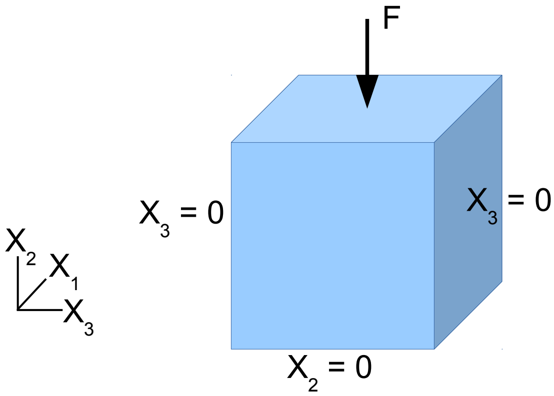
SSNV187 - Validation of the law ELAS_HYPER on a cube

Summary:

This test makes it possible to validate the behavior very-rubber band of the type Signorini (material ELAS_HYPER). One is based on an elementary test in plane deformations and in $3D$, compared to an analytical reference.

1 Problem of reference

1.1 Geometry



One considers a cube of with dimensions $1\mathrm{m}$ who rests on a plan ($x_2=0$ on the lower face), subjected to a pressure F on the higher face and in situation of deformation planes according to x_3 ($x_3=0$ on the faces right-hand side and left). The cube can thus only be stretched along the axis x_1).

1.2 Properties of materials

One tests on three different materials, corresponding to three standard models in very-elasticity.

Behavior `ELAS_HYPER`	Mooney-Rivlin	Néo-Hookéen	Signorini
`C10`	0,709	1.2345	0.1234
`C01`	2.3456	0	1.2345
`C20`	0	0	0,456
`NAKED`	0,499	0,499	0,499

1.3 Boundary conditions and loadings

- Lower face : $DY=0$
- Higher face : $F=0.876\ \mathrm{Pa}$
- Left and right face : $DZ=0$ in 3D, nothing in `D_PLAN`

The loading is increasing of $F=0$ with $F=0.876\mathrm{Pa}$, in 20 increments.

2 Reference solution

2.1 Method of calculating

One rests on the result of [bib1]. The state of plane deformations allows to very easily write the uniform field of displacement in the cube:

$$\begin{cases} u_1 = a_1 . x_1 \\ u_2 = w . x_2 \\ u_3 = 0 \end{cases} \quad (1)$$

with w the vertical displacement (negative) of the higher face and a_1 an arbitrary constant.
The condition of incompressibility makes it possible to write:

$$a_1 = \frac{-w}{1+w} \quad (2)$$

And one finds the relation between the force applied F and displacement w higher face:

$$F = 2S . \frac{w.(2+w).(1+(1+w)^2)}{(1+w)^3} . \left(\frac{\partial \Psi}{\partial J_1} + \frac{\partial \Psi}{\partial J_2} \right) \quad (3)$$

S is surface, Ψ is the potential of deformation and J_1, J_2 are the invariants of the tensor of Green-Lagrange. Potential of deformation used by ELAS_HYPER is the following:

$$\Psi = C_{10} . (J_1 - 3) + C_{01} . (J_2 - 3) + C_{20} . (J_1 - 3)^2 + \Psi_{vol} \quad (4)$$

Ψ_{vol} is the potential corresponding to the incompressibility. It depends on the invariants J_1 and J_2 and of C_{10}, C_{01} and C_{20} who are the characteristic materials. Like moreover $S=1$ one obtains:

$$F = 2 . \frac{w.(2+w).(1+(1+w)^2)}{(1+w)^3} . \left[C_{10} + C_{01} + 2 . C_{20} . \frac{w^2.(2+w)^2}{(1+w)^2} \right] \quad (5)$$

The solution of this nonlinear equation in w is done simply by dichotomy for $w<0$.

3 Bibliographical references

1 G.A. HOLZAPFEL: Nonlinear solid mechanics, 2001, Wiley.

4 Modeling A

4.1 Characteristics of modeling

It is a modeling in $2D$ with plane deformations D_PLAN, by using linear meshs.

4.2 Characteristics of the grid

Many linear elements: 207 including 132 triangles and 47 quadrangles (the rest being meshs of edge).
Many nodes: 132

4.3 Sizes tested and results

The first calculation (MOONEY-RIVLIN)

Value tested	Moment	Reference	Type	*Tolerance*
Displacement w	1.0	$-3{,}40091E\text{-}2$	Analytical	0.20%

The second calculation (NEO-HOOKEAN)

Value tested	Moment	Reference	Type	*Tolerance*
Displacement w	1.0	-0.078180	Analytical	0.20%

The third calculation (SIGNORINI)

Value tested	Moment	Reference	Type	*Tolerance*
Displacement w	1.0	-0.070936	Analytical	0.20%

5 Modeling B

5.1 Characteristics of modeling

It is a modeling in $2D$ with plane deformations D_PLAN, by using quadratic meshs.

5.2 Characteristics of the grid.

Many quadratic elements: 207 including 132 triangles and 47 quadrangles (the rest being meshs of edge).
Many nodes: 132

5.3 Sizes tested and results

The first calculation (MOONEY-RIVLIN)

Value tested	**Moment**	**Reference**	**Type**	***Tolerance***
Displacement w	1.0	$-3{,}40091\text{E-}2$	Analytical	0.20%

The second calculation (NEO-HOOKEAN)

Value tested	**Moment**	**Reference**	**Type**	***Tolerance***
Displacement w	1.0	-0.078180	Analytical	0.20%

The third calculation (SIGNORINI)

Value tested	**Moment**	**Reference**	**Type**	***Tolerance***
Displacement w	1.0	-0.070936	Analytical	0.20%

6 Modeling C

6.1 Characteristics of modeling

It is a modeling 3D.

6.2 Characteristics of the grid

Many elements: 8734 tetrahedrons and 1728 nodes.

6.3 Sizes tested and results

The first calculation (MOONEY-RIVLIN)

Value tested	Moment	Reference	Type	*Tolerance*
Displacement w	1.0	$-3{,}40091\text{E-}2$	Analytical	0.20%

The second calculation (NEO-HOOKEAN)

Value tested	Moment	Reference	Type	*Tolerance*
Displacement w	1.0	-0.078180	Analytical	0.20%

The third calculation (SIGNORINI)

Value tested	Moment	Reference	Type	*Tolerance*
Displacement w	1.0	-0.070936	Analytical	0.20%

7 Modeling D

7.1 Characteristics of modeling

It is a modeling 3D_SI (elements TETRA10 under-integrated).

7.2 Characteristics of the grid

Many elements: 271 tetrahedrons and 514 nodes.

7.3 Sizes tested and results

The first calculation (MOONEY-RIVLIN)

Value tested	Moment	Reference	Type	*Tolerance*
Displacement w	1.0	$-3,40091E-2$	Analytical	0.20%

The second calculation (NEO-HOOKEAN)

Value tested	Moment	Reference	Type	*Tolerance*
Displacement w	1.0	-0.078180	Analytical	0.20%

The third calculation (SIGNORINI)

Value tested	Moment	Reference	Type	*Tolerance*
Displacement w	1.0	-0.070936	Analytical	0.20%

8 Summary of the results

The got results are in concord with the reference solution