

## SSNV176 – Identification of the law ENDO\_ORTH\_BETON

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### Summary:

The tests of the law here are presented ENDO\_ORTH\_BETON on a single element allowing to identify the parameters of the model. Insofar as there does not exist empirical formula making it possible to gauge the parameters, the user will be able to use some of the cases tests presented here to adjust his parameters. The study of the parameters of the model is in documentation [R7.01.09]. The 5 tests suggested are the following:

- 1) simple traction
- 2) simple traction with piloting
- 3) simple compression
- 4) simple compression with piloting
- 5) simple traction, simple compression and a biaxial test

## 1 Problem of reference

### 1.1 Geometry and boundary conditions

The element used is a tetrahedron at a point of gauss. There is thus no problem of homogeneity of the fields in the element.

The conditions of blockings and the relations linear between the nodes which should be applied are summarized on [the Figure 1.1-a]. Edges  $N0N1$ ,  $N0N2$  and  $N0N3$  are length 1.

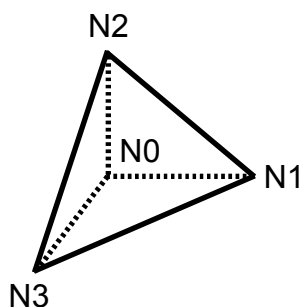
Taking into account the geometry of the element, conditions of blockings and relations linear, the deformation is directly connected to displacements of the nodes:

$$\begin{aligned}\varepsilon_{xx} &= DX(N1) \\ \varepsilon_{yy} &= DY(N2) \\ \varepsilon_{zz} &= DZ(N3) \\ \varepsilon_{xy} &= DX(N2) = DY(N1) \\ \varepsilon_{xz} &= DX(N3) = DZ(N1) \\ \varepsilon_{yz} &= DY(N3) = DZ(N2)\end{aligned}$$

If one works with imposed deformation, it is thus enough to impose displacement on the adequate nodes.

If one wishes to work with imposed force, as it is the case for modeling E, it is necessary to force the following loadings (see to it [Figure 1.1-a] for the definition of the faces  $F1$ ,  $F2$ ,  $F3$  and  $F4$ ):

$$\begin{aligned}\sigma_{xx} > 0 &: FX \text{ on } F1 \text{ and } -1/\sqrt{3} FX \text{ on } F4, FX < 0 \text{ (traction according to } x \text{)} \\ \sigma_{xx} < 0 &: FX \text{ on } F1 \text{ and } -1/\sqrt{3} FX \text{ on } F4, FX < 0 \text{ (compression according to } x \text{)} \\ \sigma_{yy} > 0 &: FY \text{ on } F2 \text{ and } -1/\sqrt{3} FY \text{ on } F4, FY < 0 \text{ (traction according to } y \text{)} \\ \sigma_{yy} < 0 &: FY \text{ on } F2 \text{ and } -1/\sqrt{3} FY \text{ on } F4, FY < 0 \text{ (compression according to } y \text{)} \\ \sigma_{zz} > 0 &: FZ \text{ on } F3 \text{ and } -1/\sqrt{3} FZ \text{ on } F4, FZ < 0 \text{ (traction according to } z \text{)} \\ \sigma_{zz} < 0 &: FZ \text{ on } F3 \text{ and } -1/\sqrt{3} FZ \text{ on } F4, FZ < 0 \text{ (compression according to } z \text{)}\end{aligned}$$



Blockings:  
 $N0: DX = DY = DZ = 0$

Linear relations:  
 $DY(N1) = DX(N2)$   
 $DZ(N1) = DX(N3)$   
 $DZ(N2) = DY(N3)$

Traction/compression in imposed displacement:  
According to  $x$   $DX$  imposed on  $N1$   
According to  $y$   $DY$  imposed on  $N2$   
According to  $z$   $DZ$  imposed on  $N3$

Definition of the faces:  
 $F1 = N0 N2 N3$   
 $F2 = N0 N1 N3$   
 $F3 = N0 N1 N2$   
 $F4 = N1 N2 N3$

Traction/compression in imposed force:  
According to  $x$ :  $FX$  on  $F1$  and  $-1/\sqrt{3} FX$  on  $F4$   
According to  $y$ :  $FY$  on  $F2$  and  $-1/\sqrt{3} FY$  on  $F4$   
According to  $z$ :  $FZ$  on  $F3$  and  $-1/\sqrt{3} FZ$  on  $F4$

**Figure 1.1-a: Geometry and boundary conditions of the uniaxial tests**

## 1.2 Material properties

The characteristic materials are identical for the 5 tests which are presented.

The elastic characteristics of materials are the following ones:

$$E = 32000 \text{ Mpa} ; \nu = 0.2$$

The breaking stresses in traction and compression are:

$$\sigma_{rupture}^{traction} = 3.2 \text{ MPa} ; \sigma_{rupture}^{compression} = -31.8 \text{ MPa}$$

One uses the game of parameter following for the law of behavior:

ALPHA	K0 (Mpa)	ECROB (MJ/m <sup>3</sup> )	ECROD (MJ/m <sup>3</sup> )	K <sub>1</sub> (Mpa)	K <sub>2</sub>
0.87	3.10 <sup>-4</sup>	1.10 <sup>-3</sup>	6.10 <sup>-2</sup>	10.5	6.10 <sup>-4</sup>

### Note:

There exist several sets of parameters which provide the same breaking stresses. The parameters were identified so that the envelope of rupture of the biaxial tests does not present swelling (cf Doc. [R7.01.09]).

The answers of the model for the uniaxial tests are represented below.

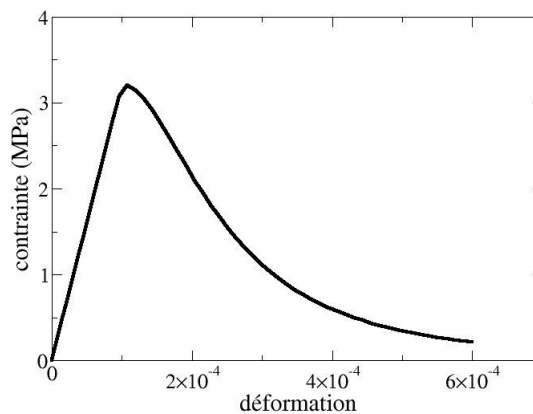


Figure 1.2-a: Answer of the law ENDO\_ORTH\_BETON in simple traction

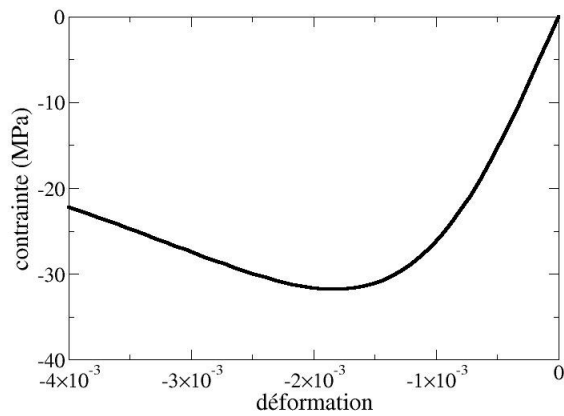


Figure 1.2-b: Answer of the law ENDO\_ORTH\_BETON in simple compression

The internal variables, which are numbered in Aster, have the following meaning:

$$V1 = D_{xx}; V2 = D_{yy}; V3 = D_{zz}; V4 = D_{xy}; V5 = D_{xz}; V6 = D_{yz}; V7 = d;$$

Where  $D$  is the tensor representing the orthotropic damage of traction, and  $d$  is the isotropic damage of compression (cf Doc. [R7.01.09]).

## 2 Reference solution

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This test is a test of nonregression.

## 3 Modeling A

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### 3.1 Characteristics of modeling

Modeling 3D

Element MECA\_TETRA4.

### 3.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 1 TETRA4

### 3.3 Features tested

The law of behavior ENDO\_ORTH\_BETON in simple traction (without piloting).

### 3.4 Way of loading

The element is subjected to a uniaxial traction in the direction  $X$ . Displacement  $DX$  is imposed on the node  $NI$ .

### 3.5 Values tested

Moment	Name of the field	Component	Place	Aster
50	DEPL	$DX$	$NI$	3.E-04
50	EPSI_ELGA	$EPXX$	VOLUME, point 1	3.E-04
50	SIEF_ELGA	$SIXX$	VOLUME, point 1	1.11388E+00
50	VARI_ELGA	$V1(D_{xx})$	VOLUME, point 1	6.59365E-01
50	VARI_ELGA	$V7(d)$	VOLUME, point 1	2.42260E-04

## 4 Modeling B

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### 4.1 Characteristics of modeling

Modeling 3D

Element MECA\_TETRA4.

### 4.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 1 TETRA4

### 4.3 Features tested

The law of behavior ENDO\_ORTH\_BETON in simple compression (without piloting of the loading).

### 4.4 Way of loading

The element is subjected to a uniaxial traction in the direction  $X$ . Displacement  $DX$  is imposed on the node  $NI$ .

### 4.5 Values tested

Moment	Name of the field	Component	Place	Aster
50	DEPL	$DX$	$NI$	-3.E-03
50	EPSI_ELGA	$EPXX$	VOLUME, point 1	-3.E-03
50	SIEF_ELGA	$SIXX$	VOLUME, point 1	-2.74465E+01
50	VARI_ELGA	$V2(D_{yy})$	VOLUME, point 1	1.30416E-01
50	VARI_ELGA	$V7(d)$	VOLUME, point 1	4.80080E-01

## 5 Modeling C

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### 5.1 Characteristics of modeling

Modeling 3D

Element MECA\_TETRA4.

### 5.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 1 TETRA4

### 5.3 Features tested

The law of behavior ENDO\_ORTH\_BETON in simple traction (with piloting of the loading).

### 5.4 Way of loading

The element is subjected to a uniaxial traction in the direction  $X$ . Displacement  $DX$  is imposed on the node  $NI$ . The difference with modeling A is that one uses the method of piloting of the loading PRED\_ELAS (cf Doc. [R5.03.80]).

### 5.5 Values tested

Moment	Name of the field	Component	Place	Aster
51	DEPL	$DX$	$NI$	1.44744E-04
51	EPSI_ELGA	$EPXX$	VOLUME, point 1	1.44744E-04
51	SIEF_ELGA	$SIXX$	VOLUME, point 1	2.89945E+00
51	VARI_ELGA	$V1(D_{xx})$	VOLUME, point 1	2.08793E-01
51	VARI_ELGA	$V7(d)$	VOLUME, point 1	2.30235E-04



## 6 Modeling D

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### 6.1 Characteristics of modeling

Modeling 3D

Element MECA\_TETRA4.

### 6.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 1 TETRA4

### 6.3 Features tested

The law of behavior ENDO\_ORTH\_BETON in simple compression (with piloting of the loading).

### 6.4 Way of loading

The element is subjected to a uniaxial traction in the direction  $X$ . Displacement  $DX$  is imposed on the node  $NI$ . The difference with modeling B is that one uses the method of piloting of the loading PRED\_ELAS (cf Doc. [R5.03.80]).

### 6.5 Values tested

Moment	Name of the field	Component	Place	Aster
51	DEPL	$DX$	$NI$	-1.17993E-03
51	EPSI_ELGA	$EPXX$	VOLUME, point 1	-1.17993E-03
51	SIEF_ELGA	$SIXX$	VOLUME, point 1	-2.86498E+01
51	VARI_ELGA	$V2(D_{yy})$	VOLUME, point 1	4.73153E-02
51	VARI_ELGA	$V7(d)$	VOLUME, point 1	1.34312E-01

## 7 Modeling E

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### 7.1 Characteristics of modeling

Modeling 3D

Element MECA\_TETRA4.

### 7.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 1 TETRA4

### 7.3 Features tested

The law of behavior here is tested ENDO\_ORTH\_BETON in 3 cases of loading:

- 1)  $U_1$  : Simple traction
- 2)  $U_2$  : Compression
- 3)  $U_3$  : Biaxial loading (traction in the direction  $y$ , compression in the direction  $x$ , with a report fixes constraints:  $\sigma_{yy} = -0.2\sigma_{xx}$ )

This case test makes it possible to check that the set of parameters chosen by the user respects the following data:

- 1) breaking stresses in traction,
- 2) breaking stresses in compression,
- 3) pas de swelling of the envelope of rupture for biaxial tests. That consists in checking that the maximum constraint in traction  $\sigma_{yy}$  biaxial test is lower than the breaking stress in simple traction.

### 7.4 Way of loading

Unlike modelings A, B, C and D, it is the force, and not the displacement, which is imposed here. One uses the method of piloting of the loading PRED\_ELAS, because the behavior is lenitive. The following loadings are applied:

- $U_1$  :  $FX$  on  $F1$ ,  $-1/\sqrt{3}FX$  on  $F4$ ,  $FX < 0$  (Traction)
- $U_2$  :  $FX$  on  $F1$ ,  $-1/\sqrt{3}FX$  on  $F4$ ,  $FX > 0$  (Compression)
- $U_3$  :  $FX$  on  $F1$ ,  $-1/\sqrt{3}FX$  on  $F4$ ,  $FX > 0$  (Compression according to the axis  $x$ );  
 $FY$  on  $F2$ ,  $-1/\sqrt{3}FY$  on  $F4$ , with  $FY = -0,2FX$  (Traction according to the axis  $y$ ).

## 7.5 Values tested

Moment	Result	Name of the field	Component	Place	Aster
42	<i>U1</i>	SIEF_ELGA	<i>SIXX</i>	VOLUME, point 1	3.20684E+00
76	<i>U2</i>	SIEF_ELGA	<i>SIXX</i>	VOLUME, point 1	-3.18000E+01
74	<i>U3</i>	SIEF_ELGA	<i>SIXX</i>	VOLUME, point 1	-1.42038E+01

One tests for each calculation, the maximum value (in absolute value) of the constraint  $\sigma_{xx}$ . One then obtains the breaking stress in traction (*U1*), in compression (*U2*), and it is checked that the tensile stress in the biaxial test (*U3*) is lower than the breaking stress in simple traction (*U1*):

- 1) *U1* :  $\sigma_{rupture}^{traction} = 3.20684 \text{ MPa}$
- 2) *U2* :  $\sigma_{rupture}^{compression} = -31.8 \text{ MPa}$
- 3) *U3* :  $\sigma_{U3}^{traction} = -0.2 \sigma_{U3}^{compression} = 0.2 \times 14.2038 \text{ Mpa}$  and  $\sigma_{U3}^{traction} < \sigma_{rupture}^{traction}$

**Warning 1** : It may be that the number of steps of time is insufficient to reach the lenitive phase. The user will thus check that for calculations *U1* and *U2*, calculation *U3* being subjected to an additional warning (cf. **warning 2**), it is well in the lenitive phase (reduction in the parameter of piloting). The maximum constraint in absolute value should not be reached for the last step of time. In the contrary case, it is necessary to continue calculation until the lenitive phase.

**Warning 2**: It is possible, for certain set of parameters, to observe difficulties of convergence for calculation *U3* at the time of the lenitive phase. Indeed, the law of behavior ensures the existence and the unicity of the solution in imposed deformation, but not in imposed force. These problems of convergence appearing only in the lenitive phase, the user will be able to consider the greatest value of the parameter of piloting reached, equal to the greatest compressive stress  $\sigma_{xx}$  attack in absolute value, like reference to gauge *K2*. This is true only if there are problems of convergence. If there is no problem of convergence for calculation *U3*, and that the maximum constraint in absolute value is reached for the last step of time, calculation should be continued.

## 8 Summary of the results

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The objective of the modelings presented in this document is to identify the parameters of the law ENDO\_ORTH\_BETON. Insofar as there does not exist empirical formula for the values of the parameters to be used, the user will have to gauge his parameters step by step on the various tests suggested. The method to gauge the parameters, which is in the document [R7.01.09], can be summarized as follows:

- 1) choice of  $ALPHA$  : (0.85 to 0.9),
- 2) calibration of  $K0$ ,  $ECROB$  on modelings A, C or E (calculation  $UI$ ). Once these gauged parameters, it should not be modified in the phase of calibration of the other parameters,
- 3) calibration of  $K1$ ,  $K2$  and  $ECROD$  on modelings B (or D) and E. In fact, modeling E (calculations  $U2$  and  $U3$ ) is enough. It makes it possible to check the value of the breaking stress in simple compression, and to ensure that the envelope of rupture for biaxial tests does not inflate. It is not necessary to gauge the parameter  $K2$  in a very fine way because it rises from a qualitative argument, and no experimental data is never available to identify it.