SSNV138 - Plate Cantilever in great rotations subjected to one moment

Summary:

Quasi-static calculation of an elastic plate embedded on a side and subjected to one bending moment at the other side, leading to great rotations of the plate.

Interest:

To test the geometrical nonlinear finite elements `COQUE_3D` (modelings `With` and `C`), `DKT` (modelings `E` and `F`), `DKTG` (modelings `G` and `H`) and `POU_D_T_GD` (modeling `B`) using the algorithm of update of great rotations 3D `GROT_GDEP` in `STAT_NON_LINE`.

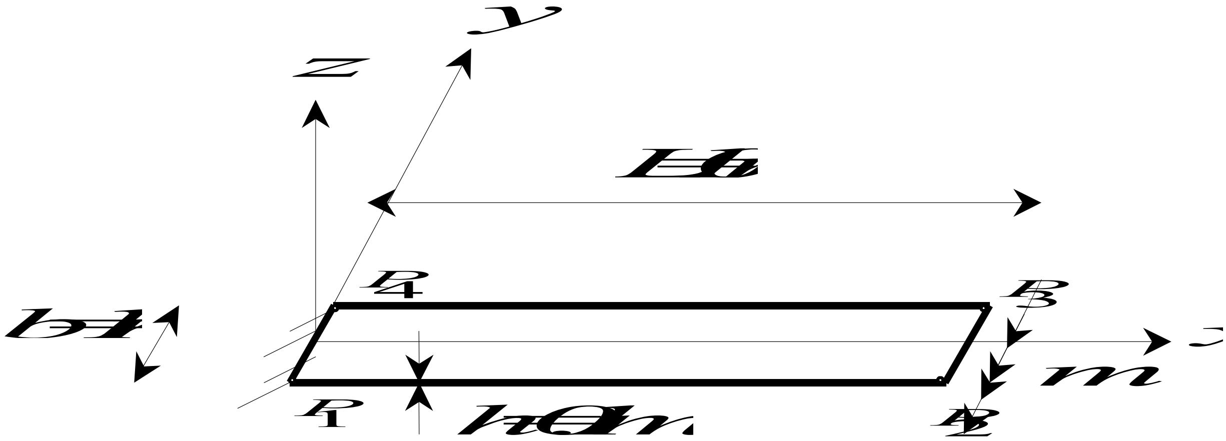
To also compare the multifibre results with the finite element of beam `POU_D_TGM` who allows to treat great rotations under the assumption of small increments of loading.

Note:

This test is the version plates case test of beam `SSNL103`. The mechanical characteristics were modified in order to support a surface modeling.

1 Problem of reference

1.1 Geometry



Rectangular plate embedded in P_1P_4 and P_1P_4 subjected in P_2P_3 with a linear couple:

$$m=-m e_y \;\; ; \;\; m>0$$

1.2 Properties of materials and characteristic of section

Elastic behavior:

$$E=12\times10^6\,Pa \;\; ; \;\; \nu=0$$

The fact that the Poisson's ratio (ν) either no one makes the solution of plate identical to that of beam.
I_y is the inertia of the section with a model of beam:

$$I_y=\frac{b\,h^3}{12}=\frac{1}{12}\times10^{-3}$$

1.3 Boundary conditions and loading

Embedding in P_1P_4 . One seeks the successive states of balance under the loading made up of the linear couple in P_2P_3 :

$$m(t)=100\,t \quad ; \quad t \text{ pseudo-time.}$$

One is interested particularly in displacements horizontal and vertical and rotation of the line P_2P_3 .

2 Reference solution

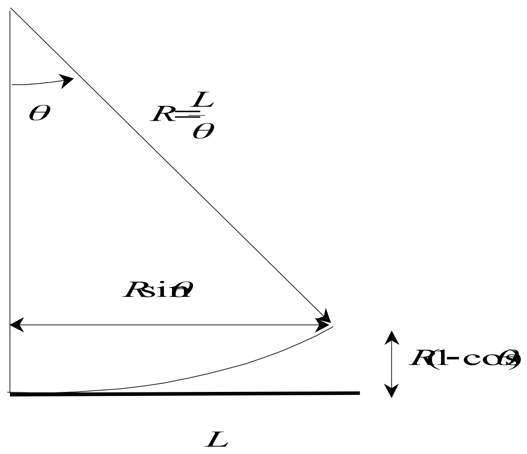
2.1 Method of calculating used for the reference solution

With a kinematics of beam and a model in resulting efforts, curve (in great rotations) of the cantilever subjected to the bending moment $M=mb$ is, with the preceding numerical data:

$$\frac{d\theta}{dx}=\frac{mb}{EI_y}=\frac{t}{L}$$

It is the solution of Euler.

2.2 Results of reference



According to the solution of Euler, the deformation is an arc of a circle. With the section P_2 P_3 $(x=L)$, rotation is worth:

$$\theta(x=L)=t\,.$$

In the absence of normal effort, average surface remains inextensible and the radius of curvature is given by:

$$R=\left(\frac{d\theta}{dx}\right)^{-1}=\frac{L}{t}$$

Horizontal displacement is then

$$u=R\sin\theta-L=L\left(\frac{\sin t}{t}-1\right)$$

and vertical displacement is

$$v=R(1-\cos\theta)=\frac{L}{t}(1-\cos t)$$

2.3 Bibliographical references

[1] MR. AL MIKDAD: Statics and Dynamics of the Beams in Great Rotations and Resolution of the Problems of Instability Non Linéaire. Doctorate, University of Technology of Compiegne (1998).

[2] J.C. SIMO and L. CONSIDERING QUOC: In Three-dimensional Finite Strain Rod Model. Share II: Computational Aspects. Comput. Meth. Appl. Mech. Engrg. 58.79 - 116 (1986).

[3] J.C. SIMO, D.D. FOX TERRIER and M.S. RIFAI: There are Resulting Stress Exact Geometrically Shell Model. Share III: Computational Aspects of the Nonlinear Theory. Comput. Meth. Appl. Mech. Engrg. 79.21 - 70 (1990).

3 Modeling A

3.1 Characteristics of modeling

Modeling COQUE_3D

3.2 Characteristics of the grid

Many nodes: 54
Number of meshs and type: 10 QUAD9 and 1 SEG3

3.3 Sizes tested and results

The incremental analysis is carried out in the interval of pseudo-time $[0:2.4]$ in fourteen steps of load.

3.3.1 History of horizontal rotation DRY with the nodes charged

Moment	Couples m	Type of Reference	Reference DRY (radians)	Tolerance (%)
0.6	60.	'ANALYTICAL'	– 0.6000E+00	1.
1.2	120	'ANALYTICAL'	– 1.2000E+00	0.1
1.8	180	'ANALYTICAL'	– 1.8000E+00	1.
2.4	240	'ANALYTICAL'	– 2.4000E+00	0.1

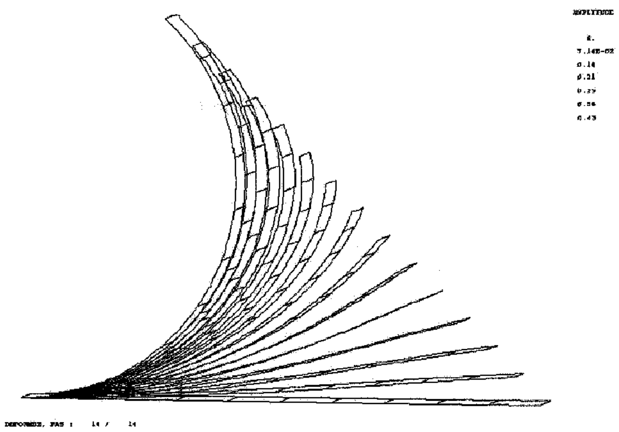
3.3.2 History of horizontal displacement DX with the nodes charged

Moment	Couples m	Type of Reference	Reference	Tolerance (%)
0.6	60.	'ANALYTICAL'	– 5.8929E-01	0.1
1.2	120	'ANALYTICAL'	– 2.23300E+00	0.1
1.8	180	'ANALYTICAL'	– 4.58973E+00	0.1
2.4	240	'ANALYTICAL'	– 7.18557E+00	0.1

3.3.3 History of vertical displacement DZ with the nodes charged

Moment	Couples m	Type of Reference	Reference	Tolerance (%)
0.6	60.	'ANALYTICAL'	2.91107E+00	0.1
1.2	120	'ANALYTICAL'	5.31368E+00	0.1
1.8	180	'ANALYTICAL'	6.81778E+00	0.1
2.4	240	'ANALYTICAL'	7.23914E+00	0.1

We present hereafter a visualization of the deformation during 14 pas de charges:



3.3.4 Remarks

One uses `COEF_RIGI_DRZ` = 0,001. The value of the angle swing reached is of 135 degrees.

4 Modeling B

4.1 Characteristics of modeling

POU_D_T_GD (beam $3D$ in great rotations).

modeling POU_D_T_GD.

4.2 Characteristics of the grid

Many nodes: 11
Number of meshs and type: 10 SEG2

4.3 Sizes tested and results

The incremental analysis is carried out in the interval of pseudo-time $[0:6]$ in 60 pas de charges.

4.3.1 History of horizontal rotation DRY (radians) with the nodes charged

Moment	Moment m	Type of Reference	Reference	Tolerance (%)
0.3	30.	'ANALYTICAL'	– 0.3000E+00	0.1
0.6	60.	'ANALYTICAL'	– 0.6000E+00	0.1
1.0	100.	'ANALYTICAL'	– 1.0000E+00	0.1
3.0	300.	'ANALYTICAL'	– 3.0000E+00	0.1
6	600	'ANALYTICAL'	-6.0000E+00	0.1

4.3.2 History of horizontal displacement $DX(m)$ with the nodes charged

Moment	Moment m	Type of Reference	Reference	Tolerance (%)
0.3	30.	'ANALYTICAL'	– 1.4932E-01	0.3
0.6	60.	'ANALYTICAL'	– 5.8934E+01	0.3
3.0	300.	'ANALYTICAL'	– 9.5296	0.3
6	600	'ANALYTICAL'	– 10.4657	0.3

4.3.3 History of vertical displacement $DZ(m)$ with the nodes charged

Moment	Moment m	Type of Reference	Reference	Tolerance (%)
0.3	30.	'ANALYTICAL'	1.4887E+00	0.1
0.6	60.	'ANALYTICAL'	2.9110E+00	0.1
3.0	300.	'ANALYTICAL'	6.6333	0.5
6	600	'ANALYTICAL'	6.638286E–02	2.0

5 Modeling C

5.1 Characteristics of modeling

Modeling COQUE_3D

5.2 Characteristics of the grid

Many nodes: 64
Number of meshs and type: 20 TRIA7 and 1 SEG3

5.3 Sizes tested and results

The incremental analysis is carried out in the interval of pseudo-time $[0:2.2]$ in eight step of load.

5.3.1 History of horizontal rotation DRY with the nodes charged

Moment	Couples m	Type of Reference	Reference	Tolerance (%)
0.6	60	'ANALYTICAL'	– 0.6000E+00	1.0
1.2	120	'ANALYTICAL'	– 1.2000E+00	1.0
1.8	180	'ANALYTICAL'	– 1.8000E+00	1.0
2.2	220	'ANALYTICAL'	– 2.1728E+00	2.0

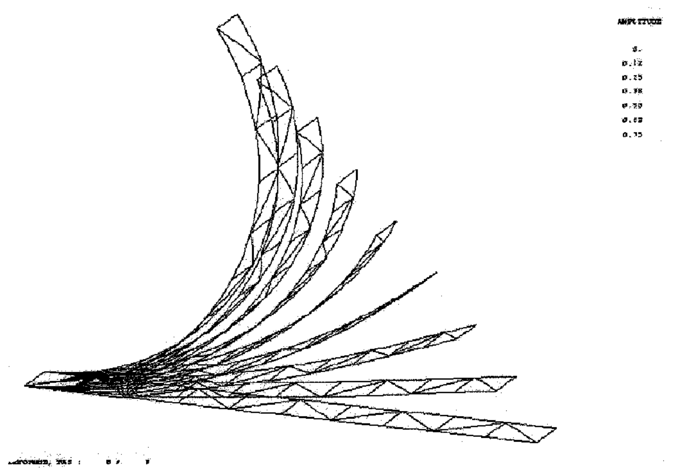
5.3.2 History of horizontal displacement DX with the nodes charged

Moment	Couples m	Type of Reference	Reference	Tolerance (%)
0.6	60	'ANALYTICAL'	– 5.8929E-01	0.5
1.2	120	'ANALYTICAL'	– 2.23300E+00	1.0
1.8	180	'ANALYTICAL'	– 4.58973E+00	2.0
2.2	220	'ANALYTICAL'	– 6.3250163463	2.0

5.3.3 History of vertical displacement DZ with the nodes charged

Moment	Couples m	Type of Reference	Reference	Tolerance (%)
0.6	60	'ANALYTICAL'	2.91107E+00	0.1
1.2	120	'ANALYTICAL'	5.31368E+00	0.5
1.8	180	'ANALYTICAL'	6.81778E+00	0.5
2.2	220	'ANALYTICAL'	7,22046E+00	1.0

We present hereafter a visualization of the deformation during 8 pas de charges:



5.3.4 Remarks

One uses COEF_RIGI_DRZ = 0,001. The value of the angle swing reached is of 125 degrees.

6 Modeling D

6.1 Characteristics of modeling

`POU_D_TGM` (beam 3D multifibre for the geometrical nonlinear analysis and material).

6.2 Characteristics of the grid

Many nodes: 11
Number of meshs and type: 10 `SEG2`

6.3 Characteristics of the grid of the transverse section

Many fibres: 160 (40 in the thickness and 4 in the width)
Number of meshs and type: 160 `QUAD4`

6.4 Sizes tested and results

The incremental analysis is carried out in the interval of pseudo-time $[0:6]$ in 1200 not of load.

6.4.1 History of horizontal rotation DRY (radians) with the nodes charged

Moment	Moment m	Type of Reference	Reference	Tolerance (%)
0.3	30.	'ANALYTICAL'	– 0.3000E+00	0.1
0.6	60.	'ANALYTICAL'	– 0.6000E+00	0.1
1.0	100.	'ANALYTICAL'	– 1.0000E+00	0.1
3.0	300.	'ANALYTICAL'	– 3.0000E+00	0.1
6	600	'ANALYTICAL'	– 6.0000E+00	0.1

6.4.2 History of horizontal displacement $\mathrm{DX}(\mathrm{m})$ with the nodes charged

Moment	Moment m	Type of Reference	Reference	Tolerance (%)
0.3	30.	'ANALYTICAL'	– 1.4932E-01	2.0
0.6	60.	'ANALYTICAL'	– 5.8934E-01	1.0
3.0	300.	'ANALYTICAL'	– 9.5296	0.3
6	600	'ANALYTICAL'	– 10.4657	0.3

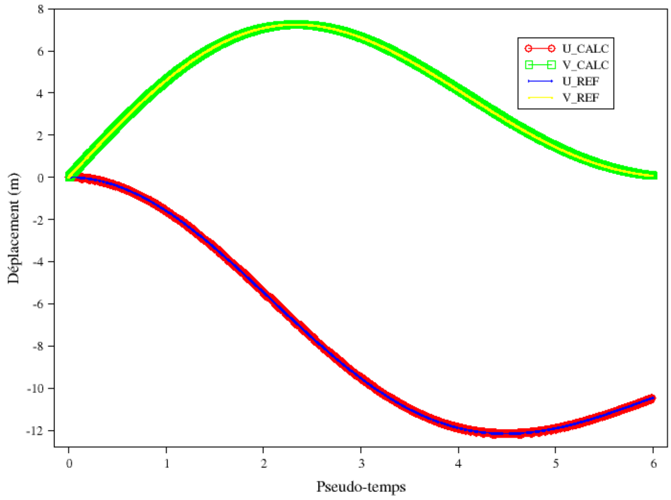
6.4.3 History of vertical displacement $\mathrm{DZ}(\mathrm{m})$ with the nodes charged

Moment	Moment m	Type of Reference	Reference	Tolerance (%)
0.3	30.	'ANALYTICAL'	1.4887E+00	0.1
0.6	60.	'ANALYTICAL'	2.9110E+00	0.1
3.0	300.	'ANALYTICAL'	6.6333	0.5
6	600	'ANALYTICAL'	6.638286E-02	0.07

With the last step of time, displacements vertical very weak compared to maximum is reached at the time of the way of loading (DZ maximum around $7\,\mathrm{m}$). A comparison with the reference solution into relative is not very relevant (there would be then almost 30% of relative error). One would prefer a test into relative compared to maximum displacement: $\frac{valeur\ calculée - valeur\ de\ référence}{DZ_{max}} < tol^{relative}$.

With this intention, one tests in absolute, with a tolerance in being worth absolute $DZ_{max} \cdot tol^{relative}$.

Déplacements U et V en fonction du pseudo-temps



7 Modeling E

7.1 Characteristics of modeling

This modeling is identical to modeling D. the only difference is at the level of the management of the step of time.

Management of the under-cutting of the step of time per vent-driven: So with convergence, the increment of displacement is such as $max(DX, DY) > 5.e-2$ on a node of the grid, then one Re-cutting the step of time.

7.2 Sizes tested and results

One tests the same values as those of modeling D, with the same tolerances.

8 Modeling F

8.1 Characteristics of modeling

Modeling DKT

8.2 Characteristics of the grid

Many nodes: 22
Number of meshs and type: 10 QUAD4 and 22 SEG2

8.3 Sizes tested and results

The incremental analysis is carried out in the interval of pseudo-time $[0:5.3]$ in $[37]$ pas de charges.

8.3.1 History of horizontal rotation DRY with the nodes charged

Moment	Couples m	Type of Reference	Value of reference (Radian)	Tolerance (%)
0.6	60.	'ANALYTICAL'	-0.6	0.01
1.2	120.	'ANALYTICAL'	-1.2	0.01
1.8	180.	'ANALYTICAL'	-1.8	0.01
3.0	300.	'ANALYTICAL'	-3.0	0.01
4.0	400.	'ANALYTICAL'	-4.0	0.01
5.3	530.	'ANALYTICAL'	-5.3	0.01

8.3.2 History of horizontal displacement DX with the nodes charged

Moment	Couples m	Type of Reference	Value of reference m	Tolerance (%)
0.6	60.	'ANALYTICAL'	-0.14932644	2.0
1.2	120.	'ANALYTICAL'	-2.23300762	2.0
1.8	180.	'ANALYTICAL'	-4.58973538	1.0
3.0	300.	'ANALYTICAL'	-9.52959997	0.5
4.0	400.	'ANALYTICAL'	-11.89200624	0.1
5.3	530.	'ANALYTICAL'	-11.57031593	0.5

8.3.3 History of vertical displacement DZ with the nodes charged

Moment	Couples m	Type of Reference	Value of reference m	Tolerance (%)
0.6	60.	'ANALYTICAL'	2.91107308	1,
1.2	120.	'ANALYTICAL'	5.31368538	0.8
1.8	180.	'ANALYTICAL'	6.81778941	0.5
3.0	300.	'ANALYTICAL'	6.63330832	0.2
4.0	400.	'ANALYTICAL'	4.13410905	1.0
5.3	530.	'ANALYTICAL'	0.84080314	1.5

8.3.4 Remarks

One uses COEF_RIGI_DRZ = 0,001. The value of the angle swing reached is of 304. degrees.

9 Modeling G

9.1 Characteristics of modeling

Modeling DKT

9.2 Characteristics of the grid

Many nodes: 22
Number of meshs and type: 20 TRIA3 and 22 SEG2

9.3 Sizes tested and results

The incremental analysis is carried out in the interval of pseudo-time $[0:5.0]$ in $[80]$ pas de charges.

9.3.1 History of horizontal rotation DRY with the nodes charged

Moment	Couples m	Type of Reference	Value of reference (radians)	Tolerance (%)
0.6	60.	'ANALYTICAL'	-0.6	0.01
1.2	120.	'ANALYTICAL'	-1.2	0.01
1.8	180.	'ANALYTICAL'	-1.8	0.01
3.0	300.	'ANALYTICAL'	-3.0	0.01
4.0	400.	'ANALYTICAL'	-4.0	0.01
5.0	500.	'ANALYTICAL'	-5.0	0.01

9.3.2 History of horizontal displacement DX with the nodes charged

Moment	Couples m	Type of Reference	Value of reference m	Tolerance (%)
0.6	60.	'ANALYTICAL'	-0.14932644	0.25
1.2	120.	'ANALYTICAL'	-2.23300762	0.25
1.8	180.	'ANALYTICAL'	-4.58973538	0.5
3.0	300.	'ANALYTICAL'	-9.52959997	0.1
4.0	400.	'ANALYTICAL'	-11.89200624	0.15
5.0	500.	'ANALYTICAL'	-11.91784855	0.1

9.3.3 History of vertical displacement DZ with the nodes charged

Moment	Couples m	Type of Reference	Value of reference m	Tolerance (%)
0.6	60.	'ANALYTICAL'	2.91107308	0.25
1.2	120.	'ANALYTICAL'	5.31368538	0.25
1.8	180.	'ANALYTICAL'	6.81778941	0.25
3.0	300.	'ANALYTICAL'	6.63330832	0.25
4.0	400.	'ANALYTICAL'	4.13410905	0.5
5.0	500.	'ANALYTICAL'	1.43267563	0.8

9.3.4 Remarks

One uses COEF_RIGI_DRZ = 0,001. The value of the angle swing reached is of 286. degrees.

10 Modeling H

10.1 Characteristics of modeling

Modeling DKTG

10.2 Characteristics of the grid

Many nodes: 22
Number of meshs and type: 10 QUAD4 and 22 SEG2

10.3 Sizes tested and results

The incremental analysis is carried out in the interval of pseudo-time $[0:1.4]$ in $[6]$ pas de charges.

10.3.1 History of horizontal rotation DRY with the nodes charged

Moment	Couples m	Type of Reference	Value of reference (radians)	Tolerance (%)
0.6	60.	'ANALYTICAL'	-0.6	0.01
1.2	120.	'ANALYTICAL'	-1.2	0.01
1.4	140.	'ANALYTICAL'	-1.5	0.01

10.3.2 History of horizontal displacement DX with the nodes charged

Moment	Couples m	Type of Reference	Value of reference m	Tolerance (%)
0.6	60.	'ANALYTICAL'	-0.14932644	2.0
1.2	120.	'ANALYTICAL'	-2.23300762	1.5
1.4	140.	'ANALYTICAL'	-2.96107336	1.0

10.3.2.1 History of vertical displacement DZ with the nodes charged

Moment	Couples m	Type of Reference	Value of reference m	Tolerance (%)
0.6	60.	'ANALYTICAL'	2.91107308	1.0
1.2	120.	'ANALYTICAL'	5.31368538	1.0
1.4	140.	'ANALYTICAL'	5.92880612	1.0

10.3.3 Remarks

One uses COEF_RIGI_DRZ = 0,001. The value of the angle swing reached is of 80. degrees.

11 Modeling I

11.1 Characteristics of modeling

Modeling DKTG

11.2 Characteristics of the grid

Many nodes: 222
Number of meshs and type: 20 TRIA3 and 22 SEG2

11.3 Sizes tested and results

The incremental analysis is carried out in the interval of pseudo-time $[0:1.5]$ in $[10]$ pas de charges.

11.3.1.1 History of horizontal rotation DRY with the nodes charged

Moment	Couples m	Type of Reference	Value of reference (radians)	Tolerance (%)
0.6	60.	'ANALYTICAL'	-0.6	0.01
1.2	120.	'ANALYTICAL'	-1.2	0.01
1.4	140.	'ANALYTICAL'	-1.5	0.01

11.3.1.2 History of horizontal displacement DX with the nodes charged

Moment	Couples m	Type of Reference	Value of reference m	Tolerance (%)
0.6	60.	'ANALYTICAL'	-0.14932644	0.5
1.2	120.	'ANALYTICAL'	-2.23300762	0.25
1.4	140.	'ANALYTICAL'	-2.96107336	0.25

11.3.1.3 History of vertical displacement DZ with the nodes charged

Moment	Couples m	Type of Reference	Value of reference m	Tolerance (%)
0.6	60.	'ANALYTICAL'	2.91107308	0.5
1.2	120.	'ANALYTICAL'	5.31368538	0.5
1.4	140.	'ANALYTICAL'	5.92880612	0.5

11.3.2 Remarks

One uses COEF_RIGI_DRZ = 0,001. The value of the angle swing reached is of 86. degrees.

12 Summary of the results

- **Modelings COQUE_3D, POU_D_T_GD and POU_D_TGM**

  One notices difficulties of convergence which disappear by multiplying the thickness by 3 or 4.

  It is necessary to increase the value of COEF_RIGI_DRZ who allots a rigidity around the normal of the elements of hull which is worth by default 10^{-5} (the smallest rigidity of inflection around the directions in the plan of the hull) in order to be able to increase the value of the swing angle which one can reach. Values of this coefficient until 10^{-3} remain licit.

  During the iterations of Newton, deformations of membrane appear and are cancelled with convergence.

  Speeds of convergence of the algorithms of NEWTON are comparable for modelings POU_D_T_GD and COQUE_3D.

  The speed of convergence of the algorithm of NEWTON in the case of modeling POU_D_TGM is much weaker than the two others because this modeling requires in this case to make very small increments of loading for describing the geometrical transformation well and remaining on the assumption of the small deformations. The cost in time CPU feels some since calculation is meadows of 10 times longer than that of modeling POU_D_T_GD. Of course the element multifibre has the advantage of being able to treat several types of behavior and not only one elastic behavior like the element POU_D_T_GD. If the precision necessary is not about 1%, one can allow oneself to use less step of time.
  In addition it is important for a problem like this one, where the inertia of the section plays a paramount role, to take care to discretize the section with sufficient fibres when one uses POU_D_TGM, to obtain inertia nearest possible to the theoretical value (this is why one has with a grid with nearly 40 fibres in the thickness).

- **Modelings DKT and DKTG**
  Modeling DKT allows to reach a great swing angle $300°$, more important than that obtained with modeling DKTG $86°$.
  Compared to the analytical solution, the maximum change in displacement is of 2% for modelings DKT and DKTG