
SSNV146 - Regularized limiting analysis. Spherotoric bottom reserve

Summary

This test makes it possible to qualify the operators used analyzes regularized limit of it.

One calculates the limiting load by a kinematic approach regularized by the method of Norton-Hoff-Friaâ. Compared to index A of this document, it should be noted that the method of calculating was modified in Code_Aster which does not use now any more the Norton-Hoff material but called on the more general resolution with incompressible elements.

The problem of reference is resulting from a European benchmark carried out within the framework of a project Brite EuRam BE97-4547 "LISA", in 1998.

One considers an axisymmetric spherotoric bottom reserve. The constitutive material checks the criterion of Von Mises and the structure is subjected to an internal pressure.

The structure is modelled by incompressible elements.

The resolution by the regularized method of Norton-Hoff-Friaâ is carried out in the order `STAT_NON_LINE`. A postprocessing in the order `POST_ELEM` allows to obtain the estimates of the high delimiters and lower of the limiting load.

The reference solution is digital and the results are in perfect agreement with the values of reference. Certain details of implementation of this CAS-test are presented in the document of assistance to the use [U2.05.04].

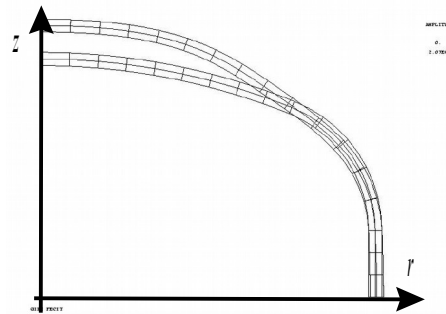
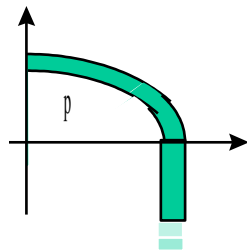
1 Problem of reference

The test is CAS-test LA6 of a benchmark of the European project Brite EuRam BE97-4547 "LISA" studied by three organizations:

- EDF (Norton-Hoff method with three values of the parameter of regularization: the Norton-Hoff coefficient n);
- LTAS in Liege (regularized kinematic method of the dedicated university software "ELSA");
- ForschungZentrum de Jülich (method static approached by finite elements in displacements, reduced representation of the fields of auto-contraintes and algorithm of optimization of the code **Permas**).

One considers an axisymmetric spherotoric bottom reserve subjected to an internal pressure.

The same grid is used by the three participating organizations. It contains two Q8 elements in the thickness, and in all there are 34 elements, and 141 nodes. The constitutive material is homogeneous and checks the criterion of Von Mises with for threshold the limit elastic (σ_y) of 100 MPa .



Results EDF LISA: initial grid and deformation obtained for the Norton-Hoff coefficient $N=31$

The calculation of the limiting load by the Norton-Hoff-Friaâ method is detailed in the reference document [R7.07.01].

Briefly let us recall that one aims at seeking the limiting load λ_{lim} , for which the structure, made up of a material of the perfect elastoplastic type with threshold of von Mises, can support the surface loadings $\lambda_{lim} F$ and loadings of volume $\lambda_{lim} f$ which one can possibly add constant loadings (F_0 and f_0). Code_Aster makes it possible to calculate, for each moment of calculation, i.e. for increasingly weak regularizations, two parameters:

- an estimate of an upper limit of the limiting load::

$$\hat{\lambda}_m = \int_{\Omega} \sigma_y \sqrt{\frac{2}{3} \varepsilon(u_m) \cdot \varepsilon(u_m)} d\Omega - L_0(u_m)$$

- and, in the absence of constant loading, an estimate of a lower limit $\underline{\lambda}_m$:

$$\underline{\lambda}_m = \int_{\Omega} \frac{A(m)}{m} \cdot \left(\sqrt{\varepsilon(u_m) \cdot \varepsilon(u_m)} \right)^m d\Omega \cdot \left(\sup_{x \in \Omega} \left(\sqrt{\frac{\frac{3}{2} \sigma^D(u_m) \cdot \sigma^D(u_m)}{\sigma_y}} \right) \right)^{-1} \leq \hat{\lambda}_m$$

if a constant loading is present, there is the power of the constant loading in the field speed solution of the problem.

It is noted that the tensor deviatoric of the constraints checks the relation of behavior of Norton-Hoff:

$$\sigma^D(u) = A(m) \cdot \left(\sqrt{\varepsilon^D(u) \cdot \varepsilon^D(u)} \right)^{m-2} \cdot \varepsilon^D(u) \Leftrightarrow \sigma^D(u) = A(m)^n \cdot \left(\sqrt{\sigma^D(u) \cdot \sigma^D(u)} \right)^{1-n} \cdot \varepsilon^D(u)$$

with $\text{tr } \varepsilon(u) = 0$, and: $A(m) = k^{1-m} \left(\frac{2}{3}\right)^{m/2} \sigma_y^m$ and $n = \frac{1}{m-1}$.

1.1 Data for modeling

Geometry	The axisymmetric spherotoric bottom reserve has the following characteristics: <ul style="list-style-type: none"> • ray interns cylindrical part: 49 mm ; • thickness: 2 mm ; • ray of the spherical part to the apex: 98 mm ; • ray of the torus of connection: 20 mm .
• Material properties	Elastic limit: $\sigma_y = 100 \text{ MPa}$
Boundary conditions	Axial displacement DY no one on the end $BORD_{INF}$ cylindrical part (conditions of symmetry)
Loadings	Pressure interns 1 MPa applied to the internal wall B_D

1.2 Results of reference

For this case test, one does not have of analytical results but only the digital values resulting from the calculations carried out within the framework of the benchmark LISA and pointed out Table 1.2-1.

The methods regularized kinematics of EDF and the University of Liege give very nearby results. The lower limit provided by FZJ is more important than the preceding hight delimiters, which is an anomaly.

The results selected as values of reference are thus those provided by Ulg and EDF with the old method of calculating of the limiting load established in Code_Aster (using the Norton-Hoff material) which we will identify below like "EDF LISA".

Modeling		Higher value estimated	Lower value estimated
EDF ⁽¹⁾	$m = 1,0476$ $n = 21$	3.9514	3.6049
	$m = 1,0322$ $n = 31$	3.9456	3.7090
	$m = 1,0141$ $n = 71$	3.9404	3.8372
	$m = 1,0099$ ⁽²⁾ $n = 101$ ⁽²⁾	3.9396	3.8673
Univ. of Liège/LTAS		3.931	nothing
Research centre FZJ		nothing	3,997

Table 1.2-1 : Results of the benchmark LISA

Foot-note ⁽¹⁾ : the results provided by EDF were got with an old version of Code_Aster who used, for calculation limiting load, the Norton-Hoff material $n = (m-1)^{-1}$

Foot-note ⁽²⁾ : value obtained later on with the benchmark

1.3 Bibliographical references

- [1] Voldoire F.: Calculation of load limits with *Code_Aster* and benchmark of Brite EuRam "LISA". Note HI-74/98/026/A.
- [2] Heitzer M . "Traglast- und Einspielanalyse zur Bewertung der Sicherheit to passive Komponenten." Thesis., RWTH Aachen (1999).
- [3] Direct Yan A.M. "Contributions to the limit state analysis of plastified and cracked structures". Thesis, Univ. Liege, (1999).

2 Modeling A

For the limiting analysis, Code_Aster uses:

- quasi-incompressible finite elements;
- a kinematic approach regularized (method of regularization of Norton-Hoff-Friaâ, cf [R7.07.01]) for the criterion of resistance of Von Mises (adjustment by a coefficient of regularization of which the limiting value led to convergence);
- a nonlinear static resolution by parametric piloting;
- postprocessing to obtain a higher estimate of the values λ_{lim}^{sup} and lower λ_{lim}^{inf} who frame the limiting value λ_{lim} .

List of moments has is used to control the method of regularization as Norton-Hoff via a coefficient t , ($m = 1 + 10^{1-t}$), and not evolution of the loading as during an ordinary calculation.

2.1 Characteristics of modeling

One considers a cylinder modelled by incompressible axisymmetric elements of type QUAD8.

2.2 Characteristics of the grid

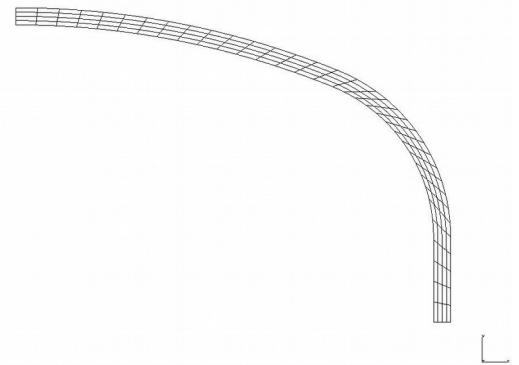
Many nodes: 485

Many meshes: 212

Type of meshes:

136 meshes of the type QUAD8 incompressible;

76 meshes of the type SEG3 for the application of the pressure.



2.3 Sizes tested and results

With the grid considered, the calculation of the CAS-test is stopped with $t = 2,85126 s$. Additional details on calculation are indicated in the document of assistance to the use [U2.05.04].

The Norton-Hoff coefficient corresponding to this moment is $m = 1,0141$ that is to say $n = 71$.

Identification	Reference	Aster	% difference
	EDF LISA N = 71		
Load limits higher	3.9404	3.9351	-0.133
Estimated limiting load	3.8372	3.8245	-0.331
	EDF LISA N = 101		
Load limits higher	3.9396	3.9351	-0.1142
Estimated limiting load	3.8673	3.8245	-1.0861

	Univ, of Liège/LTAS		
Load limits higher	3.931	3.9351	0.106

Table 2.3-1 : Results of the comparison

3 Summary of the results

Digital results of *Code_Aster* are in concord with the values of the digital references.