SSNV195 - Bar in multi-cracking with X-FEM

Summary

The purpose of this test is to in the case of test and validate the features of method X-FEM the structures $2D$ and $3D$ multi-fissured. Broadly it is a question of remaking essential tests SSNV173 [V6.04.173] and SSNV182 [V6.04.182] for a multi-fissured structure this time.

This test brings into play a parallelepipedic bar, whose median section is embedded, presenting two cracks the beam completely (one will speak then about interfaces), symmetrically placed coast and other of the embedded section. The bar is subjected to imposed displacements, which has as a consequence the total opening of the interfaces and the separation of the structure in three parts when she is requested in traction, or the appearance of contact pressure when the request is of compression.

Two types of interfaces are considered, horizontal (only one stage of elements is cut) and inclined (several stages of elements are cut, in order to test the good management of the under-cutting of the elements at the time of the presence of several cracks in the structure.

The case of a plate $2D$ presenting two interfaces is also studied.

1 Problem of reference

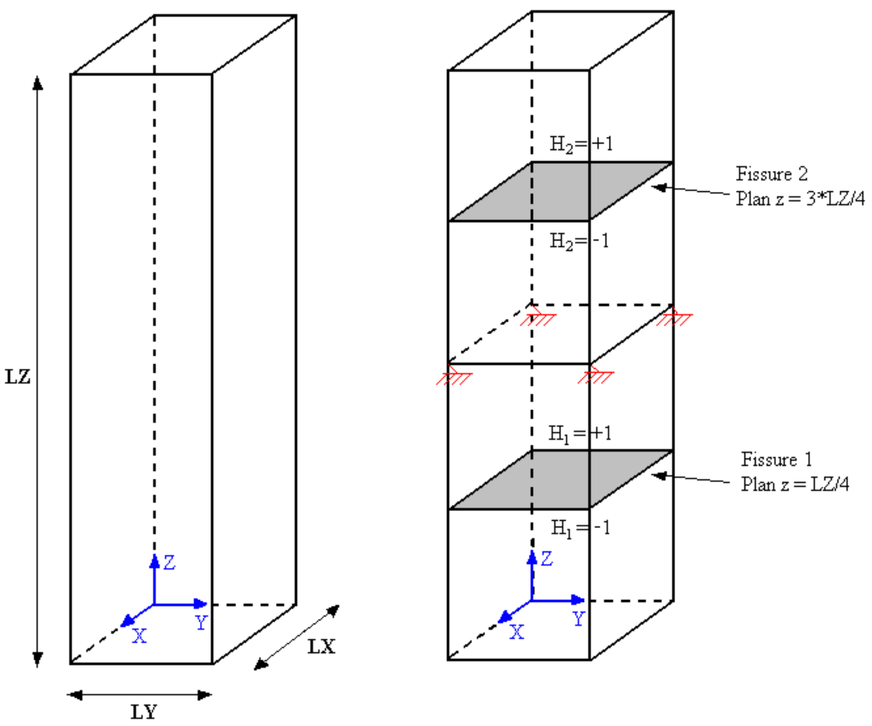
1.1 Geometry 3D

The structure is a right at square base and healthy parallelepiped. Dimensions of the bar (see [Figure 1.1-1]) are: $LX=5\,m$, $LY=5\,m$ and $LZ=50\,m$. It does not comprise any crack.

The interfaces will be introduced by functions of levels (level noted sets LN for level set normal) directly in the command file using the operator DEFI_FISS_XFEM [U4.82.08]. Initially, the interfaces horizontal, and will be introduced each one in the middle of the blocks inferior and superior compared to the section embedded (see [Figure 1.1-2]). The equations of the functions of levels for the two horizontal interfaces are thus the following ones:

$$\mathrm{LN1}=Z-LZ/4 \quad \textbf{éq 1.1-1}$$
$$\mathrm{LN2}=Z-3\cdot LZ/4 \quad \textbf{éq 1.1-2}$$

No level set tangential is necessary since the keyword was used TYPE_DISCONTINUITE=' INTERFACE', which makes it possible to have a structure completely cut in three parts.



1) 2)

Figure 1.1. Geometry of the bar and positioning of the crack

For modeling C the two interfaces will be tilted undergoing rotations along the axis OX (see [Figure 4.1-1]). With this occasion one tests also the good performance of the operators of post treatment in X-FEM which underwent modifications at the time of the passage to multi-cracking.

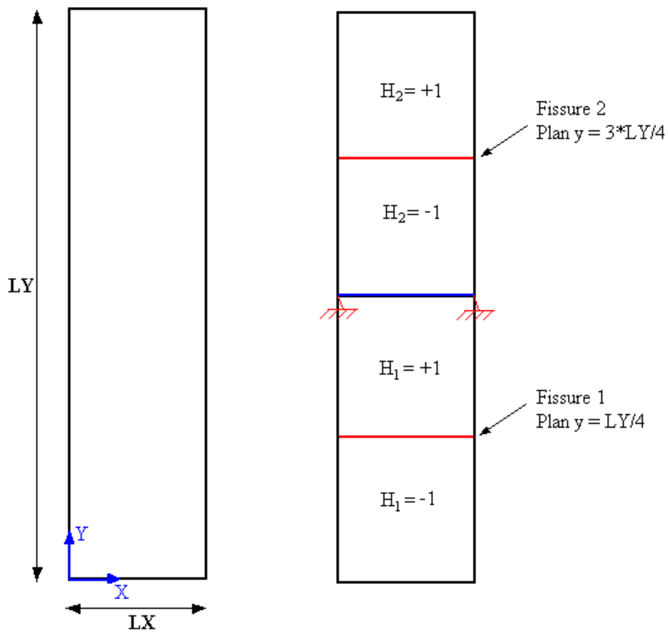
1.2 Geometry 2D

The structure is a healthy rectangle. Dimensions of the bar (see [Figure 1.2-1]) are: $LX=5\,m$ and $LY=50\,m$. It does not comprise any crack.

The interfaces will be introduced by functions of levels (level sets) directly into the file orders using the operator `DEFI_FISS_XFEM` [U4.82.08]. By analogy with the case 3D , the interfaces are present at the mediums of the two parts of the plate, separated by the line from embedding (see Figure 1.2-a). The corresponding equations of the functions of levels are:

$$\mathrm{LN1}=\mathrm{Y}-LY/4 \quad \textbf{éq 1.2-1}$$
$$\mathrm{LN2}=Y-3\cdot LY/4 \quad \textbf{éq 1.2-3}$$

No level set tangential is necessary since the keyword was used `TYPE_DISCONTINUITE='INTERFACE'`, which makes it possible to have a structure completely cut in three parts.



1) 2)

Figure 1.2. Geometry of the plate and positioning of the crack

1.3 Properties of material

Young modulus: $E=100\,MPa$

Poisson's ratio: $\nu=0.0$

1.4 Boundary conditions and loadings

The nodes of the median surface of the bar are embedded (see them [Figure 1.1-2] and [Figure 1.2-2]) while displacements are imposed on those of surfaces lower and higher. One wishes to initially show the possibility of separating a structure in 3 parts following the introduction of two interfaces (imposed displacements will open the interfaces) and in the second time one will prove the taking into account of the contact on the lips of two interfaces (imposed displacements will close the interfaces).

2 Modeling a: opening horizontal Interfaces

2.1 Characteristics of the grid

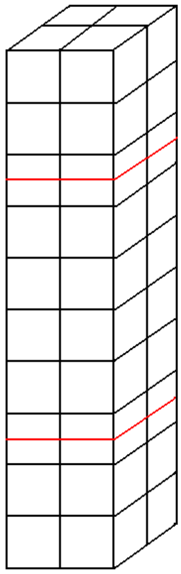


Figure 2.1-1: Grid of modeling A

One discretizes the structure using the finite elements HEXA8. According to the three directions of the frame of reference chosen, one has $2\times2\times10$ elements thus a total of 40 finite elements (see [Figure 2.1-1])

The number of stages of elements according to the direction Z was selected to avoid the enrichment of the same element by the two interfaces. Indeed, in the implementation of multi-cracking (see manual) one avoids approaching two cracks with less than 4 healthy elements in order to avoiding the conflicts during the management of the degrees of freedom nouveau riches.
Thus, by choosing 10 stages, one will be able at the same time to impose boundary conditions on the extreme meshs in the usual way and to prevent that enrichments characteristic of the X-FEM are touched. The first interface will be introduced in the middle of the third stage while the second in the middle of the eighth.

2.2 Boundary conditions

In order to open the two interfaces and to prevent the movements of rigid body, one embeds the nodes belonging to median surface:

```
GROUP_NO=SURFMED: DX = 0, DY = 0 and DZ = 0
```

For the nodes belonging to extreme surfaces (lower and higher) one imposes displacements as follows:

```
GROUP_NO=SURFINF: DX = 0, DY = 0 and DZ = - DEPZ
GROUP_NO=SURFSUP: DX = 0, DY = 0 and DZ = DEPZ
```

The value of imposed displacement is $DEPZ=3.10^{-3}m$. One will be able in makes move the two extreme blocks of the bar following the three directions by simple assignment of a nonworthless value for the degrees of freedom corresponding to DX or DY .

2.3 Analytical resolution

The solution of such a problem is of course obvious. As explained for the case test SSNV173 [V6.04.173], the solution is the following one: all displacements according to x and y are worthless, all displacements according to z below the level-set lower are equal to imposed displacement u_z at the base of the structure, all displacements according to z between the two level-set are worthless and all displacements according to z with the top of the level set higher are equal to imposed displacement u_z at the top of the structure.

2.4 Sizes tested and results

One tests the values of displacement after the convergence of the iterations of the operator `STAT_NON_LINE`. It is checked that one finds well the values determined with [the §2.3].

Identification	Reference	Tolerance
`DX` for all the nodes just below the lower interface	0.00	1.0E-15
`DX` for all the nodes just with the top of the lower interface	0.00	1.0E-15
`DY` for all the nodes just below the lower interface	0.00	1.0E-15
`DY` for all the nodes just with the top of the lower interface	0.00	1.0E-15
`DZ` for all the nodes just below the lower interface	-3.E-3	1.0E-09%
`DZ` for all the nodes just with the top of the lower interface	0.00	1.0E-15
`DX` for all the nodes just below the higher interface	0.00	1.0E-15
`DX` for all the nodes just with the top of the higher interface	0.00	1.0E-15
`DY` for all the nodes just below the higher interface	0.00	1.0E-15
`DY` for all the nodes just with the top of the higher interface	0.00	1.0E-15
`DZ` for all the nodes just below the higher interface	0.00	1.0E-15
`DZ` for all the nodes just with the top of the higher interface	3.E-3	1.0E-09%

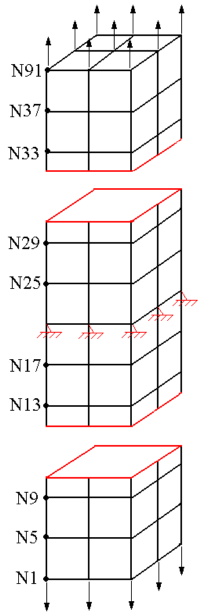


Figure 2.4-1: Final grid and positioning of the nodes tested

To test all the nodes in only once, one tests the minimum and the maximum of column.

2.5 Remarks

It is noticed that the values of the degrees of freedom tested are those expected, the interfaces open thus separating the bar in three parts. The result is displayed using the operators of postprocessing.

3 Modeling b: horizontal Interfaces in closing

3.1 Characteristics of the grid and the interface

The grid is the same one as that of modeling A. the objective being to test the management of the contact on the interfaces represented with X-FEM, only the boundary conditions changed.

3.2 Boundary conditions

The boundary conditions will allow this time to test the taking into account of the contact without friction on the lips of the interfaces. One thus keeps the embedding of the nodes belonging to median surface but the displacements imposed on the nodes of the two extremes (lower and higher) will close the interfaces. One will thus have:

```
GROUP_NO=SURFMED: DX = 0, DY = 0 and DZ = 0
GROUP_NO=SURFINF: DX = 0, DY = 0 and DZ = DEPZ
GROUP_NO=SURFSUP: DX = 0, DY = 0 and DZ = - DEPZ
```

3.3 Analytical resolution

The interfaces being horizontal and the state of uniaxial pressing and normal to the interface, there is no possible slip. One checks the value of the contact pressure and the solution of the problem is that of the same problem without interface, for each block of a side and other of embedded surface. As shown for the case test SSNV182 [V6.04.182] and taking into account the geometry and of the boundary conditions described with the §2.2, the contact pressure is given by:

$$\lambda = E\frac{DEPZ}{LZ/2}$$ **éq 4.3-1**

With the digital values previously introduced, $\lambda = -12.10^3 \text{ Pa}$.

3.4 Sizes tested and results

The good progress of calculation allows *a priori* to validate the good management of the contact for the multi cracking. Components of `SD FISS_XFEM` concerning the contact for each crack are managed correctly on the level as of operators `MODI_MODELE_XFEM` and `STAT_NON_LINE`. One tests the values of the contact pressure for the nodes medium pertaining to the elements cut by the cracks.

Identification	Reference
`LAGS_C` for all the nodes of the two cracks	-1.2E4
`LAGS_F1` for all the nodes of the two cracks	0.00
`LAGS_F2` for all the nodes of the two cracks	0.00

4 Modeling C – tilted Interfaces

The purpose of this modeling is to prove the taking into account of the contact rubbing for tilted interfaces as well as the good performance of the algorithms of cutting and postprocessing.
One chose an angle of inclination $\theta=30°$ according to rotation around the axis OX, for the two interfaces. The normal with the interfaces thus create is noted $\mathbf{n}$ and the tangent vector is noted $\boldsymbol{\tau}$:

$$\mathbf{n}=\begin{pmatrix}0\\-1/2\\\sqrt{3}/2\end{pmatrix},\quad \boldsymbol{\tau}=\begin{pmatrix}0\\\sqrt{3}/2\\1/2\end{pmatrix}$$ **éq 6-1**

The new functions of level introducing the two cracks into the command file are:

$$\mathrm{LN1}=\mathrm{Z}-\tan\theta\cdot Y-(LZ/4-\tan\theta\cdot LX/2)$$ **éq 6-2**

$$\mathrm{LN2}=\mathrm{Z}-\tan\theta\cdot Y-(3\cdot LZ/4-\tan\theta\cdot LX/2)$$ **éq 6-3**

4.1 Characteristics of the grid

Compared to modelings A and B, the number of elements according to the direction Z (height of the bar) was multiplied by 2 in order to avoid the adjacency of the elements nouveau riches by the two tilted interfaces. There is thus a grid $2\times2\times20$ elements `HEXA8` (see [Figure 4.1-1]).

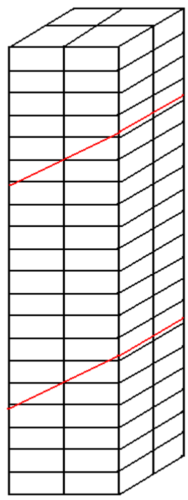


Figure 4.1-1: Grid for modeling C

4.2 Boundary conditions

The boundary conditions make it possible to carry out a mixed request: compression on the level of the first interface and traction for the second.
Thus, besides the embedding of the nodes belonging to median surface (as for the first two modelings), one imposes positive displacements according to the direction Z for the nodes belonging to extreme surfaces (lower and higher). One will thus have:

```
GROUP_NO=SURFMED: DX = 0, DY = 0 and DZ = 0
GROUP_NO=SURFINF: DX = 0, DY = 0 and DZ = DEPZ
GROUP_NO=SURFSUP: DX = 0, DY = 0 and DZ = DEPZ
```

4.3 Analytical resolution

The analytical solution concerns, for the lower block (interface 1), the values of the contact pressure and the semi-multipliers of friction. For the higher block (interface 2), the analytical solution relates to the values of displacements of the nodes. The solution for this last is identical to that of modeling A and thus one will detail only the analytical solution for the lower block.

The interfaces being inclined, one will be able to have slip. To avoid that, one forces adherence by choosing a coefficient of friction of sufficiently high Coulomb. Theoretically, it is enough to take:

$$\mu > \tan(\theta) \qquad \text{éq 6.3-1}$$

Thus, the solution of the problem remains identical to that of the same problem without interface. The analytical resolution for this problem is presented in [V604182 - §4.1]. One finds for the contact pressure:

$$\lambda = n_z \cdot E \frac{DEPZ}{LZ/2} \cdot n_z \qquad \text{éq 6.3-2}$$

and for the semi-multipliers of friction:

$$\Lambda = \left(\frac{1}{\mu} \frac{\tau_z}{n_z} \right) \tau \qquad \text{éq 6.3-3}$$

With the digital values previously introduced and considering $\mu = 1$, one obtains for the contact pressure $\lambda = -9.10^3 Pa$ and for the semi-multiplier of friction following the direction τ, $\Lambda \cdot \tau = 1/\sqrt{3}$.

4.4 Sizes tested and results

One tests the values of the contact pressure and the semi-multipliers of friction for interface 1. For interface 2 one tests the values of displacements to the top and below the level-set.

Identification	Reference
DX for all the nodes just below the lower interface	0.00
DX for all the nodes just with the top of the lower interface	0.00
DY for all the nodes just below the lower interface	0.00
DY for all the nodes just with the top of the lower interface	0.00
LAGS_C for all the nodes lower interface	-9E3
LAGS_F1 for all the nodes lower interface	0
LAGS_F2 for all the nodes lower interface	$1/\sqrt{3}$
DX for all the nodes just below the higher interface	0.00
DX for all the nodes just with the top of the higher interface	0.00
DY for all the nodes just below the higher interface	0.00
DY for all the nodes just with the top of the higher interface	0.00
DZ for all the nodes just below the higher interface	0.00
DZ for all the nodes just with the top of the higher interface	3.0E-3

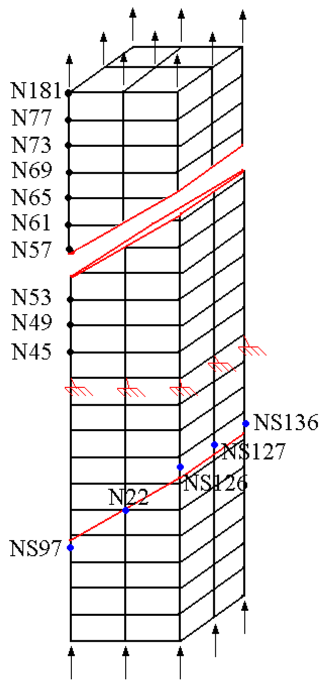


Figure 4.4-1: Final grid and positioning of the nodes tested

To test all the nodes in only once, one tests the minimum and the maximum of column.

4.5 Remarks

It is checked that the digital values obtained for the degrees of freedom tested, following the analysis with several interfaces X-FEM, are quite identical with the values of the analytical solution for the two interfaces.

5 Modeling D – Interfaces multiple in 2D

The purpose of this modeling is to test the operation of the X-FEM multi-cracking for structures $2D$.

5.1 Characteristics of the grid

One chooses to model the structure $2D$ by a grid made up of 2×10 elements QUAD4, as shown on [Figure 5.4-1].

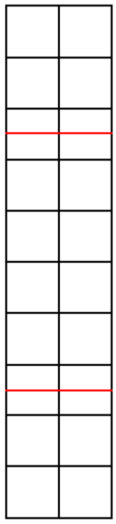


Figure 5.1-1: Grid for modeling D

5.2 Boundary conditions

The boundary conditions are similar to those imposed for modeling A, obviously adapted for the case $2D$. One embeds the nodes belonging the line of centers and one imposes displacements on the nodes belonging to the extreme lines (lower and higher). One has thus:

```
GROUP_NO=LIGMED: DX = 0 and DY = 0
GROUP_NO=LIGINF: DX = 0 and DY = - DEPY
GROUP_NO=LIGSUP: DX = 0 and  DY = DEPY
```

One considers $DEPY=2.E-3$.

5.3 Analytical resolution

The solution of such a problem is of course still obvious: all displacements according to x are worthless, all displacements according to y below the level-set lower are equal to imposed displacement u_y at the base of the structure, all displacements according to y between the two level set are worthless and all displacements according to y with the top of the level-set higher are equal to imposed displacement u_y at the top of the structure.

5.4 Sizes tested and results

One tests the values of displacement after convergence of the iterations of the operator STAT_NON_LINE. One does not test with each time only one node of each stage and one obtains the following table or the position of each node tested is indicated on [Figure 5.4-1].

Identification	Reference
DX for all the nodes just below the lower interface	0.00
DX for all the nodes just with the top of the lower interface	0.00
DY for all the nodes just below the lower interface	-2.00E-3
DY for all the nodes just with the top of the lower interface	0.00
DX for all the nodes just below the interface higher	0.00
DX for all the nodes just with the top of the interface higher	0.00
DY for all the nodes just below the interface higher	0.00
DY for all the nodes just with the top of the interface higher	2.00E-3

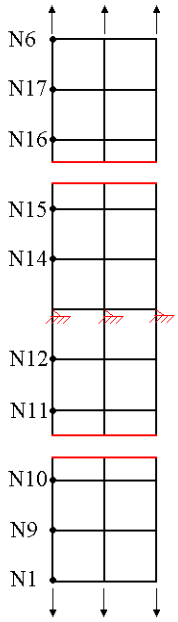


Figure 5.4-1: Final grid and positioning of the nodes tested

6 Summaries of the results

The goals of this test are achieved. It was a question of validating the operation of the evolution of the X-FEM towards multi-cracking. One could note the good taking into account of two interfaces in several situations: models $3D$ and $2D$, horizontal interfaces, inclined interfaces, contact without or with friction, etc.

All the operators who underwent modifications for the passage to multi-cracking function correctly.