2.7992999999999998E-3	0.5

Table 5.3.1-3: Results of modeling C

Evolution of the variable of isotropic work hardening viscoplastic, p , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
20	'ANALYTICAL'	1.6461E-3	0.5
2000	'ANALYTICAL'	2.2344999999999999E-3	0.5
200000	'ANALYTICAL'	2.6329000000000001E-3	0.5
1000000	'ANALYTICAL'	2.762699E-3	0.5
1600000	'ANALYTICAL'	2.8137000000000001E-3	0.5

Table 5.3.1-4: Results of modeling C

5.3.2 Cas NEWTON + TANGENT Matrix

The sizes tested are the same ones as in the preceding case. On the other hand the tolerance is of 4 % (for all the values tested).

6 Modeling D

6.1 Characteristics of modeling

A modeling is used `AXIS`.

One carries out two calculations which differ only by the algorithm from integration: `'NEWTON'` and `'RUNGE_KUTTA'`.

In this case, the coefficient K_D is selected only depend on the temperature.

T (°C)	900	1000	1025
K_D	15	15	15

Table 6.1-1: K_D of modeling D

The discretization in time is rather fine:

```
( JUSQU_A = 2,          NUMBER = 10 ),  
( JUSQU_A = 2. ,      NUMBER = 10 ),  
( JUSQU_A = 20. ,     NUMBER = 10 ),  
( JUSQU_A = 200. ,    NUMBER = 10 ),  
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( JUSQU_A = 200000. , NUMBER = 10 ),  
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( JUSQU_A = 1600000. , NUMBER = 30 ),  
( JUSQU_A = 1700000. , NUMBER = 40 ),  
( JUSQU_A = 1800000. , NUMBER = 40 ),  
( JUSQU_A = 1900000. , NUMBER = 40 ),  
( JUSQU_A = 2000000. , NUMBER = 40 ),  
( JUSQU_A = 2100000. , NUMBER = 40 ),  
( JUSQU_A = 2200000. , NUMBER = 40 ),  
( JUSQU_A = 2300000. , NUMBER = 40 ),  
( JUSQU_A = 2400000. , NUMBER = 40 ),  
( JUSQU_A = 2500000. , NUMBER = 40 ),
```

6.2 Characteristics of the grid

One chooses to represent the cylindrical test-tube by a quadrangle with 8 nodes in axisymetry.

Many nodes: 8

Many meshes: 1 (QUAD8)

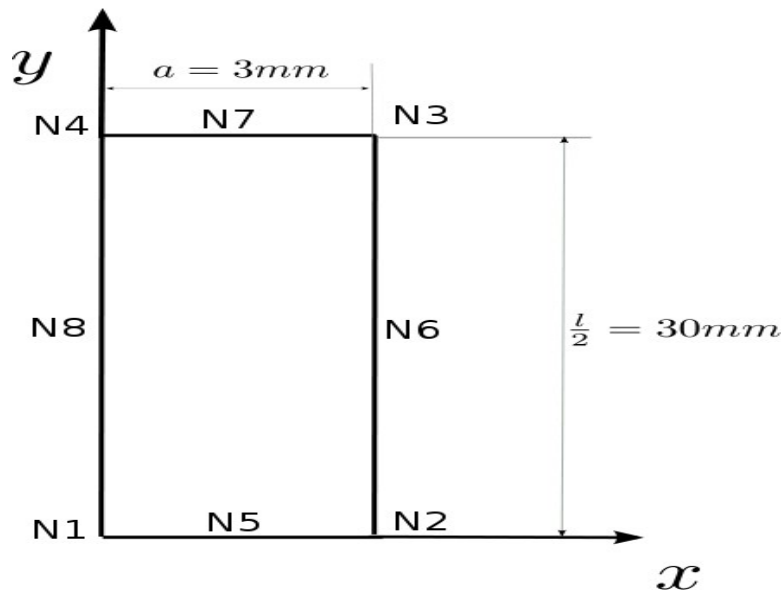


Image 6.2-1: Grid of modeling D

6.3 Sizes tested and results

6.3.1 Case ELASTIC RUNGE_KUTTA + Matrix

Evolution of the constraint, σ_0 , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
1000000	'ANALYTICAL'	79.9228999999999998	0.5
1600000	'ANALYTICAL'	70.9021999999999993	1.0

Table 6.3.1-1: Results of modeling D

Evolution of the variable of damage, D according to time. One tests this value at various moments according to modeling:

Moment	Type of reference	Reference	Tolerance (%)
1000000	'ANALYTICAL'	0.0662665800000000005	1.0
1600000	'ANALYTICAL'	0.0902788000000000006	1.0

Table 6.3.1-2: Results of modeling D

Evolution of the variable of isotropic work hardening viscoplastic, r , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
1000000	'ANALYTICAL'	2.747799999999999999E-3	0.5
1600000	'ANALYTICAL'	2.799359999999999999E-3	0.5

Table 6.3.1-3: Results of modeling D

Evolution of the variable of isotropic work hardening viscoplastic, p , according to time. One tests this value at various moments:

Moment	Type of reference	Reference	Tolerance (%)
1000000	'ANALYTICAL'	2.752290000000000001E-3	0.5
1600000	'ANALYTICAL'	2.813743799999999999E-3	0.5

Table 6.3.1-4: Results of modeling D

6.3.2 Cas NEWTON + TANGENT Matrix

The sizes tested are the same ones as in the preceding case. On the other hand the tolerance is of 4 % (for all the values tested).

7 Summary of the results

Results got with *Code_Aster* are close to the reference solution obtained on Mathematica.