# SSNV163 - Clean calculation of creep with the models BETON_UMLV_FP and BETON_BURGER_FP

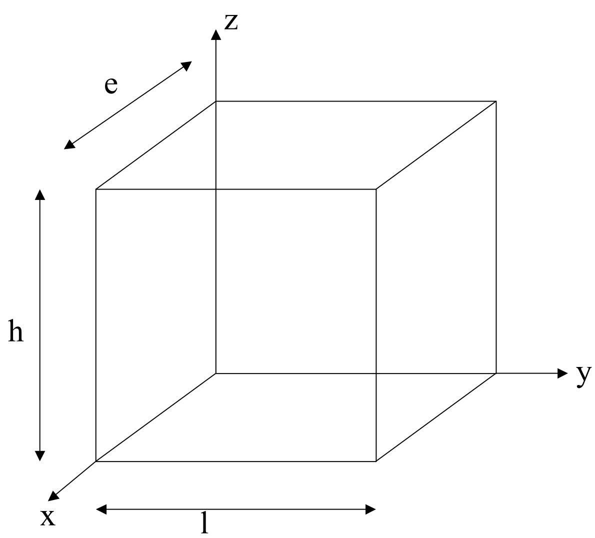
**Summary:**

This test makes it possible to validate the models of creep clean BETON_UMLV_FP and BETON_BURGER_FP. The results of this test are compared with the solutions analytical (BETON_UMLV_FP) or obtained according to a diagram of integration clarifies (BETON_BURGER_FP) for three types of modelings: 3D, hasxisymetric and plane constraints.

Modeling a: clean Creep test with the model BETON_UMLV_FP and a modeling 3D.
Modeling b: clean Creep test with the model BETON_UMLV_FP and a modeling AXIS.
Modeling C: Clean creep test with the model BETON_UMLV_FP and a modeling C_PLAN.
Modeling D: Clean creep test with the model BETON_BURGER_FP and a modeling 3D.
Modeling E: Clean creep test with the model BETON_BURGER_FP and a modeling AXIS.
Modeling F: Clean creep test with the model BETON_BURGER_FP and a modeling C_PLAN.

# 1 Problem of reference

## 1.1 Geometry




height: $h=1,00[m]$
width: $l=1,00[m]$
thickness: $e=1,00[m]$

## 1.2 Properties of material

$E=31GPa$
$\nu=0.2$

Here one informs also the curved sorption-desorption which connects the water content $C$ with the hygroscopy $h$.
In this case one supposed that the digital values of $C$ and of $h$ are the same ones.

Parameters specific to the clean creep of BETON_UMLV_FP :

| | |
|---|---|
| $k_r^s=2,0E+5$ [ $MPa$ ] | spherical part: rigidity connects associated with the skeleton formed by blocks with hydrates on a mesoscopic scale |
| $k_i^s=5,0E+4$ [ $MPa$ ] | spherical part: rigidity connects intrinsically associated with the hydrates on a microscopic scale |
| $k_r^d=5,0E+4$ [ $MPa$ ] | deviatoric part: rigidity associated with the capacity with water adsorbed to transmit loads (*load bearing toilets*) |
| $\eta_r^s=4,0E+10$ [ $MPa.s$ ] | spherical part: viscosity connects associated with the mechanism with diffusion within capillary porosity |
| $\eta_i^s=1,0E+11$ [ $MPa.s$ ] | spherical part: viscosity connects associated with the mechanism with diffusion interlamellaire |
| $\eta_r^d=1,0E+10$ [ $MPa.s$ ] | deviatoric part: viscosity associated with the water adsorbed by the layers with hydrates |

$\eta_i^d = 1,0E+11$ [ $MPa.s$ ] deviatoric part: viscosity of free water.

Parameters specific to the clean creep of BETON_BURGER_FP :

| | |
|---|---|
| $k_r^s = 2{,}0E+5$ [ $MPa$ ] | spherical part: rigidity connects associated with the reversible field with the differed deformations |
| $k_r^d = 5{,}0E+4$ [ $MPa$ ] | deviatoric part: rigidity associated associated with the reversible field with the differed deformations |
| $\eta_r^s = 4{,}0E+10$ [ $MPa.s$ ] | spherical part: viscosity connects associated with the reversible field with the differed deformations |
| $\eta_i^s = 1{,}0E+11$ [ $MPa.s$ ] | spherical part: viscosity connects associated with the irreversible mechanism of diffusion |
| $\eta_r^d = 1{,}0E+10$ [ $MPa.s$ ] | deviatoric part: viscosity associated with the reversible field with the differed deformations |
| $\eta_i^d = 1{,}0E+11$ [ $MPa.s$ ] | deviatoric part: viscosity connects associated with the irreversible mechanism of diffusion |
| $\kappa = 3.0 \times 10^{-3}$ | Normalizes unrecoverable deformations controlling to it not linearity applied to the module of the long-term deformations |

## 1.3 Boundary conditions and loadings

In this test, one creates a homogeneous field of drying invariant in the structure, moisture is worth $100\%$ (condition of a sealed test-tube). The mechanical loading corresponds to an one-way compression according to the vertical direction ( $z$ in $3D$ or $y$ in $2D$ ); its intensity is of $1[MPa]$. The load is applied in $1s$ and is maintained constant for 100 days.

## 1.4 Initial conditions

The beginning of calculation is supposed the moment $-1$ . At this moment there is neither field of drying, nor forced mechanical.

To moment 0, one applies a field of drying corresponding to $100\ \%$ of hygroscopy.

# 2 Reference solution

## 2.1 Solutions obtained for the model BETON_UMLV_FP

### 2.1.1 Method of calculating

This section presents the analytical resolution supplements problem of a body of test subjected to a homogeneous and one-way stress field applied instantaneously to the initial moment and maintained constant thereafter (case of a creep test in simple compression):

$$\underline{\underline{\sigma}} = \sigma_0 \underline{e}_z \otimes \underline{e}_z \quad \text{éq 2.1-1}$$

Whose partly spherical and deviatoric decomposition is written:

$$\underline{\underline{\sigma}} = \underbrace{\frac{1}{3}\sigma_0 \underline{\underline{1}}}_{\text{partie sphérique}} + \underbrace{\frac{2}{3}\sigma_0 \underline{e}_z \otimes \underline{e}_z - \frac{1}{3}\sigma_0 \left(\underline{e}_x \otimes \underline{e}_x + \underline{e}_y \otimes \underline{e}_y\right)}_{\text{partie déviatorique}} \quad \text{éq 2.1-2}$$

By operating a spherical/deviatoric decomposition identical to that of the constraints, the axial deformation is written in the form:

$$\varepsilon_{zz} = \varepsilon^{fs}(\sigma_0/3) + \varepsilon^{fd}(2\sigma_0/3) \quad \text{éq 2.1-3}$$

It is thus necessary successively to solve the response to a level of spherical constraint and a level of deviatoric constraints.

### 2.1.2 Resolution of the equations constitutive of spherical creep [bib2]

The process of deformation spherical of creep is controlled by the system of equations coupled according to (equations [éq 2.2-1] and [éq 2.2-2], cf [R7.01.06]):

$$\dot{\varepsilon}^{fs} = \frac{1}{\eta_r^s} \cdot \left[h \cdot \sigma^s - k_r^s \cdot \varepsilon_r^{fs}\right] - \dot{\varepsilon}_i^{fs} \quad \text{éq 2.2-1}$$

where $k_r^s$ indicate rigidity connect associated with the skeleton formed by blocks with hydrates on a mesoscopic scale;

and $\eta_r^s$ viscosity connects associated with the mechanism with diffusion within capillary porosity.

$$\dot{\varepsilon}_i^{fs} = \frac{1}{\eta_i^s} \left\langle \left[k_r^s \cdot \varepsilon^{fs} - \left(k_r^s + k_i^s\right) \cdot \varepsilon_i^{fs}\right] - \left[h\sigma^s - k_r^s \cdot \varepsilon_r^{fs}\right] \right\rangle^+ \quad \text{éq 2.2-1}$$

where $k_i^s$ indicate rigidity connect intrinsically associated with the hydrates on a microscopic scale;

and $\eta_i^s$ viscosity connects associated with the interfoliaceous mechanism of diffusion.

In [éq 2.2-2], hooks $\langle\ \rangle^+$ appoint the operator of Mac Cauley: $\langle x \rangle^+ = \frac{1}{2}\left(x + |x|\right)$

The resolution of the preceding system of coupled equations requires to distinguish two cases according to the sign from the quantity ranging between the hooks from Mac Cauley. In the continuation, one presents the analytical resolution of the answer to a level of constraint $\sigma^s$. The relative humidity is supposed to be invariant; the medium is saturated with water.

#### 2.1.2.1 Case of short-term creep

At the initial moment, $t=0$, a spherical constraint is applied $\sigma^s$ positive. The reversible and irreversible deformations of creep are equal to zero (initial conditions). The equation of the system [éq 2.2-2] is thus written:

$$\dot{\varepsilon}_i^{fs}(t=0)=\frac{1}{\eta_i^s}\left\langle\left[2\cdot k_r^s\cdot 0-k_i^s\cdot 0-\sigma^s\right]\right\rangle^+=\frac{1}{\eta_i^s}\left\langle\left[-\sigma^s\right]\right\rangle^+=0 \quad \text{éq 2.2.1-1}$$

The speed of irreversible deformation of creep is thus equal to zero. One from of deduced that the irreversible deformation of creep is also equal to zero. The speed of deformation unrecoverable remains equal to zero until the moment $t=t_0$, defined by the relation [éq 2.2.1-2]:

$$2\cdot k_r^s\cdot\varepsilon_r^{fs}(t_0)-\sigma^s=0\Rightarrow\varepsilon_r^{fs}(t_0)=\frac{\sigma^s}{2\cdot k_r^s} \quad \text{éq 2.2.1-2}$$

Until the moment $t=t_0$, the reversible deformation of creep is defined by the following relation:

$$\dot{\varepsilon}_r^s=\frac{1}{\eta_r^s}\cdot\left[\sigma^s-k_r^s\cdot\varepsilon_r^s\right]\Rightarrow\varepsilon_r^s(t)=\frac{\sigma^s}{k_r^s}\cdot\left(1-\exp\left(-\frac{t}{\tau_r^s}\right)\right) \quad \text{éq 2.2.1-3}$$

$\tau_r^s=\frac{\eta_r^s}{k_r^s}$ is the characteristic time associated with the reversible deformation of creep. The moment $t_0$ is thus defined by the relation [éq 2.2.1-4]:

$$\varepsilon_r^{fs}(t_0)=\frac{\sigma^s}{2\cdot k_r^s}=\frac{\sigma^s}{k_r^s}\cdot\left(1-\exp\left(-\frac{t_0}{\tau_r^s}\right)\right)\Rightarrow t_0=\ln(2)\cdot\tau_r^s\approx 0.69\cdot\tau_r^s \quad \text{éq 2.2.1-4}$$

The reversible and irreversible deformations of creep are thus determined by:

$$\begin{cases}\varepsilon_r^{fs}(t)=\frac{\sigma^s}{k_r^s}\cdot\left(1-\exp\left(-\frac{t}{\tau_r^s}\right)\right)\\ \varepsilon_i^{fs}(t)=0\end{cases} \quad \text{éq 2.2.1-5}$$

During the calculation of the deformations of creep for $t>t_0$, the new initial conditions are thus:

$$\begin{cases}\varepsilon_r^{fs}(t_0)=\frac{\sigma^s}{2\cdot k_r^s}\\ \varepsilon_i^{fs}(t_0)=0\end{cases} \quad \text{éq 2.2.1-6}$$

#### 2.1.2.2 Case of long-term creep

By expressing speeds of reversible and irreversible deformations of creep according to the deformations of creep, one obtains the relation then:

$$\begin{cases} \dot{\varepsilon}_r^{fs} = \left(-\frac{k_r^s}{\eta_r^s} - 4\cdot\frac{k_r^s}{\eta_i^s}\right)\cdot\varepsilon_r^{fs} + \left(2\cdot\frac{k_i^s}{\eta_i^s}\right)\cdot\varepsilon_i^{fs} + \left(\frac{1}{\eta_r^s} + \frac{2}{\eta_i^s}\right)\cdot\sigma^s \\ \dot{\varepsilon}_i^{fs} = \left(2\cdot\frac{k_r^s}{\eta_i^s}\right)\cdot\varepsilon_r^{fs} + \left(-\frac{k_i}{\eta_i^s}\right)\cdot\varepsilon_i^{fs} + \left(-\frac{1}{\eta_i^s}\right)\cdot\sigma^s \end{cases} \quad \text{éq 2.2.2-1}$$

In order to simplify calculations, the following intermediate variables are defined:

$$u_{rr} := \frac{k_r}{\eta_r} = \frac{1}{\tau_r},\ u_{ii} := \frac{k_i}{\eta_i} = \frac{1}{\tau_i} \ \text{ et } \ u_{ri} := \frac{k_r}{\eta_i} \quad \text{éq 2.2.2-2}$$

The system of equations [éq 2.2.2-1] can be put then in the following matric form:

$$\underline{\dot{\varepsilon}}^{fs} = \begin{pmatrix} \dot{\varepsilon}_r^{fs} \\ \dot{\varepsilon}_i^{fs} \end{pmatrix} = \underbrace{\begin{pmatrix} -u_{rr} - 4\cdot u_{ri} & 2\cdot u_{ii} \\ 2\cdot u_{ri} & -u_{ii} \end{pmatrix}}_{\underline{\underline{A}}} \cdot \underbrace{\begin{pmatrix} \varepsilon_r^{fs} \\ \varepsilon_i^{fs} \end{pmatrix}}_{\underline{\varepsilon}^s} + \sigma^s \cdot \frac{1}{k_r^s} \cdot \underbrace{\begin{pmatrix} u_{rr} + 2\cdot u_{ri} \\ -u_{ri} \end{pmatrix}}_{\underline{B}} \quad \text{éq 2.2.2-3}$$

I.e.:

$$\underline{\dot{\varepsilon}}^{fs} = \underline{\underline{A}}\cdot\underline{\varepsilon}^{fs} + \sigma^s \cdot \underline{B} \quad \text{éq 2.2.2-4}$$

Let us suppose that the matrix $\underline{\underline{A}}$ that is to say *diagonalisable* (this property will be checked thereafter): $\underline{\underline{A}} = \underline{\underline{P}}\cdot\underline{\underline{D}}\cdot\underline{\underline{P}}^{-1}$ where $\underline{\underline{D}}$ indicate the diagonal matrix of the eigenvalues of the matrix $\underline{\underline{A}}$, $\underline{\underline{P}}$ the matrix of the clean vectors of the matrix $\underline{\underline{A}}$ and $\underline{\underline{P}}^{-1}$ the matrix reverses matrix $\underline{\underline{P}}$. By carrying out term in the long term the product by the quantity $\underline{\underline{P}}^{-1}$, [éq 2.2.2-4] can put itself in the form:

$$\underline{\dot{\varepsilon}}^{fs,*} = \underline{\underline{D}}\cdot\underline{\varepsilon}^{fs,*} + \sigma^s \cdot \underline{B}^* \ \text{ avec } \ \underline{\varepsilon}^{fs,*} = \underline{\underline{P}}^{-1}\cdot\underline{\varepsilon}^{fs} \ \text{ et } \ \underline{B}^* = \underline{\underline{P}}^{-1}\cdot\underline{B} \quad \text{éq 2.2.2-5}$$

That is to say $\lambda_1$ and $\lambda_2$ eigenvalues of the matrix $\underline{\underline{A}}$. The quantities are defined:

$$\underline{\varepsilon}^{fs,*} := \begin{pmatrix} \varepsilon_1^* \\ \varepsilon_2^* \end{pmatrix} \ \text{ et } \ \underline{B}^* := \begin{pmatrix} b_1^* \\ b_2^* \end{pmatrix}$$

[éq 2.2.2-5] is written then:

$$\begin{cases} \dot{\varepsilon}_1^*(t) = \lambda_1 \cdot \varepsilon_1^*(t) + \sigma^s \cdot b_1^* \\ \dot{\varepsilon}_2^*(t) = \lambda_2 \cdot \varepsilon_2^*(t) + \sigma^s \cdot b_2^* \end{cases} \quad \text{éq 2.2.2-6}$$

System whose solution is written:

$$\begin{cases} \varepsilon_1^*(t) = -\dfrac{\sigma^s \cdot b_1^*}{\lambda_1} + \mu_1 \cdot \exp(\lambda_1 \cdot t) \\ \varepsilon_2^*(t) = -\dfrac{\sigma^s \cdot b_2^*}{\lambda_2} + \mu_2 \cdot \exp(\lambda_2 \cdot t) \end{cases} \begin{pmatrix} \lambda_1 \neq 0 \\ \lambda_2 \neq 0 \end{pmatrix} \quad \text{éq 2.2.2-7}$$

One can then return to initial space, by the means of the matrix of passage; the reversible and irreversible deformations of creep are linear combinations of $\varepsilon_1^*$ and $\varepsilon_2^*$. Eigenvalues of the matrix $\underline{\underline{A}}$, $\lambda_1$ and $\lambda_2$ are obtained while solving:

$$\det(\underline{\underline{A}} - \lambda_i \cdot \underline{\underline{1}}) = 0$$
$$\Rightarrow \begin{vmatrix} -u_{rr} - 4 \cdot u_{ri} - \lambda_i & 2 \cdot u_{ii} \\ 2 \cdot u_{ri} & -u_{ii} - \lambda_i \end{vmatrix} = 0 \Rightarrow \lambda_i^2 + (u_{rr} + 4 \cdot u_{ri} + u_{ii}) \cdot \lambda_i + u_{rr} \cdot u_{ii} = 0 \quad \text{éq 2.2.2-8}$$

By noticing that $u_{rr}$, $u_{ri}$ and $u_{ii}$ are strictly positive, the discriminant is thus always strictly positive. The eigenvalues are thus real and distinct, the matrix $\underline{\underline{A}}$ is thus *diagonalisable*. In addition, none of the two eigenvalues is equal to zero $(\lambda_1 \cdot \lambda_2 = u_{rr} \cdot u_{ii} \neq 0)$. The two eigenvalues are defined by:

$$\begin{cases} \lambda_1 = \dfrac{-(u_{rr} + 4 \cdot u_{ri} + u_{ii}) - \sqrt{\Delta}}{2} \\ \lambda_2 = \dfrac{-(u_{rr} + 4 \cdot u_{ri} + u_{ii}) + \sqrt{\Delta}}{2} \end{cases} \quad \text{éq 2.2.2-9}$$

One can show that the two eigenvalues are indeed negative. Let us show that the second eigenvalue is negative. The spherical deformation of creep is thus asymptotic, hypothesis put forth in the model of clean creep spherical [bib1]. Let us determine a base of the clean vectors now $(\underline{X}_1, \underline{X}_2)$ associated with the eigenvalues $\lambda_1$ and $\lambda_2$. It is determined by solving the equation $(\underline{\underline{A}} - \lambda_i \cdot \underline{\underline{1}}) \cdot \underline{X}_i = \underline{0}$.
A particular base of clean vectors is written:

$$\underline{X}_1 = \begin{pmatrix} x_1 \\ 1 \end{pmatrix} \text{ et } \underline{X}_2 = \begin{pmatrix} 1 \\ x_2 \end{pmatrix} \text{ avec } x_1 = \frac{\lambda_1 + u_{ii}}{2 \cdot u_{ri}} \text{ et } x_2 = \frac{2 \cdot u_{ri}}{\lambda_2 + u_{ii}} \quad \text{éq 2.2.2-10}$$

After having checked that $\underline{\underline{P}}$ can be reversed, one from of deduced indeed the solution in physical space:

$$\begin{cases} \varepsilon_r^{fs}(t) = \left[ -\sigma^s \cdot \left( x_1 \cdot \frac{b_1^*}{\lambda_1} + \frac{b_2^*}{\lambda_2} \right) + x_1 \cdot \mu_1 \cdot \exp(\lambda_1 \cdot t) + \mu_2 \cdot \exp(\lambda_2 \cdot t) \right] \\ \varepsilon_i^{fs}(t) = \left[ -\sigma^s \cdot \left( \frac{b_1^*}{\lambda_1} + x_2 \cdot \frac{b_2^*}{\lambda_2} \right) + \mu_1 \cdot \exp(\lambda_1 \cdot t) + x_2 \cdot \mu_2 \cdot \exp(\lambda_2 \cdot t) \right] \end{cases}$$ éq 2.2.2-11

avec

$$\begin{cases} b_1^* = \frac{1}{x_1 \cdot x_2 - 1} \cdot \left[ x_2 \cdot (u_{rr} + 2 \cdot u_{ri}) + u_{ri} \right] \\ b_2^* = \frac{1}{x_1 \cdot x_2 - 1} \cdot \left[ -(u_{rr} + 2 \cdot u_{ri}) - x_1 \cdot u_{ri} \right] \end{cases}$$

Lastly, $\mu_1$ and $\mu_2$ are defined by the relations:

$$\begin{cases} \mu_1 = -\frac{1}{(x_1 \cdot x_2 - 1) \cdot \exp[(\lambda_1 + \lambda_2) \cdot t_0]} \cdot \left[ \frac{1}{2 \cdot k_r} \cdot x_2 \cdot \exp(\lambda_2 \cdot t_0) - \frac{1}{k_i} \cdot \exp(\lambda_2 \cdot t_0) \right] \\ \mu_2 = -\frac{1}{(x_1 \cdot x_2 - 1) \cdot \exp[(\lambda_1 + \lambda_2) \cdot t_0]} \cdot \left[ -\frac{1}{2 \cdot k_r} \cdot \exp(\lambda_1 \cdot t_0) + \frac{1}{k_i} \cdot x_1 \cdot \exp(\lambda_1 \cdot t_0) \right] \end{cases}$$ éq 2.2.2-12

### 2.1.3 Resolution of the equations constitutive of creep deviatoric

The deviatoric constraints comprise a reversible part and an irreversible part (cf [R7.01.06]):

$$\underbrace{\underline{\underline{\varepsilon}}^{fd}}_{\text{déformation déviatorique totale}} = \underbrace{\underline{\underline{\varepsilon}}_r^{fd}}_{\text{contribution eau absobée}} + \underbrace{\underline{\underline{\varepsilon}}_i^{fd}}_{\text{contribution eau libre}}$$ éq 2.3-1

*J*ème principal component of the total deviatoric deformation is governed by the equations [éq 2.3 - 2] and [éq 2.3-3]:

$$\eta_r^d \dot{\varepsilon}_r^{d,j} + k_r^d \varepsilon_r^{d,j} = h \cdot \sigma^{d,j}$$ éq 2.3-2

where $k_r^d$ indicate rigidity associated with the capacity with water adsorbed to transmit loads (*load bearing toilets*);

and $\eta_r^d$ viscosity associated with the water adsorbed by the layers with hydrates.

$$\eta_i^d \dot{\varepsilon}_i^{d,j} = h \cdot \sigma^{d,j}$$ éq 2.3-3

where $\eta_i^d$ indicate the viscosity of free water. The system of equations [éq 2.3-2] and [éq 2.3-3] is simpler to solve than that governing the spherical behavior owing to the fact that it is uncoupled. It is always supposed that moisture remains equal to 1 during all the loading. The equation [éq 2.3-2] corresponds to the viscoelastic model of Kelvin whose answer to a level of constraint is of exponential type. As for the equation [éq 2.3-3], the answer in deformation is linear with time. The total deformation of creep is thus written as the sum of the contribution of a chain of Kelvin and contribution of a shock absorber and series:

$$\varepsilon^{d,j}(t)=\left(\frac{t}{\eta_i^d}+\frac{1}{k_r^d}\left(1-e^{-\frac{k_r^d}{\eta_r^d}t}\right)\right)\cdot\sigma^{d,j}H(t) \qquad \text{éq 2.3-4}$$

### 2.1.4 Summary of the analytical solution

For a uniaxial loading the analytical solutions of the two components of deformation are known. The contribution of the deviatoric part is written:

$$\varepsilon^{fd}(t)=\frac{2}{3}\sigma_0\cdot\left(\frac{t}{\eta_i^d}+\frac{1}{k_r^d}\left(1-\exp\left(-\frac{k_r^d t}{\eta_r^d}\right)\right)\right) \qquad \text{éq 2.4-1}$$

As for the contribution of the spherical part, the solution is defined on two intervals:

$$\varepsilon^{fs}(t)=\begin{cases}\dfrac{\sigma_0}{3k_r^s}\cdot\left(1-\exp\left(-\dfrac{k_r^s t}{\eta_r^s}\right)\right) & t\le\dfrac{\eta_r^s}{k_r^s}\ln 2\\[2ex] \dfrac{\sigma_0}{3}\left(\left(\dfrac{1}{k_r^s}+\dfrac{1}{k_i^s}\right)+\mu_1(1+x_1)\exp(\lambda_1 t)+\mu_2(1+x_2)\exp(\lambda_2 t)\right) & t>\dfrac{\eta_r^s}{k_r^s}\ln 2\end{cases} \qquad \text{éq 2.4-2}$$

The axial deformation is a linear function of the two preceding contributions:

$$\varepsilon_{zz}=\varepsilon^{fs}(\sigma_0/3)+\varepsilon^{fd}(2\sigma_0/3) \qquad \text{éq 2.4-3}$$

## 2.2 Solutions obtained for the model BETON_BURGER_FP

The analytical solution was not developed for this loading of uniaxial creep. The reference solution is obtained numerically by using a script python (accessible under the repertoire astest: SSNV163D.44). The diagram of integration used is explicit and sensitive to the temporal discretization employed.

## 2.3 Sizes and results of reference

The test is homogeneous. One tests the deformation in an unspecified node.

## 2.4 Uncertainties on the solution

Exact analytical result for BETON_UMLV_FP.
Result depend on the temporal discretization employed for BETON_BURGER_FP.

## 2.5 Bibliographical references

1. BENBOUDJEMA, F.: Modeling of the deformations differed from the concrete under biaxial requests. Application to the buildings engines of nuclear power plants, Memory of D.E.A. Advanced materials – Engineering of the Structures and the Envelopes, 38 p. (+ additional) (1999).

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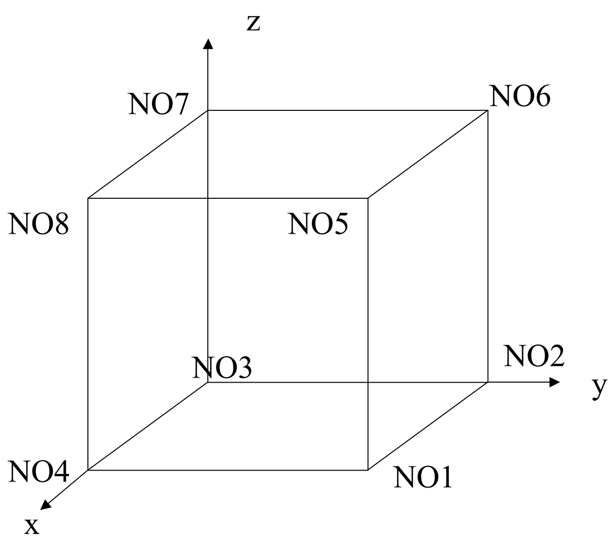
3. THE POPE, Y.: Relation of behavior UMLV for the clean creep of the concrete, Reference material of *Code_Aster* [R7.01.06], 16 p (2002).

4. FOUCAULT, A.: Relation of behavior `BETON_BURGER_FP` for the clean creep of the concrete, Reference material of *Code_Aster* [R7.01.35] (2011).

# 3 Modeling A

## 3.1 Characteristics of modeling

Modeling `3D`




## 3.2 Characteristics of the grid

Many nodes: 8
Many meshs: 1 of type `HEXA 8`
6 of type `QUAD 4`

The following meshs are defined:

| | | | | |
|---|---|---|---|---|
| $S_{ARR}$ | $NO3$ | $NO7$ | $NO8$ | $NO4$ |
| $S_{AVT}$ | $NO1$ | $NO2$ | $NO6$ | $NO5$ |
| $S_{DRT}$ | $NO1$ | $NO5$ | $NO8$ | $NO4$ |
| $S_{GCH}$ | $NO3$ | $NO2$ | $NO6$ | $NO7$ |
| $S_{INF}$ | $NO1$ | $NO2$ | $NO3$ | $NO4$ |
| $S_{\mathrm{SUP}}$ | $NO5$ | $NO6$ | $NO7$ | $NO8$ |

The boundary conditions in displacement imposed are:

On the nodes $NO1$ , $NO2$ , $NO3$ and $NO4$ : $DZ=0$
On the nodes $NO3$ , $NO7$ , $NO8$ and $NO4$ : $DY=0$
On the nodes $NO2$ , $NO6$ , $NO7$ and $NO8$ : $DX=0$

The loading is consisted by the same field of drying and of the same nodal force $1/4$ applied to the four nodes of $S_{\mathrm{SUP}}$ .

## 3.3 Sizes tested and results

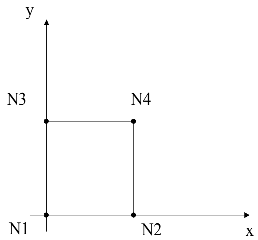
The component $\varepsilon_{zz}$ with the node $NO6$ was tested.

| Moment | Reference | *Aster* | % difference |
|---|---|---|---|
| 0. | 0. | 0. | - |
| 1.0000E+00 | – 3.225814D-05 | – 3.225810D-05 | – 1.37E-04 |
| 9.7041E+04 | – 3.867143D-05 | – 3.867140D-05 | – 8.95E-05 |
| 1.8389E+06 | – 6.088552D-05 | – 6.088554D-05 | 3.25E-05 |
| 8.6400E+06 | – 1.100478D-04 | – 1.100473D-04 | – 7.27E-06 |

# 4 Modeling B

## 4.1 Characteristics of modeling

Modeling $2D$ axisymmetric.




## 4.2 Characteristics of the grid

Many nodes: 4
Many meshs: 1 of type `QUAD 4`
4 of type `SEG2`

The following meshs are defined:

| | | |
|---|---|---|
| $L_{INF}$ | $NO1$ | $NO2$ |
| $L_{DRT}$ | $NO2$ | $NO4$ |
| $L_{\mathrm{SUP}}$ | $NO4$ | $NO3$ |
| $L_{GCH}$ | $NO3$ | $NO1$ |

The boundary conditions in displacement imposed are:

On $L_{GCH}$ : $DY=0$
On $L_{INF}$ : $DX=0$

The loading is consisted by the same field of drying and of the same nodal force $1/2$ applied to the two nodes of $L_{\mathrm{SUP}}$ .

## 4.3 Sizes tested and results

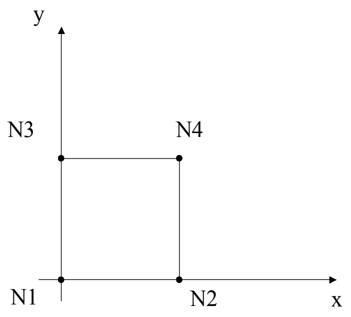
The component $\varepsilon_{yy}$ with the node $NO3$ was tested

| Moment | Reference | *Aster* | % difference |
|---|---|---|---|
| 0. | 0. | 0. | - |
| 1.0000E+00 | – 3.225814D-05 | – 3.225810D-05 | – 1.37E-04 |
| 9.7041E+04 | – 3.867143D-05 | – 3.867140D-05 | – 8.95E-05 |
| 1.8389E+06 | – 6.088552D-05 | – 6.088554D-05 | 3.25E-05 |
| 8.6400E+06 | – 1.100478D-04 | – 1.100473D-04 | – 7.27E-06 |

# 5 Modeling C

## 5.1 Characteristics of modeling

Modeling in Plane Constraints.




## 5.2 Characteristics of the grid

Many nodes: 4
Many meshs: 1 of type `QUAD 4`
4 of type `SEG2`

The following meshs are defined:

| | | |
|---|---|---|
| $L_{INF}$ | $NO1$ | $NO2$ |
| $L_{DRT}$ | $NO2$ | $NO4$ |
| $L_{\text{SUP}}$ | $NO4$ | $NO3$ |
| $L_{GCH}$ | $NO3$ | $NO1$ |

The boundary conditions in displacement imposed are:

On $L_{GCH}$ : $DY=0$
On $L_{INF}$ : $DX=0$

The loading is consisted by the same field of drying and of the same nodal force $1/2$ applied to the two nodes of $L_{\text{SUP}}$ .

## 5.3 Sizes tested and results

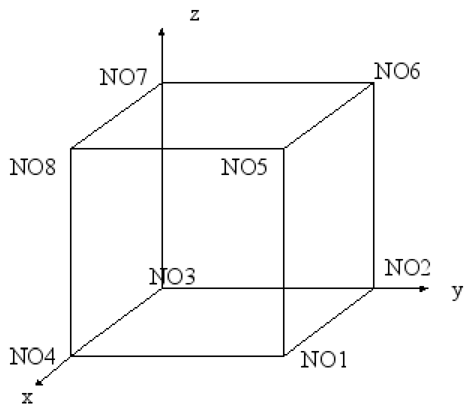
The component $\varepsilon_{yy}$ with the node $NO3$ was tested

| Moment | Reference | *Aster* | % difference |
|---|---|---|---|
| 0. | 0. | 0. | - |
| 1.0000E+00 | – 3.225814D-05 | – 3.225810D-05 | – 1.40E-04 |
| 9.7041E+04 | – 3.867143D-05 | – 3.867140D-05 | – 9.225E-05 |
| 1.8389E+06 | – 6.088552D-05 | – 6.088554D-05 | 3.08E-05 |
| 8.6400E+06 | – 1.100478D-04 | – 1.100478D-04 | – 8.22E-06 |

# 6 Modeling D

## 6.1 Characteristics of modeling

Modeling 3D




## 6.2 Characteristics of the grid

Many nodes: 8
Many meshs: 1 of type HEXA 8
6 of type QUAD 4

The following meshs are defined:

| | |
|---|---|
| $S_{ARR}$ | $NO3\ NO7\ NO8\ NO4$ |
| $S_{AVT}$ | $NO1\ NO2\ NO6\ NO5$ |
| $S_{DRT}$ | $NO1\ NO5\ NO8\ NO4$ |
| $S_{GCH}$ | $NO3\ NO2\ NO6\ NO7$ |
| $S_{INF}$ | $NO1\ NO2\ NO3\ NO4$ |
| $S_{\mathrm{SUP}}$ | $NO5\ NO6\ NO7\ NO8$ |

The boundary conditions in displacement imposed are:

- On the nodes $NO1$ , $NO2$ , $NO3$ and $NO4$ : $DZ=0$
- On the nodes $NO3$ , $NO7$ , $NO8$ and $NO4$ : $DY=0$
- On the nodes $NO2$ , $NO6$ , $NO7$ and $NO8$ : $DX=0$

The loading is consisted by the same field of drying and of the same nodal force $1/4$ applied to the four nodes of $S_{\mathrm{SUP}}$ .

## 6.3 Sizes tested and results

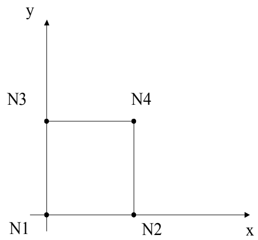
The component $\varepsilon_{zz}$ with the node $NO6$ was tested.

| Moment | Type of Reference | Reference | % Tolerance |
|---|---|---|---|
| 0. | SOURCE_EXTERNE | 0. | - |
| 1.0000E+00 | SOURCE_EXTERNE | – 3.22581D-05 | 0.5 |
| 9.7041E+04 | SOURCE_EXTERNE | – 3.89947D-05 | 0.5 |
| 1.8389E+06 | SOURCE_EXTERNE | – 6.55895D-05 | 0.5 |
| 8.6400E+06 | SOURCE_EXTERNE | – 1.32437D-04 | 0.5 |

# 7 Modeling E

## 7.1 Characteristics of modeling

Modeling $2D$ AXIS.




## 7.2 Characteristics of the grid

Many nodes: 4
Many meshs: 1 of type QUAD 4
4 of type SEG2

The following meshs are defined:

| | | |
|---|---|---|
| $L_{INF}$ | $NO1$ | $NO2$ |
| $L_{DRT}$ | $NO2$ | $NO4$ |
| $L_{\mathrm{SUP}}$ | $NO4$ | $NO3$ |
| $L_{GCH}$ | $NO3$ | $NO1$ |

The boundary conditions in displacement imposed are:

On $L_{GCH}$ : $DY=0$
On $L_{INF}$ : $DX=0$

The loading is consisted by the same field of drying and of the same nodal force $1/2$ applied to the two nodes of $L_{\mathrm{SUP}}$ .

## 7.3 Sizes tested and results

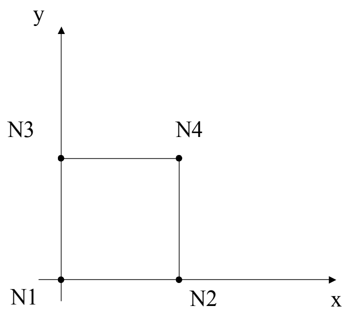
The component $\varepsilon_{yy}$ with the node $NO3$ was tested

| Moment | Type of Reference | Reference | % Tolerance |
|---|---|---|---|
| 0. | SOURCE_EXTERNE | 0. | - |
| 1.0000E+00 | SOURCE_EXTERNE | – 3.22581D-05 | 0.5 |
| 9.7041E+04 | SOURCE_EXTERNE | – 3.89947D-05 | 0.5 |
| 1.8389E+06 | SOURCE_EXTERNE | – 6.55895D-05 | 0.5 |
| 8.6400E+06 | SOURCE_EXTERNE | – 1.32437D-04 | 0.5 |

# 8 Modeling F

## 8.1 Characteristics of modeling

Modeling in Plane Constraints.




## 8.2 Characteristics of the grid

Many nodes: 4
Many meshs: 1 of type `QUAD 4`
4 of type `SEG2`

The following meshs are defined:

| | | |
|---|---|---|
| $L_{INF}$ | $NO1$ | $NO2$ |
| $L_{DRT}$ | $NO2$ | $NO4$ |
| $L_{SUP}$ | $NO4$ | $NO3$ |
| $L_{GCH}$ | $NO3$ | $NO1$ |

The boundary conditions in displacement imposed are:

On $L_{GCH}$ : $DY=0$
On $L_{INF}$ : $DX=0$

The loading is consisted by the same field of drying and of the same nodal force $1/2$ applied to the two nodes of $L_{SUP}$ .

## 8.3 Sizes tested and results

The component $\varepsilon_{yy}$ with the node $NO3$ was tested

| Moment | Type of Reference | Reference | % Tolerance |
|---|---|---|---|
| 0. | SOURCE_EXTERNE | 0. | - |
| 1.0000E+00 | SOURCE_EXTERNE | – 3.22581D-05 | 0.5 |
| 9.7041E+04 | SOURCE_EXTERNE | – 3.89947D-05 | 0.5 |
| 1.8389E+06 | SOURCE_EXTERNE | – 6.55895D-05 | 0.5 |
| 8.6400E+06 | SOURCE_EXTERNE | – 1.32437D-04 | 0.5 |

# 9 Summary of the results

Values obtained with *Code_Aster* are in agreement with the values of reference. This same test was turned with Castem at the Laboratory of Mechanics with the University of the Marne the Valley, the same results were obtained for the model BETON_UMLV_FP.