SSNV166 – Cylinder fissured under multiple loadings

Summary

The purpose of this test is calculation of the stress intensity factors along the bottom of crack for a cylinder comprising an axisymmetric crack.

The influence of the degree of the elements and the type of the method is studied through various modelings.

1) Modeling A test $K1$ and $K3$ with a linear grid $3D$ and a method with the finite elements classical (FEM).
2) Modeling B test $K1$ and $K3$ with a quadratic grid $3D$ (elements of Barsoum) around the bottom of crack and one FEM .
3) Modeling C test $K1$ and $K3$ with a linear grid $3D$ with a classical resolution but an extraction of the factors of intensity based on an energy calculation.

Moreover, for each modeling, various cases of loadings are studied:

- traction (request in mode I);
- torsion (request in mode III);
- inflection (opening of dimensioned, closing of the other) with and without taking into account of the contact.

The cases of traction and torsion do not put concerned the contact.

Although symmetries exist in certain cases (axisymetry for case 1, symmetry planes for the 2ème) the representation is made in $3D$ to make the test generalizable under multiple loading.

1 Problem of reference

1.1 Geometry

The crack is a circular ring in an orthogonal plan with the axis of the cylinder [Figure 1.1-a]. Parameters a and b the ray of the cylinder and the depth of the crack determine. [Figure 1.1-b] of the cylinder in the plan of crack (plan is a cut Oyz). So that the medium is regarded as infinite, the height of the cylinder is $h=10b$.

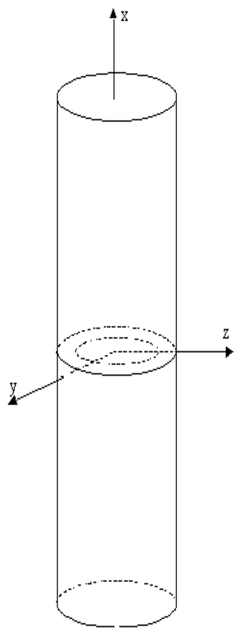

Figure 1.1-has : Geometry of the fissured cylinder

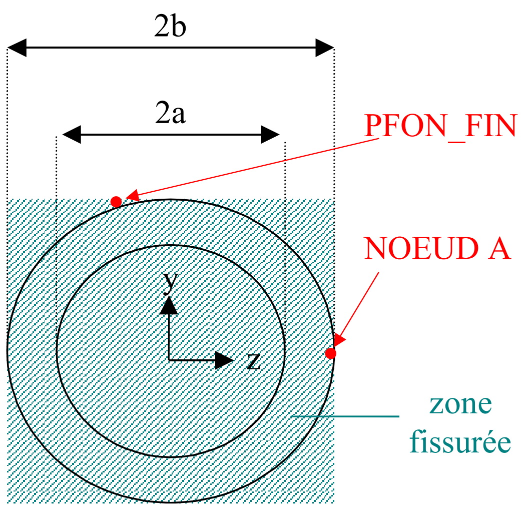

Figure 1.1-B : Plan of fissuration

1.2 Material properties

Young modulus: $E=205000\,MPa$

Poisson's ratio: $\nu=0.3$

1.3 Boundary conditions and loadings

Three loadings will be applied in order to calculate the stress intensity factors $K1$ and $K3$ in 3D by using the operator `POST_K1_K2_K3`.

Loading 1 tests $K1$ and $K3$.
Loading 2 tests $K2$ without taking into account of the contact.
Loading 3 tests $K2$ with taking into account of the contact.

One expects that $K1$ and $K3$ are constant along the bottom of crack and so that $K2$ vary.

Note: the cases of traction and torsion can be treated indifferently with or without contact (here, without contact) because there is never closing of the crack.

	Case 1: traction and torsion	Case 2: inflection without contact	Case 3: inflection with contact
Higher face	$N_x=6\,MN$ $T_x=3\,MN$	$M_y=1.5\,MN$	$M_y=1.5\,MN$

Table 1.3- 1 : Case of loadings

The preceding efforts are applied to the structure via elements 3D discrete located at the center of the higher face. It is noted that the point of maximum opening due to the imposed inflection (next moment Oy) will be the node A (see [Figure 1.1 - B].

The rigid movements of body are blocked by the same process with embedding of the center of the lower face.

2 Reference solution

2.1 Method of calculating used for the reference solution

For an axisymmetric crack in a cylinder infinite length, method of the Singular Integral equations and Asymptotic Developments [feeding-bottle1] allows to calculate the values of the stress intensity factors.

1) Case 1: Traction and Torsion

Traction induces an opening in mode 1. $K1$ is given by the following formula:

$$K_I = \frac{P}{\pi a^2}\sqrt{\pi a}\,F_1(a/b)$$

where P is the effort applied to the higher and lower face and F_1 a given function [Figure 2.1-a].

Torsion induces an opening in mode 3. $K3$ is given by the following formula:

$$K_{III} = \frac{2T}{\pi a^3}\sqrt{\pi a}\,F_3(a/b)$$

where T is the moment applied to the higher and lower face and F_3 a given function [Figure 2.1 - has].

1) Case 2: Inflection without contact

The inflection induces an opening in mode 1. The value of $K1$ at the point of maximum opening A is given by the following formula:

$$K_{I_A} = \frac{4M}{\pi a^3}\sqrt{\pi a}\,F_2(a/b)$$

where M is the moment applied to the higher and lower face and F_2 a given function [Figure 2.1 - has].

1) Case 3: Inflection with contact

There does not exist analytical solution with this problem. One expects on the one hand that $K1$ that is to say near to the case without contact on the part of the crack in opening, and in addition that $K1$ that is to say no one on the part of the crack in closing.

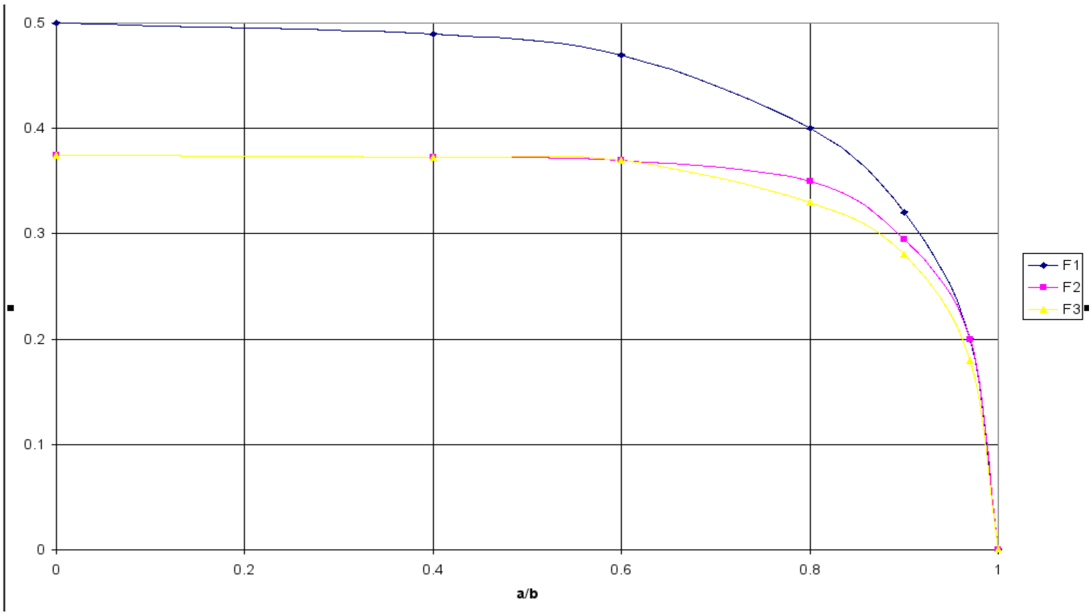

Figure 2.1 - has : Functions $F1$ **,** $F2$ **and** $F3$

These three functions come from [feeding-bottle1].

2.2 Results of reference

Digital application:

Except contrary mention, in the continuation of this document, the parameters retained for a and b are:

$$a=0.4\,m$$
$$b=0.5\,m$$

Case 1: Traction and torsion	Case 2: Inflection
$K1=5.35\ MPa.m^{1/2}$ $K3=11.22\ MPa.m^{1/2}$	$K1_A=11.71\ MPa.m^{1/2}$

Table 2.2 - 1 : Values of reference

2.3 Bibliographical references

•TADA, PARIS, IRWIN: The Stress Analysis Of Cracks Handbook, Del Research Corporation, Hellertoxn, Pennsylvania (1973).
•Calculation of the factors of intensity of the constraints by extrapolation of the field of displacements, Handbook of reference of *Code_Aster*, R7.02.08
•CORNELIU: Quarter-point elements for curved ace faces, Computers & Structures vol. 17, No 2, pp. 227-231, 1983

3 Linear modeling a: Grid, classical formulation

3.1 Characteristics of modeling

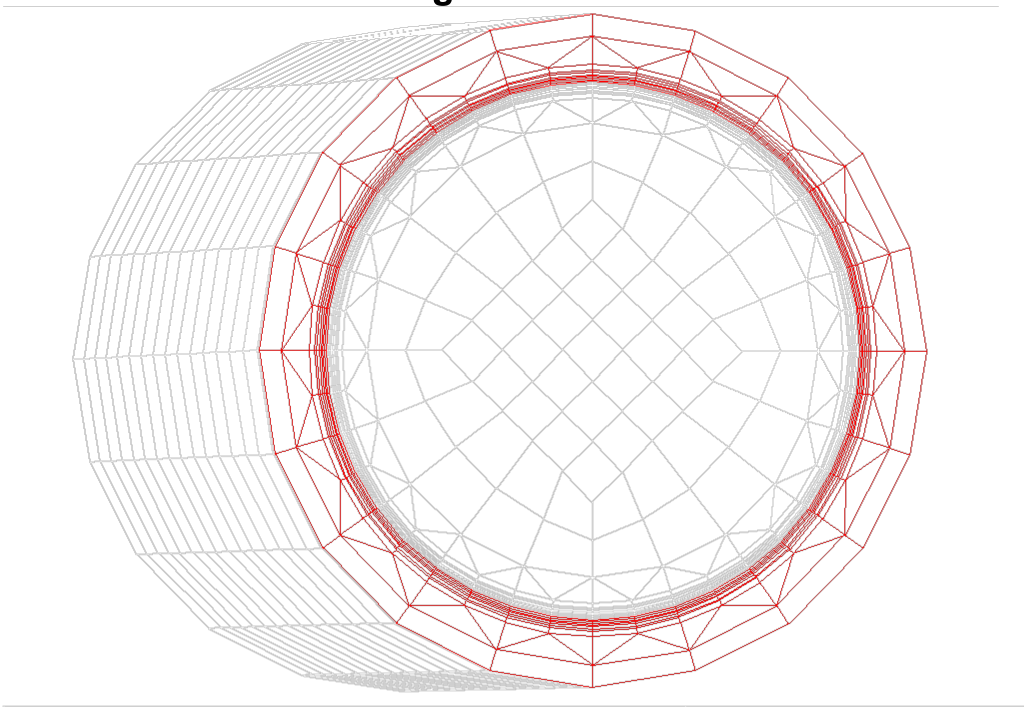

Figure 3.1 - has : Cut of the grid in the plan of the crack

The elements are all of order 1.

The interest of this modeling is to be used as a basis for more evolved formulations, and thus, to be able to note the contribution and the improvements of the other methods.

3.2 Characteristics of the grid

Many nodes: 11310
Many meshs: 14453

Type of meshs	Many meshs
POI1	4
SEG2	39
TRIA3	360
QUAD4	930
PENTA6	5440
HEXA8	7680

Table 3.2 - 1: Characteristics of the meshs

3.3 Notice

The calculation of the stress intensity factors is done using `POST_K1_K2_K3` (method of extrapolation of displacements on the lips of the crack) [feeding-bottle2].

3.4 Values tested and Results of modeling A

Procedure `POST_K1_K2_K3` allows to identify the values of the stress intensity factors except for a coefficient. It is pointed out that this method identifies the stress intensity factor $K1$ (respectively $K2$, $K3$) starting from the jump of displacement by a method of least squares.

3.4.1 Results in the case of a loading in traction (K1) and torsion (K3)

Identification	Reference	*Aster*	% difference
$K1$ with the node $PFON_{FIN}$	$5.35\ 10^{6}$	$4.52\ 10^{6}$	15
$K3$ with the node $PFON_{FIN}$	$-11.22\ 10^{6}$	$-9.54\ 10^{6}$	15

Values of $K1$ and $K3$ must be identical [Figure 4.2-has] for all the nodes of the bottom of crack because there is an axisymmetric configuration. Here, we test only the values with the node $PFON_{FIN}$.

3.4.2 Results in the case of a loading in inflection (K1) without contact

Identification	Reference	*Aster*	% difference
$K1$ with the node A	$11.71\ 10^{6}$	$9.18\ 10^{6}$	22

One compares the value of $K1$ with the reference solution only to the point of maximum opening (node A) because it is the only analytical value available in the literature.

3.4.3 Results in the case of a loading in inflection (K1) with contact

Identification	Reference	*Aster*	% difference
$K1$ with the node A	$10.17\ 10^{6}$	$8.38\ 10^{6}$	20

One compares the result got with that obtained by *Code_Aster* without taking into account of the contact (not-regression). This taking into account is carried out by the method of the active constraints.

3.4.4 Evolutions of K1, K2, K3 along the bottom of crack

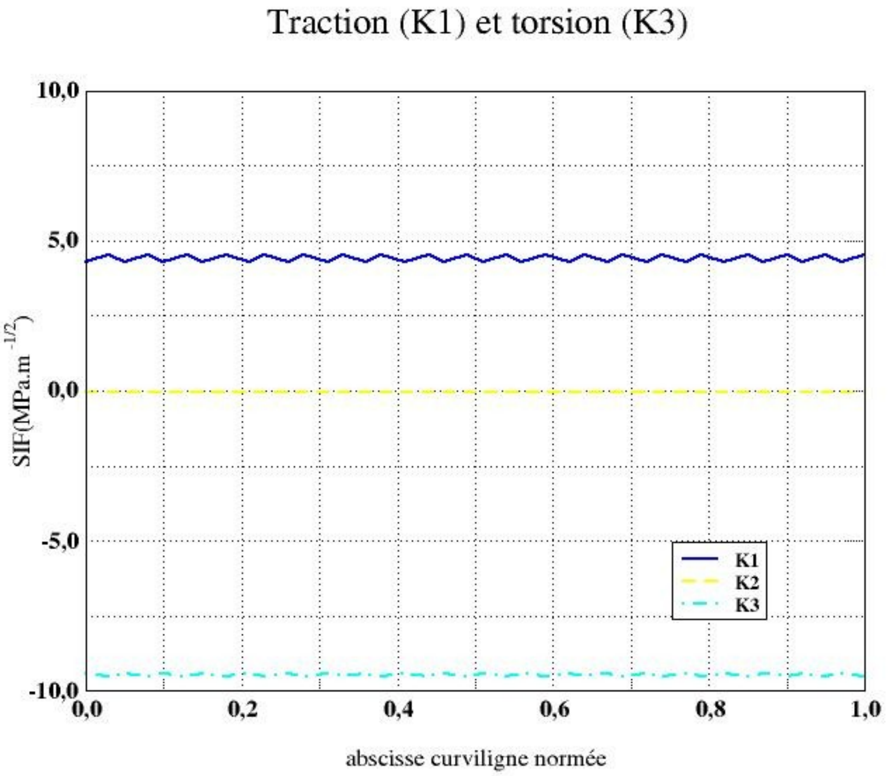

Figure 4.2 - has : $K1$ **,** $K2$ **and** $K3$ **along the bottom of crack (in** $MPa.m^{1/2}$ **)**

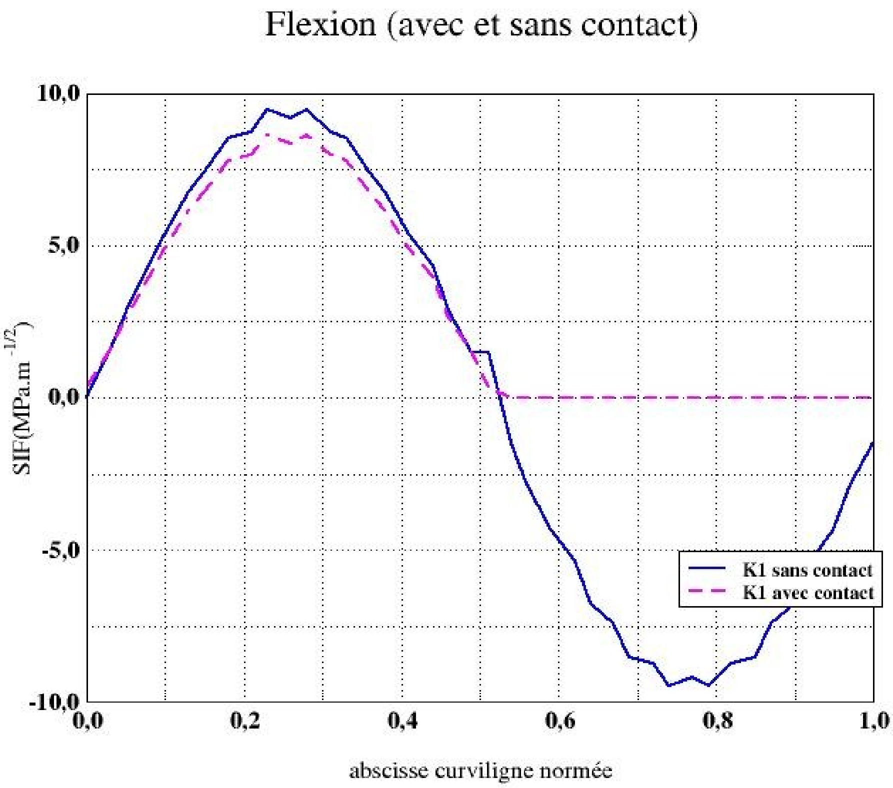

Figure 4.2 - B : $K1$ **along the bottom of crack (in** $MPa.m^{1/2}$ **)**

Comments on the results:

[the Figure 4.2-has] watch evolution of the factors of intensity of the constraints along the bottom of crack of the axisymmetric crack of depth $100\,mm$ subjected to traction and torsion. One observes many axisymmetric results (with the miscalculations near). Moreover, it is noted that the crack is not requested in mode II .
On [the Figure 4.2-B], one highlights the taking into account of the contact. On half of crack in opening, $K1$ has weaker values with taking into account of the contact, because the contact rigidifies the structure. On half in closing, $K1$ is null.
In fact, the contact does not take place on all the higher half of the crack [Figure 4.2-C] but on a surface a little smaller. On [the Figure 4.2-C] the zone in red represents the zone of contact and the zone in blue that of noncontact.

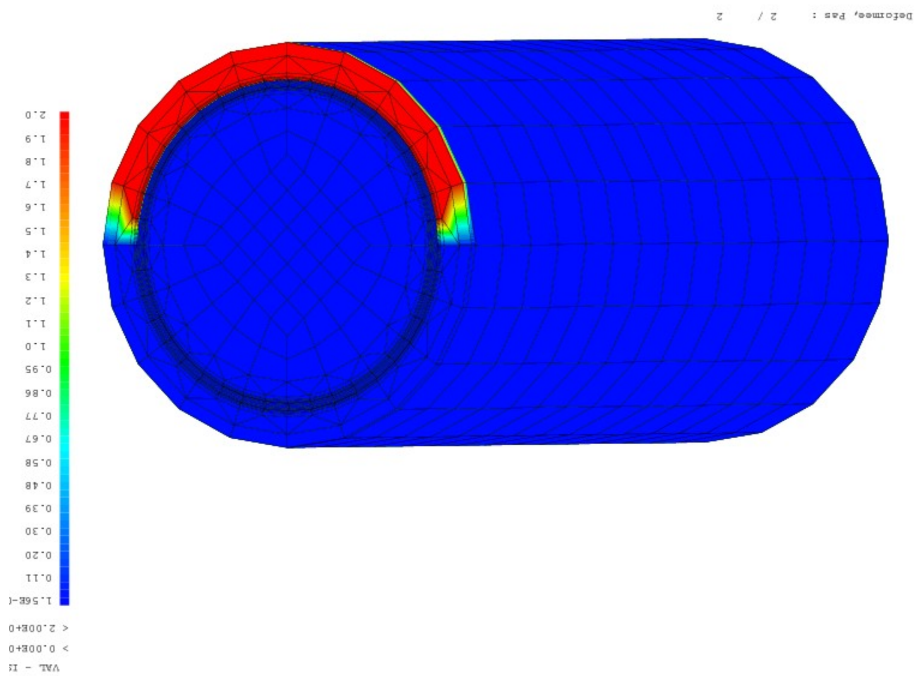

Figure 4.2 - C : Contact

4 Quadratic modeling b: Grid, classical formulation

4.1 Characteristics of modeling

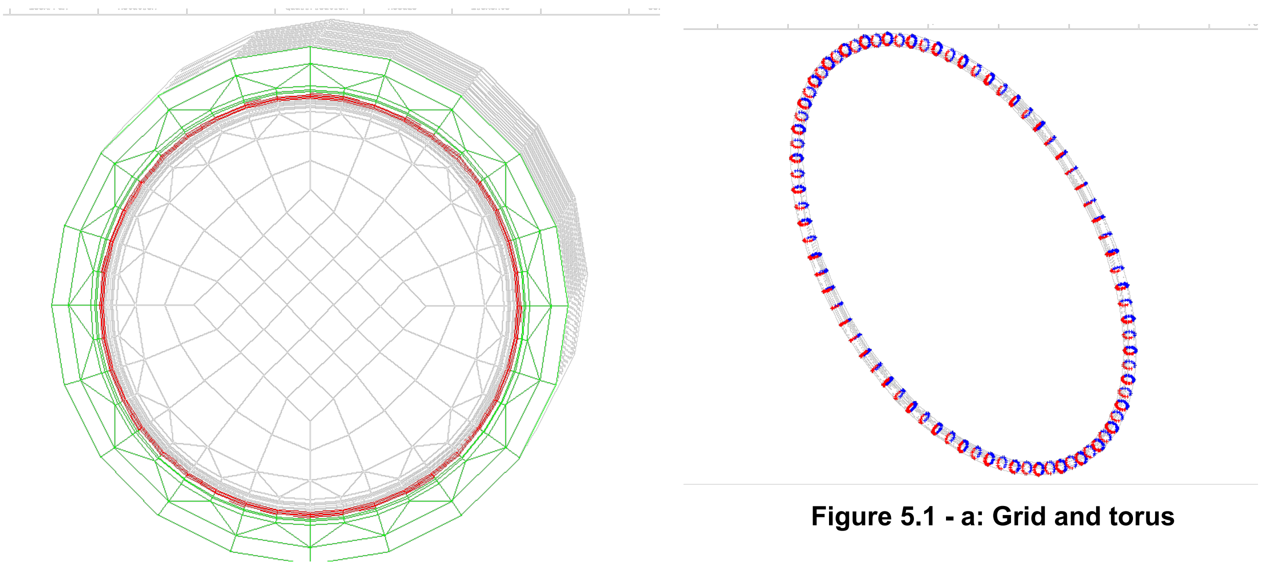

Figure 5.1 - a: Grid and torus

A torus is created around the crack. The elements of the torus are quadratic elements. The elements apart from the torus are linear. Moreover, one uses elements of BARSOUM (nodes mediums moved with the quarter) for the meshs having an edge pertaining to the bottom of crack [feeding-bottle3].

The interest of the use of a grid of the type BARSOUM is obtaining more precise results.

4.2 Characteristics of the grid

Many nodes: 20030
Many meshs: 16449

Type of meshs	Many meshs
POI1	2000
SEG3	39
TRIA3	360
QUAD4	610
QUAD8	320
PENTA6	4800
PENTA15	640
HEXA8	5760
HEXA20	1920

Table 5.2 - 1: Characteristics of the meshs

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision.

4.3 Values tested and results of modeling B

4.3.1 Results in the case of a loading in traction ($K1$) and torsion ($K3$)

Identification	Reference	*Aster*	% difference
$K1$ with the node $PFON_{FIN}$	5.35 10^6	5.04 10^6	5,7
$K3$ with the node $PFON_{FIN}$	-11.22 10^6	-10.80 10^6	3,8

Values of $K1$ and $K3$ must be identical [Figure 6.2-has] for all the nodes of the bottom of crack because there is an axisymmetric configuration. Here, we test only the values with the last node of the crack ($PFON_{FIN}$).

4.3.2 Results in the case of a loading in inflection ($K1$) without contact

Identification	Reference	*Aster*	% difference
$K1$ with the node A	11.71 10^6	10.29 10^6	12

One compares the value of $K1$ with the reference solution only to the point of maximum opening (Node A) because it is the only analytical value available in the literature.

4.3.3 Results in the case of a loading in inflection ($K1$) with contact

Identification	Reference	*Aster*	% difference
$K1$ with the node A	10.59 10^6	9.24 10^6	13

One compares the result got with that obtained by calculation *Aster* without taking into account of the contact (not - regression). The method of resolution of the contact is that of the active constraints.

4.4 Evolutions of K1, K2, K3 along the bottom of crack

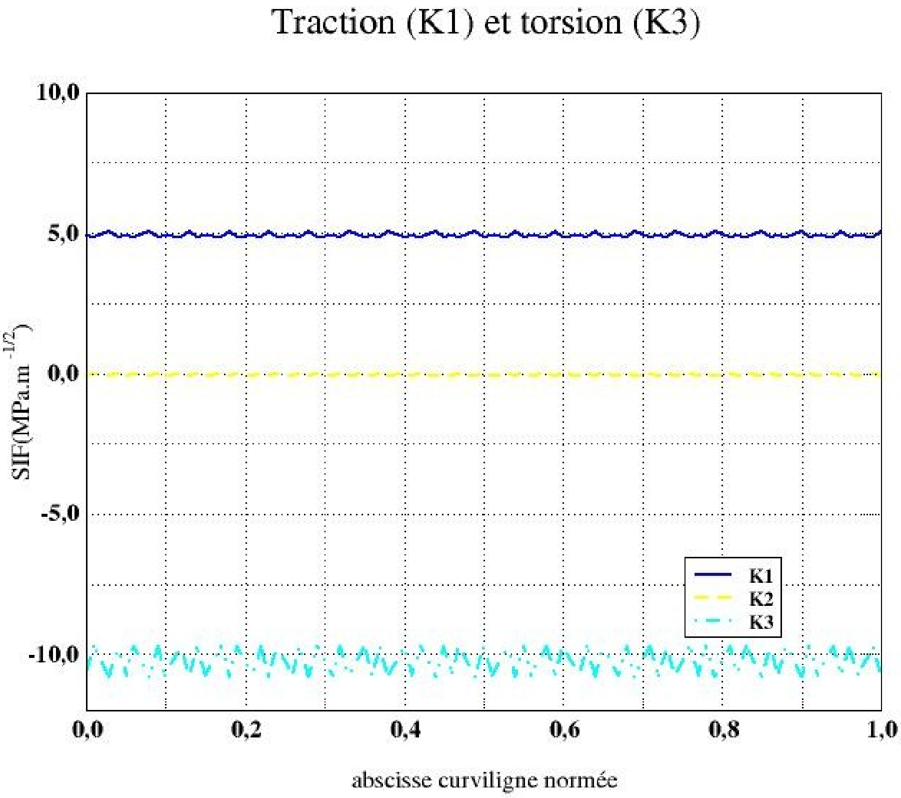

Figure 4.4-a : $K1$ **,** $K2$ **and** $K3$ **along the bottom of crack (in** $MPa.m^{1/2}$ **)**

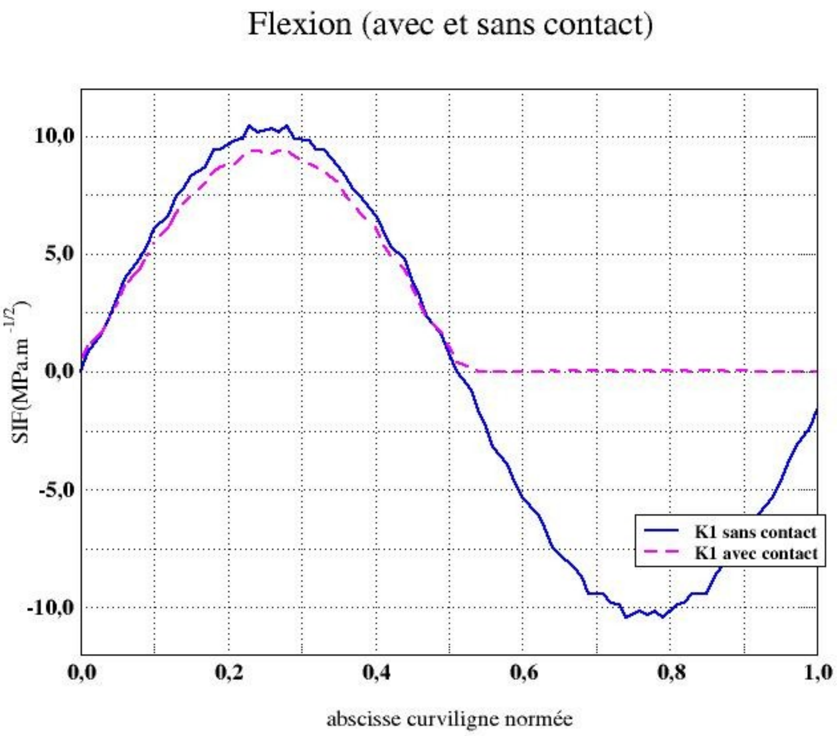

Figure 4.4-b : $K1$ **along the bottom of crack (in** $MPa.m^{1/2}$ **)**

5 Modeling C: Linear grid, classical formulation and energy method

5.1 Characteristics of modeling

The modeling of the problem is the same one as that used in A . All the elements are of order 1.

5.2 Characteristics of the grid

The grid is similar to that used in A .

Many nodes: 13630
Many meshs: 17013

Type of meshs	Many meshs
POI1	4
SEG2	39
TRIA3	360
QUAD4	1090
PENTA6	5760
HEXA8	9760

Table 7.2 - 1 : Characteristics of the meshs

5.3 Values tested and results of modeling C

One calculates the rate of refund of energy and the factors of intensity of the constraints with the order `CALC_G`, option `CALC_K_G`. This method is more general than the method of extrapolation of displacements (`POST_K1_K2_K3`) because it can be used an unspecified crack in the case of (not-plane crack, at bottom not-right).

5.3.1 Results in the case of a loading in traction (K1) and torsion (K3)

Identification	Reference	*Aster*	% difference
$Max(K1)$	$5.35\ 10^6$	$5.11\ 10^6$	4.47
$Max(K3)$	$11.22\ 10^6$	$10.52\ 10^6$	6.24

Values of $K1$ and $K3$ must be identical [Figure 5.4-a] for all the nodes of the bottom of crack because there is an axisymmetric configuration. Here, we test the maximum of $K1$ and $K3$ for all the points of the bottom of crack.

5.3.2 Results in the case of a loading in inflection (K1) without contact

Identification	Reference	*Aster*	% difference
$K1$ with the node A	$11.71\ 10^6$	$10.32\ 10^6$	11.88

One compares the value of $K1$ with the reference solution only to the point of maximum opening because it is the only analytical value available in the literature. This point is not any more one “node” but a “point” of the bottom of crack, it should then be located by its number in the list of the points of the bottom of crack. It is the point located by `NUM_PT=11`.

5.3.3 Results in the case of a loading in inflection (K1) with contact

Identification	Reference	*Aster*	% difference
$K1$ with the node A	$10.32\ 10^6$	$9.43\ 10^6$	8.57

One compares the result got with that obtained by *Code_Aster* without taking into account of the contact (not - regression). This taking into account is carried out by the method of the active constraints.
[Figure 5.4-b] compares the values of $K1$ along the bottom of crack in the case of inflection with and without contact.

5.4 Evolutions of K1 and K3 along the bottom of crack

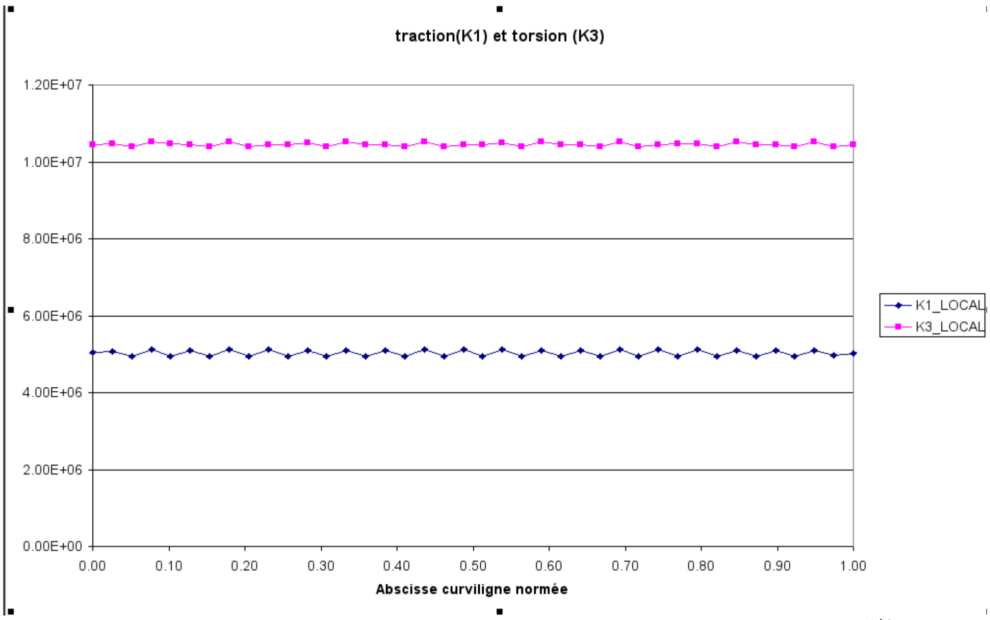

Figure 5.4-a : $K1$ and $K3$ along the bottom of crack (in $MPa.m^{1/2}$)

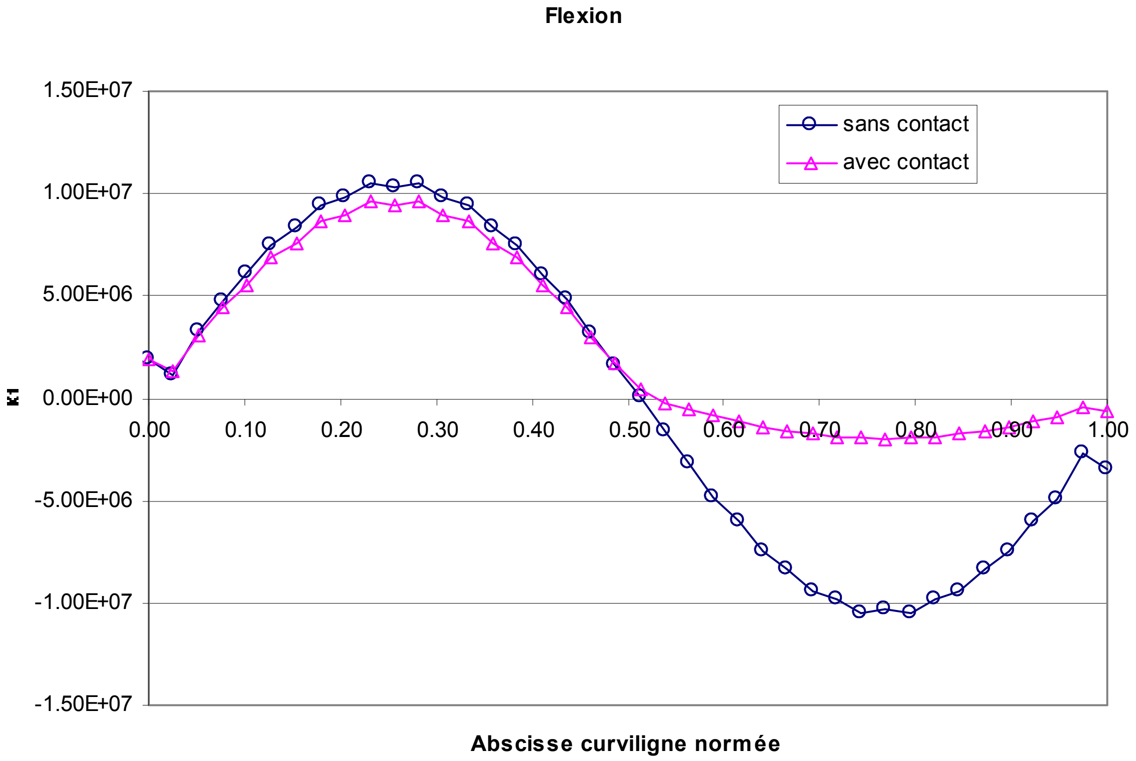

Figure 5.4-b : $K1$ **along the crack (in** $MPa.m^{1/2}$ **)**

Note:

> *It is noted that if the contact is taken into account (see [Figure 5.4-b]),* $K1$ *is not really null on the segments of the bottom of crack where there is closing. That comes owing to the fact that the energy method of calculation from* K *project the field of solution displacement on the singular auxiliary fields of displacement of an infinitely long crack in opening. However these auxiliary fields are not compatible with the mode of closing present.*

6 Summary of the results

The goals of this test are achieved:

- It is a question of validating the taking into account of the contact on the lips of the crack with quadratic elements (and elements of Barsoum). The results better, are compared with those obtained with a linear grid.
- This test shows the interest of the method *« $G-thêta$ »* for the calculation of the stress intensity factors. This energy method has the advantage of being more general than that using the jump of displacements (`POST_K1_K2_K3`) because it can apply to cracks of unspecified geometry, whereas `POST_K1_K2_K3` is restricted with the plane cracks. Moreover, method *« $G-thêta$ »* give better results (compared with the analytical solution) that `POST_K1_K2_K3` for the same linear grid.